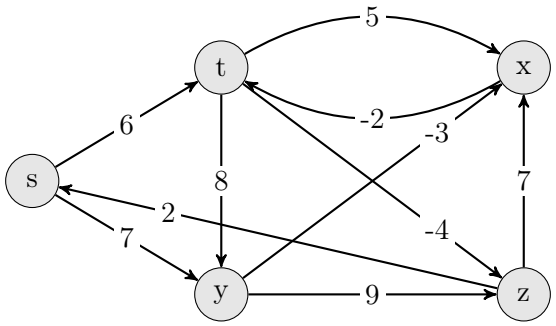
Exercise X, Algorithms 2024-2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. There are many problems on this set, solve as many as you can and ask for help if you get stuck for too long. Problems marked * are more difficult but also more fun :).

1 Consider the following edge-weighted directed graph:

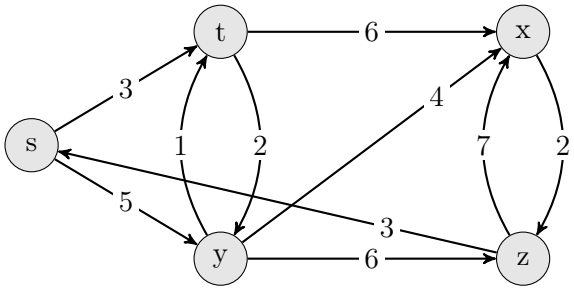


Run the Bellman-Ford algorithm using vertex z as a source. In each pass, relax edges in the order $(t,x),(t,y),(t,z),(x,t),(y,x),(y,z),(z,x),(z,s),(s,t),(s,y)$, and show the d and π values after each pass. Now, change the weight of edge (z,x) to 4 and run the algorithm again, using s as the source.

2 (Exercise 24.1-3) Given a weighted, directed graph $G=(V,E)$ with no negative-weight cycles, let m be the maximum over all vertices $v \in V$ of the minimum number of edges in a shortest path from the source s to v. (Here, the shortest path is by weight, not the number of edges.) Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m+1$ passes, even if m is not known in advance.

3 (*, Exercise 24.1-6) Suppose that a weighted directed graph $G=(V,E)$ has a negative-weight cycle. Give an efficient algorithm to list the vertices of one such cycle. Argue that your algorithm is correct.

4 (Exercise 24.3-1) Run Dijkstra's algorithm on the following edge-weighted directed graph. Use first vertex s as a source and then vertex z as a source.



5 (half a *) Suppose you are standing at the top station of Mount-Everest and you wish to ski down. There are several different stations where you can rest (vertices) and different routes between stations (directed edges). All routes go down hill so they are directed from the station of higher altitude to the station of lower altitude. Although there are different routes from the top station to other stations, if one continues to ski down one will sooner or later hit the base camp. As the view is very nice, we wish to make the ski route as long as possible. In other words, design and analyze an efficient algorithm that calculates the *longest* route to ski down starting at the top station of Mount-Everest and ending in the base camp.