# Statistical Signal Processing

# MidTerm Exam

Thursday, XX April 2011

**Read Me First!**

*Only the personal cheat sheet is allowed.*
*No class notes, no exercise text or exercise solutions.*

**Write solutions on separate sheets,**
***i.e.*** **no more than one solution per paper sheet**.

**Return your sheets ordered according to problem (solution) numbering**.

*Return the text of the exam.*

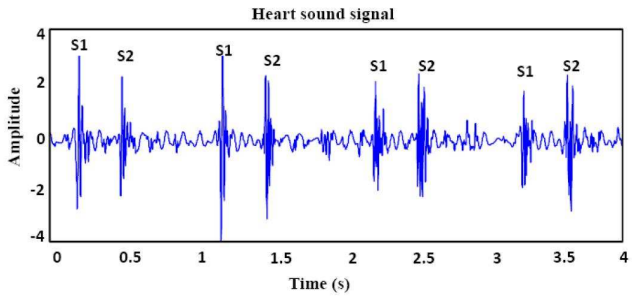
## A quick question

**Exercise 1**. How many parameters? (3 pts)

How many parameters are necessary to characterize a Markov Chain $X[n]$, $n = 1, \ldots, 1000$, with 4 possible states? (*i.e.*, $X[n]$ takes 4 possible values)

**Exercise 2**. Have you already tried to hear your heart with a microphone? (20 pts)

If we place a microphone on the chest, the signal that we can record pretty much looks like this




where S1 corresponds to the systole and S2 to the diastole. Such a signal is called phonocardiogram - PCG.

We model the recorded signal as a stochastic process $X[n]$ that we assume to be wide sense stationary - w.s.s. over windows of 1500 samples.

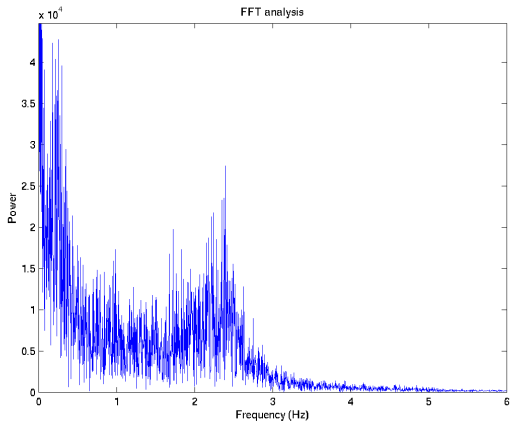
Due to movement artifacts (not easy to keep the hand still), the measurement is affected by noise that we suppose to be a white Gaussian noise $W[n]$ with variance $\sigma^2$. That is, what we measure is a signal $Y[n]$ given by

$$Y[n] = X[n] + W[n]\,.$$

Given that we measure 30000 samples of $Y[n]$,

1) Describe how we can de-noise the 30000 samples of $Y[n]$ using the Wiener filter and precisely detail the steps necessary to obtain the de-noised values (hint: don' t forget that the Wiener filter works only on w.s.s. signals ...) .

Thanks to the de-noising, we have now access to 30000 samples of $X[n]$ (de-noised $Y[n]$). When one tries to compute the spectrum of each w.s.s. window of samples using the periodogram, what he gets looks like



Your boss ask you to use a periodogram to compute the spectrum for each windows of 1500 samples and he wants to know the performance of the periodogram

2) Compute the spectral resolution of the periodogram given that you have observed 1500 samples of $X[n]$.

3) Describe in details the advantages and disadvantages of the periodogram.

4) Given that you have 30000 samples, is there a way to improve the periodogram? If yes, how?

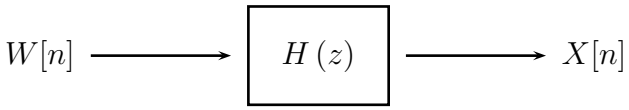
You finally decide not to use the periodogram. Based on the spectrum shown in the figure above, you are asked to:

5) Propose a parametric spectral estimation method and precisely justify you choice;

6) Describe in detail the proposed spectral estimation method and in particular the steps necessary to obtain the values of the spectrum from the 1500 values of the PCG signal $Xn$).

**Exercise 3**. A BIT OF THEORY (7PTS)

We consider a voice synthesizer based on filtering of a white noise




where the input $W[n]$ is a real Gaussian white noise, centered, with correlation $R_W[n] = \delta_n \alpha$ ($\alpha > 0$),

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

is a minimum phase filter with real coefficients $a_1$ and $a_2$, and $X[n]$ is the process describing the synthesized voice.

1) Is X[n] a wide-sense stationary process? Justify **precisely** your answer.

We are now interested in computing the second order properties of $X[n]$

2) Compute the power spectral density of $X[n]$, $S_X(\omega)$.