Dynamical Systems
for Engineers: Exercise Set 6

Exercise 1

Determine the stability of the equilibrium point(s) of the following one dimensional continuous-time systems.

$$\dot{x} = -x^3 \quad (1)$$
$$\dot{x} = x^2 - 1 \quad (2)$$
$$\dot{x} = x^2 + 1 \quad (3)$$
$$\dot{x} = x^2. \quad (4)$$

Exercise 2

Consider the autonomous nonlinear system

$$\begin{aligned} \dot{x_1} &= -x_2 + \alpha x_1(x_1^2 + x_2^2 - 1) \\ \dot{x_2} &= x_1 + \alpha x_2(x_1^2 + x_2^2 - 1). \end{aligned}$$

1. For which value(s) of α can you guarantee that the origin is asymptotically stable?
2. Is there a value of α for which the origin is stable, but is not asymptotically stable?

Exercise 3

Consider the discrete-time nonlinear system $x(t+1) = F(x(t))$ where the map F is given by $F(x) = x^3 - x/4$. Find all the fixed points of this system and characterize their stability.

Exercise 4

The system we consider here is an oscillator that is used in front ends of some mobile phones. A model for this oscillator is given by the following three differential equations

$$\begin{aligned} \dot{x_1} &= 20\left(-n(x_2) + x_3\right) \\ \dot{x_2} &= 20x_3 \\ \dot{x_3} &= \frac{1}{40}\left(x_1 + x_2\right) - x_3 \end{aligned}$$

where $n(x_2) = e^{-x_2} - 1$.

1. Compute the equilibrium point(s) of the system.
2. Characterize the stability of the equilibrium point(s).