Dynamical Systems
for Engineers: Exercise Set 11

Exercise 1

Show that the first-order continuous-time system

$$\dot{x} = r - x - e^{-x}$$

undergoes a bifurcation as r is varied. What is the type of the bifurcation? Hint: you cannot compute explicitely the equilibirum points as a function of r, but can discuss the existence and stability of the fixed point, as a function of r, by plotting the curves $r - x$ and $\exp(-x)$ as a function of x. What is the value of r at the bifurcation point?

Exercise 2

Does the second-order autonomous nonlinear system

$$
\begin{aligned}
\dot{x_1} &= \mu x_1 - x_2 + x_1 x_2^2 \\
\dot{x_2} &= x_1 + \mu x_2 + x_2^3
\end{aligned}
$$

undergo an Andronov-Hopf bifurcation at the origin when μ is varied ? If so, is the bifurcation sub-critical or super-critical? Hint: use polar coordinates.