Statistical Signal Processing

Midterm Exam

Thursday, 13 April 2017

You will hand in this sheet together with your solutions.

Write your personal data (please make it readable!).

Seat Number:

Family Name:

Name:

Read Me First!

Only the personal handwritten cheat sheet is allowed.
No class notes, no exercise text or exercise solutions.

Write solutions on separate sheets,
***i.e.* no more than one solution per paper sheet.**

Return your sheets ordered according to problem (solution) numbering.

Return the text of the exam.

Warmup exercises

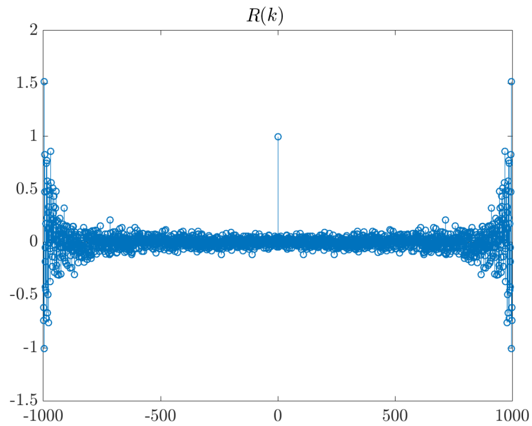
This is a warm up problem .. do not spend too much time on it.

Please provide justified, rigorous, and simple answers.

Exercise 1. Correlation (4pts)
Let $W[n]$ be a centered white noise with $\sigma^2 = 1$, taking real values. Given that the process is i.i.d. and centered, we know that, theoretically,

$$R(k) = \mathrm{E}\left[W[k+n]W[n]\right] = \begin{cases} \sigma^2 = 1 & k = 0\,; \\ 0 & k \neq 0\,; \end{cases}$$

We have measured $N = 1000$ samples of the noise $w[1], \ldots, w[1000]$ and then we have computed the correlation $R(k)$, $k = -999, \ldots, 0, \ldots, 999$. Here's the plot of the correlation.



Can you tell if the plotted correlation has been computed using:
The empirical un-biased correlation

$$R(k) = \frac{1}{N-|k|} \sum_{n=1}^{N-|k|} w[n+k]w[n]\,, \quad k = -(N-1), \ldots, (N-1)\,,$$

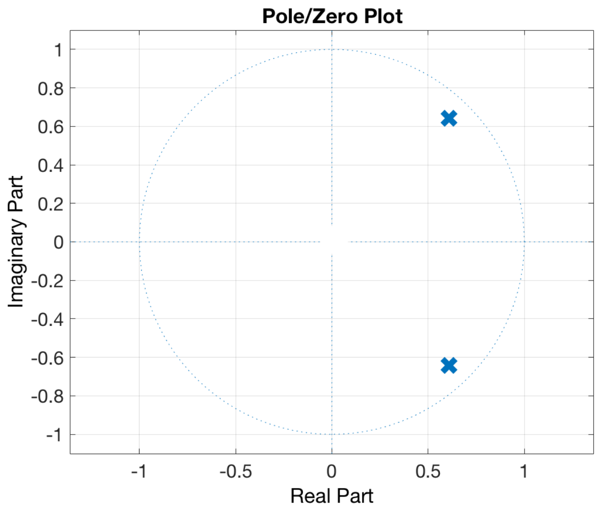
or the empirical biased correlation

$$R(k) = \frac{1}{N} \sum_{n=1}^{N-|k|} w[n+k]w[n]\,, \quad k = -(N-1), \ldots, (N-1)\,.$$

Precisely justify your answer.

Exercise 2. A SIMPLE SYSTEM (4 PTS)

The figure below depicts the poles of a causal LTI system $P(z)$. the two poles have magnitude $a = 0.9$ and phase $\varphi = \pm\pi/4$.



1) Is the system $P(z)$ stable?

2) Sketch the magnitude of the transfer function $|P(e^{jw})|$.

3) Is the inverse system $H(z) = 1/P(z)$ stable?

4) Give the impulse response $h(n)$ of $H(z)$.

Main exercises

Here comes the core part of the exam .. take time to read the introduction and each problem statement.

Please provide justified, rigorous, and simple answers.

Exercise 3. Activity classification for kids (25pts)

In order to motivate kids to do physical activities several devices, like the ActiSmile, have been invented

It basically consists in an accelerometer with an embedded software analyzing the activity. The more the kid does physical activities, the more the device smiles!

Consider now a kid that is jumping on a trampoline

We consider here a simplified model of the jump, where the measured accelerometer signal is modeled as a harmonic **real** signal $X[n]$, composed of

- A main sinusoidal signal, which frequency f_1 corresponds to the main jumping frequency (typical 35-50 strides, *e.g.* periods, per minute)
- Two secondary sinusoidal signals, mostly due to the form of the jump mouvement. These components have frequencies f_2 and f_3 that typically differ from the main jump frequency

f_1 by $\pm\Delta f_2$, with $\Delta f_2 \in [1.2, 1.6]Hz$ and $\pm\Delta f_3$, with $\Delta f_3 \in [2.4, 3.2]Hz$, respectively. That is $f_2 = f_1 \pm \Delta f_2$ and $f_3 = f_1 \pm \Delta f_3$

The accelerometer signal is sampled at $f_s = 50Hz$.

1) Assuming the accelerometer signal $X[n]$ to be a w.s.s., write the mathematical expression for the simplified model

As a first approach we use the Periodogram $P_X(\omega)$.

2) How many samples are necessary to be able to distinguish the three sinusoidal components of the signal?

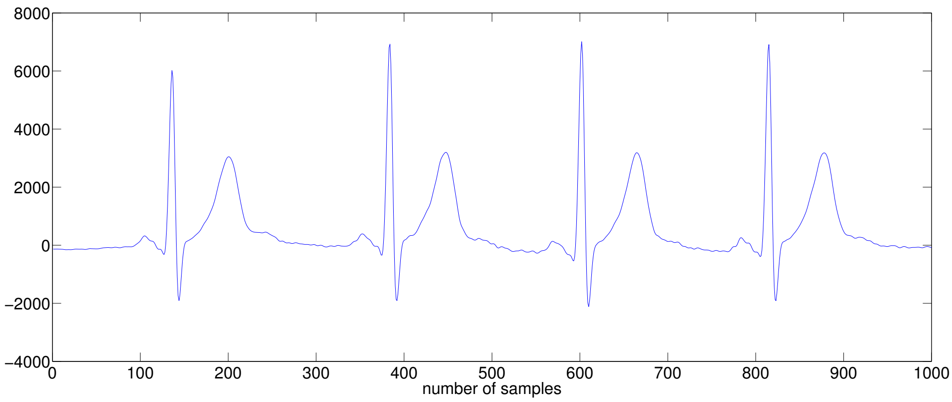
Weakness of the Periodogram are well known. We therefore use a parametric method.

3) Propose a parametric spectral estimation method than enables to estimate the frequencies f_1, f_2, and f_3 of the three components of the signal, as well as the corresponding amplitudes a_1, a_2, and a_3.

4) Given that we have measured $x[1], \ldots, x[100000]$, describe in detail the method, step by step, from the measured samples to the estimated spectrum, like if each step has to be interpreted and executed by a computer (in particular the input, the executed operation with corresponding equations, and the output of each step has to be clear).

Exercise 4. Electrocardiogram - ECG (15 pts)

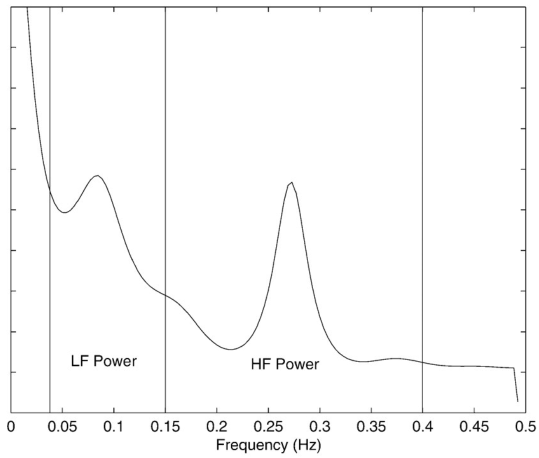
The electrocardiogram (ECG) is a signal representing the electrical activity of the heart.



ECG signal.

It can be seen as a sequence of main pulses, where each pulse corresponds to a heart beat.

One of the goal of measuring an athlete's ECG is to compute the intervals between each heart beat (called RR signal) and to analyze the corresponding spectrum (that is, the spectrum of the RR signal). In particular we want to analyze the spectral power (integral of the spectrum) over two frequency intervals. A typical spectrum of the RR signal is depicted in the figure below, with the frequency interval of interest, namely LF and HF.



Call $RR[n]$ the RR signal and $rr[1], \ldots, rr[10000]$ the samples obtained via the ECG measurement.

1) Propose a parametric spectral estimation method to estimate the spectrum of the RR signal. Justify precisely your answer.

2) Given that we have measured $rr[1], \ldots, rr[10000]$, describe in detail the method, step by step, from the measured samples to the estimated spectrum, like if each step has to be interpreted and executed by a computer (in particular the input, the executed operation with corresponding equations, and the output of each step has to be clear)