EPFL

Lomb-Scargle Periodogram

COM500 Mini-project presentation

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Introduction

- In statistical signal processing, the goal of frequency estimation is the **process of estimating complex frequency components from a noisy signal**

- **Classical periodogram** is a method based on non-parametric model
  - only assume that signal is WSS
  - dependant on the number of samples used

- **Lomb-Scargle periodogram** → advanced method of classical periodogram
  - known for working well with **unevenly sampled** time-series
  - Efficient computation even with unpleasant data

- Analysis of this mini-project made with self-made data and real ECG signal

Data

- Three signals were generated by ourselves

1. Evenly standard sampled data

$$X_{\text{even}}[n] = \sum_{n=1}^{4} \sin(2\pi f n t)$$

$$Y_{\text{even}}[n] = X_{\text{even}}[n] + W[n]$$

2. Unevenly sampled data

$$Y_{\text{uneven}}[n] = \text{random } 50\% \text{ of } Y_{\text{even}}[n]$$

3. Interpolated data from the unevenly sampled
   a. Cubic interpolation of the Y_uneven

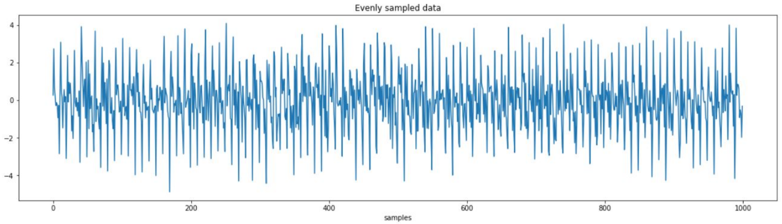


Fig. 1: Representation of the evenly sampled data created $Y_{\text{even}}[n]$

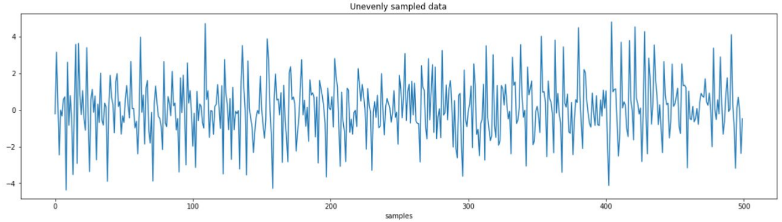


Fig. 2: Representation of the unevenly sampled data created $Y_{\text{uneven}}[n]$

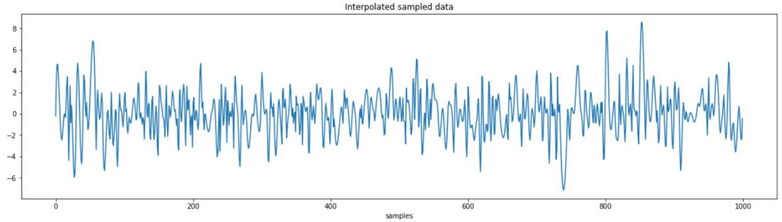


Fig. 3: Representation of the interpolated data created $Y_{\text{interp}}[n]$

Classical Periodogram

- The periodogram is a **non-parametric estimate of the Power Spectral Density** of a WSS random process

$$P_X^N(\omega) = \frac{1}{N}\left|\sum_{n=1}^{N} x[n]e^{-j\omega n}\right|^2 = \frac{1}{N}\left|\hat{x}_N(\omega)\right|^2$$

- **also the Discrete Time Fourier Transform** of the observed samples

- PSD is defined as the Fourier transform of the auto-correlation $R_X[k]$
- The periodogram is the DTFT of the correlation estimator $\hat{R}_X[k]$
  - both are linked when looking at the correlation

- The periodogram **behaves like the Fourier transform of the correlation** convoluted with the square of the “pseudo” sinc function

Lomb-Scargle Periodogram

- Lomb-Scargle is a developed classical periodogram with a more general use and the possibility to work with time-shift

$$P_X^N(\omega) = \frac{1}{N}\left|\sum_{n=1}^{N} x[n]e^{-j\omega n}\right|^2$$
$$= \frac{1}{N}\left[\left(\sum_n x[n]\cos(\omega n)\right)^2 + \left(\sum_n x[n]\sin(\omega n)\right)^2\right]$$

- Full general form $P_X^N(\omega) = \frac{A^2}{2}\left(\sum_n x[n]\cos(\omega[n-\tau])\right)^2 + \frac{B^2}{2}\left(\sum_n x[n]\sin(\omega[n-\tau])\right)^2$

$$= \frac{1}{2}\left[\frac{\left(\sum_n x[n]\cos(\omega[n-\tau])\right)^2}{\sum_n \cos^2(\omega[n-\tau])} + \frac{\left(\sum_n x[n]\sin(\omega[n-\tau])\right)^2}{\sum_n \sin^2(\omega[n-\tau])}\right] \qquad \tau = \frac{1}{2\omega}\tan^{-1}\left(\frac{\sum_n \sin(2\omega n)}{\sum_n \cos(2\omega n)}\right)$$

- Its results are nearly identical to the **fitting of a model of simple sinusoids** to unevenly sampled data at each frequency
- The τ function value serves now as a **shift to orthogonalize** the equation used for fitting

Results & Discussion

- Low-pass effect not as present with the LS periodogram
- Freq. peaks of LS periodogram are at the same location as the ground truth compared to classical periodogram
- Better results with LS periodo. when half the samples
- More during the demo !

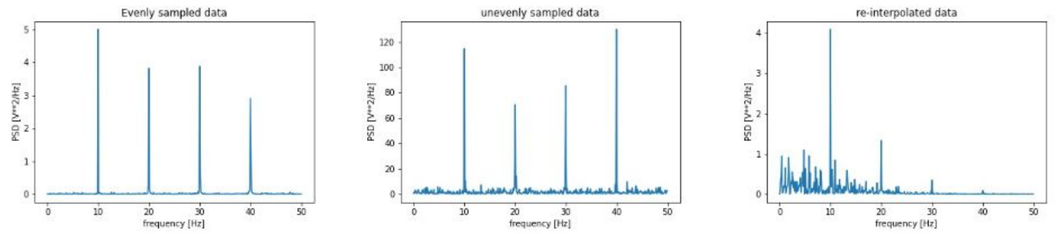


Fig. 6: Periodogram applied to evenly and unevenly sampled harmonic signal, and to the re-interpolated signal

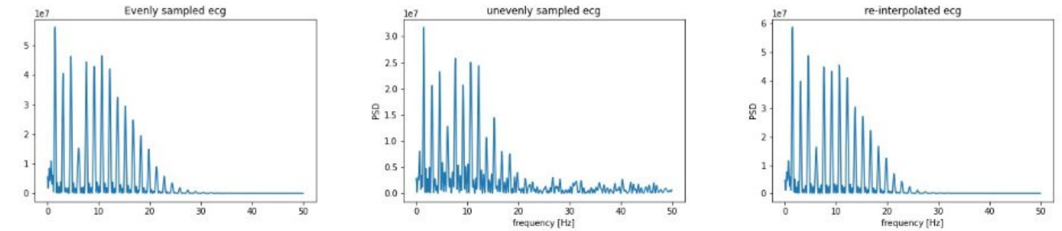


Fig. 7: Periodogram applied to 5 seconds of evenly and unevenly sampled ECG, and to the re-interpolated signal

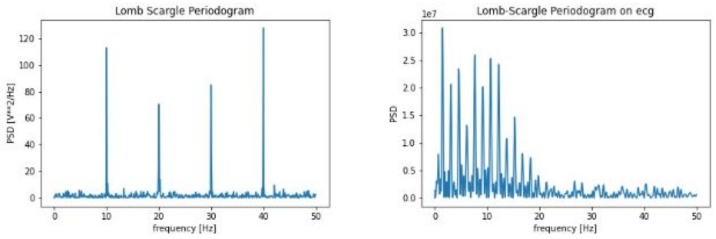
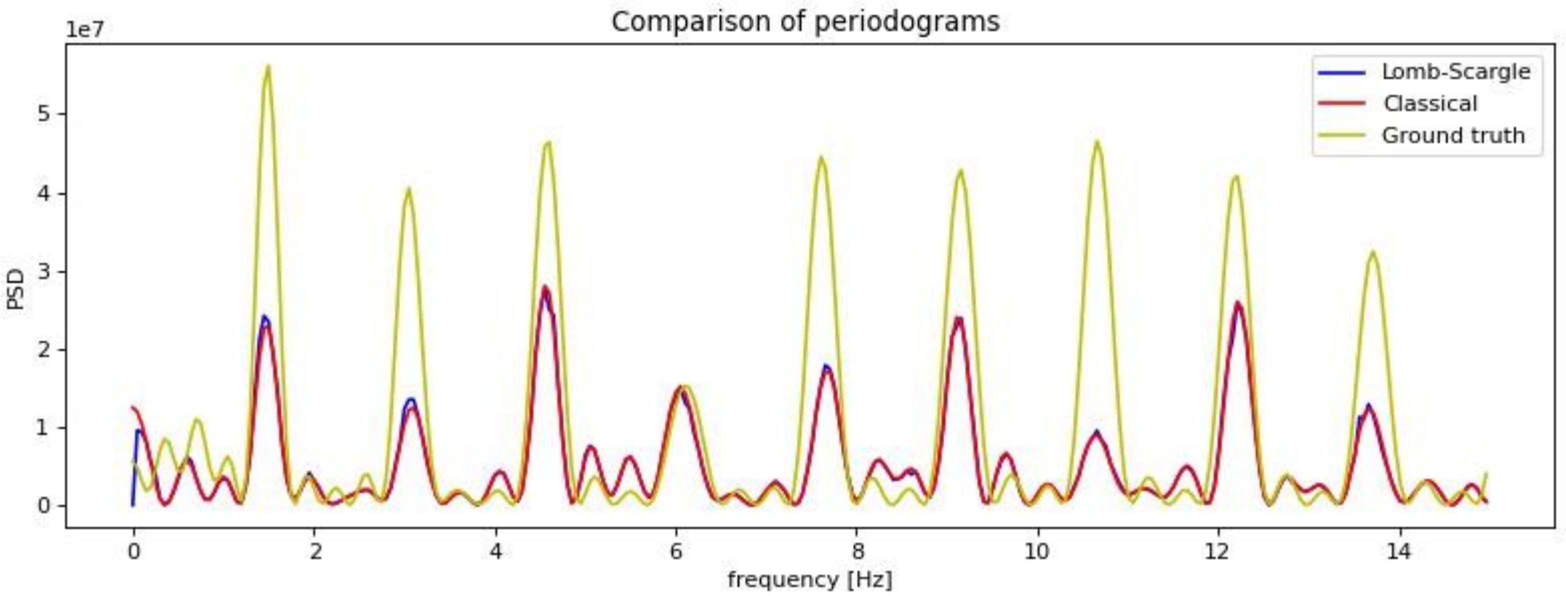


Fig. 8: Lomb-Scargle periodogram applied to the unevenly harmonic signal and ECG

Spectrum of Unevenly sampled ECG



Questions ?

Thank you for your attention !