Dynamical Systems for Engineers: Exercise Set 8

Exercise 1

This continues the last exercise of the exercise set 7.

We consider now a pendulum in friction. In addition to the gravity force F as in the previous exercise, there is a friction force proportional to the angular velocity of m but in the direction opposite to the velocity, i.e., $F_k = -km\dot{\varphi}$, with $k > 0$.

1. Give the state equations of the pendulum system with friction.

2. Compute the equilibrium point(s) of the system.

3. Characterize the stability of the equilibrium point(s).

4. Using a Lyapunov function, try to obtain some information on the basin of attraction of the asymptotically stable equilibrium point. Hint: remember that the total energy of the system (kinetic and potential energy) can be a good candidate for a Lyapunov function.

Exercise 2

Consider the autonomous nonlinear system

$$\begin{aligned}\dot{x_1} &= -x_1 + 2x_2^3 - 2x_2^4 \\ \dot{x_2} &= -x_1 - x_2 + x_1 x_2.\end{aligned}$$

1. Is the origin an asymptotically stable equilibrium?

2. Is the origin a globally asymptotically stable equilibrium?

Exercise 3

Let us consider the gradient system

$$\dot{x} = -\nabla_x V(x),$$

where $x = (x_1, x_2)$.

1. If $V(x_1, x_2) = x_1^2 x_2^2$, find all the equilibrium point(s) of the system. Verify that all solutions converge towards an equilibrium point. Is the line $x_1 = x_2$ an invariant set?

2. If $V(x_1, x_2) = x_1^4/4 + x_2^4/4 - x_1 x_2$, , find all the equilibrium point(s) of the system. Verify that all solutions converge towards an equilibrium point. Is the line $x_1 = x_2$ an invariant set?