Dynamical Systems for Engineers: Exercise Set 4

Exercise 1

Consider the two-dimensional discrete-time linear system described by

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

where

$$A = \begin{bmatrix} 1/4 & 0 \\ 2 & 1/2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1/4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

1. Characterize the stability of this system.

2. Find the free (or zero-input) solution $x(t)$ of the system subject to the initial conditions $(x_1(0), x_2(0)) = (1, 1)$ (and therefore $u(t) \equiv 0$ for all $t \in \mathbb{N}$).

3. Find the free (or zero-input) response $y(t)$ of the system subject to the initial conditions $(x_1(0), x_2(0)) = (1, 1)$.

4. Find the forced (or zero-state) solution $x(t)$ of the system subject to the input (which is the impulse function in discrete time)

$$u(t) = \begin{cases} 1 & \text{if} \quad t = 0 \\ 0 & \text{if} \quad t \neq 0 \end{cases} \tag{1}$$

(and therefore with the initial conditions $(x_1(0), x_2(0)) = (0, 0)$.

5. Find the forced (or zero-state) response $y(t)$ of the system subject to the input (1).

6. Find the complete response $y(t)$ of the system subject to the input (1) with the initial conditions $(x_1(0), x_2(0)) = (1, 1)$.

Exercise 2

Consider the autonomous three-dimensional continuous-time linear system $\dot{x} = Ax$ where

$$A = \begin{bmatrix} 2 & 4 & 0 \\ -4 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

1. Characterize the stability of this system.

2. Sketch the phase portrait of this system, discuss the nature of the equilibrium.

Exercise 3

Consider the LC series circuit of Figure 1. The input voltage $u(t)$ is applied as shown in the figure, and the output $y(t)$ is the voltage across the capacitor. Is this system B.I.B.O. stable?

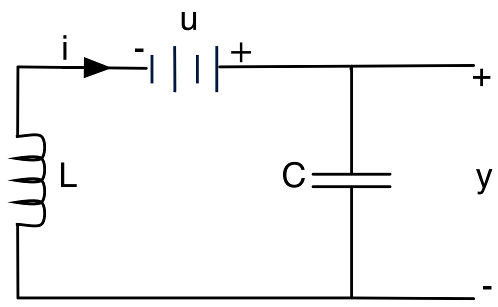

Figure 1: LC circuit.

Exercise 4

Suppose that the n-dimensional system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

is both observable and controllable. Let P by a $n \times n$ invertible matrix, and let $A' = PAP^{-1}$, $B' = PB$, $C' = CP^{-1}$ and $D' = D$. Is the n-dimensional system

$$\begin{aligned} \dot{x} &= A'x + B'u \\ y &= C'x + D'u \end{aligned}$$

always observable and/or controllable?

Exercise 5

Determine the condition on b_1, b_2, c_1, c_2 such that the two-dimensional discrete-time linear system described by

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

where

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

is completely observable and controllable.

Exercise 6

Consider a linear system in the usual ABCD representation, with

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

1. Is this system always B.I.B.O. stable?

2. Is this system observable?

3. Is this system controllable?

Exercise 7

Consider a three-dimensional continuous-time linear system whose input-output transfer function is

$$H(s) = \frac{s + \alpha}{s^3 + 7s^2 + 14s + 8}$$

where $\alpha \in \mathbb{R}$.

1. Is this system always B.I.B.O. stable, or are there values of α so that the system is B.I.B.O unstable?

2. Determine the value(s) of α so that the system is unobservable and/or uncontrollable.