Music & Co.

COM-500 Mini-project

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Introduction and procedure

- Frequency estimation → estimating the complex frequency components of a signal in the presence of noise, given assumptions of components
- Different algorithms tested :
  - MUSIC
  - ESPRIT
  - SAMV

- Tested with real and fake data
  - Fake data : two signals with AWGN added
  - One signal with clearly different frequencies Y_1
  - One signal with 2 frequencies really close to each other Y_2

  - Real data was given

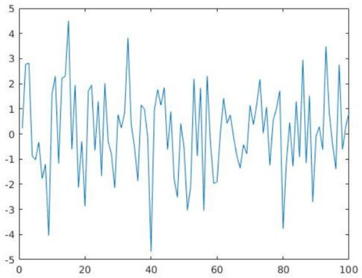

Fig. 2: Plot of $Y_1[n]$ for 100 samples

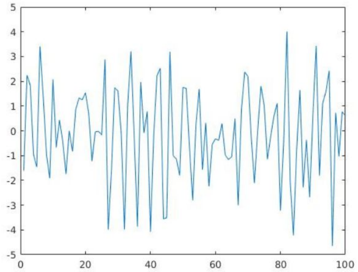

Fig. 4: Plot of $Y_2[n]$ for 100 samples

MUSIC

- Based on an eigenspace method
- First define **E** and **A**

$$\mathbf{E}^{NK} = [\mathbf{e}^{N1}(\omega_1)\dots\mathbf{e}^{N1}(\omega_K)] = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j\omega_1} & e^{-j\omega_2} & \dots & e^{-j\omega_K} \\ \vdots & \vdots & & \vdots \\ e^{-j(N-1)\omega_1} & e^{-j(N-1)\omega_2} & \dots & e^{-j(N-1)\omega_K} \end{bmatrix} \qquad \mathbf{A}^{KK} = \begin{bmatrix} \alpha_1^2 & 0 & \dots & 0 \\ 0 & \alpha_2^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \alpha_K^2 \end{bmatrix}$$

- $\mathbf{Y}^{N1}[n] = \mathbf{E}^{NK}\mathbf{X}^{K1}[n] + \mathbf{W}^{N1}[n]$
- $R_{\mathbf{Y}}^{NN} = E[\mathbf{Y}^{N1}[n]\mathbf{Y}^{N1}[n]^H]$ $\rightarrow$ Correlation matrix
- $\mathbf{R}_{\mathbf{Y}}^{NN} = \mathbf{E}^{NK}\mathbf{A}^{KK}\mathbf{E}^{NK^H} + \sigma_W^2\mathbf{I}^{NN}$
- Empirical covariance matrix : $\hat{\mathbf{R}}_{\mathbf{Y}}^{NN} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{Y}^{N1}[n]\mathbf{Y}^{N1H}[n]$
- Extraction of frequencies by finding peaks : $\frac{1}{\mathbf{a}(\omega)^H\hat{\mathbf{G}}^{N(N-k)}\hat{\mathbf{G}}^{N(N-k)^H}\mathbf{a}(\omega)}$

ESPRIT

- Estimation of Signal Parameters via Rotational Invariant Techniques
- We define two vectors $S_1 = \begin{bmatrix} I_{n-1} & 0 \end{bmatrix} S, \quad S_2 = \begin{bmatrix} 0 & I_{n-1} \end{bmatrix} S$
- The goal is to find φ such that $S_2 = S_1\varphi$
- Using the Moore-Penrose inverse for φ : $\phi = (S_1^* S_1)^{-1} S_1^* S_2$
- The angular frequencies can be found with : $\omega_n = -\arg(\lambda_n)$
  λ being the eigenvalues of φ

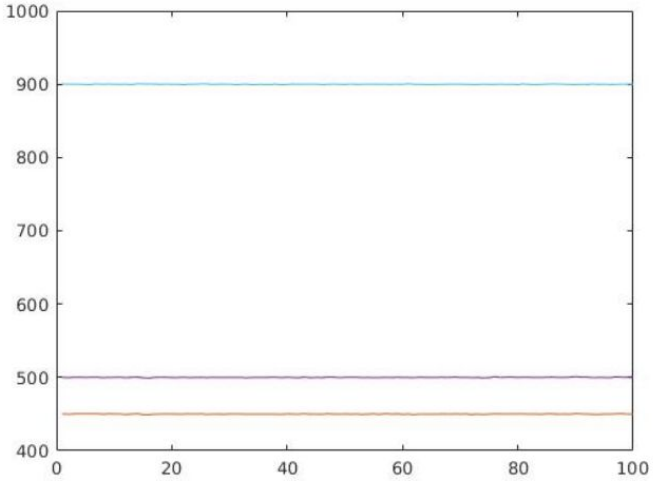

Fig. 10: Results for 100 experiments of $Y_2[n]$ using ESPRIT

SAMV

- Sparse Asymptotic Minimum Variance, iterative algorithm
- The idea is to generate a matrix **A**, for potential frequencies $[\omega_0, \ldots \omega_K]$ ⁻ a large K so there are many possibilities, and to update the power vectors $\hat{p}_k$ ›r each of the frequency
    - When the algo converges, the matrix **P** becomes sparse and only relevant the power vectors are non-zero
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$$\mathbf{A} = [\mathbf{a}(\omega_1) \ldots \mathbf{a}(\omega_K)] \qquad \mathrm{diag}(\mathbf{P}^{(0)}) = [\hat{p}_1^{(0)} \ldots \hat{p}_k^{(0)}]^T = \frac{1}{N}\|\mathbf{A}^H\mathbf{y}\| \qquad \hat{\sigma}^{(0)} = \frac{1}{N}\sum_{n=1}^{N}\|Y[n]\|^2$$

- At each iteration *i*, we update the power vector $\hat{p}_k$

$$\mathbf{R}^{(i)} = \mathbf{A}\mathbf{P}^{(i)}\mathbf{A}^H + \sigma^{(i)}\mathbf{I}$$

$$\hat{p}_k^{(i+1)} = \frac{a_k^H \mathbf{R}^{-1(i)} \mathbf{R}_N \mathbf{R}^{-1(i)} a_k}{(a_k^H \mathbf{R}^{-1(i)} a_k)^2}$$

$$\hat{\sigma}(0) = \frac{\mathrm{Tr}(\mathbf{R}^{-2(i)}\mathbf{R}_N)}{\mathrm{Tr}(\mathbf{R}^{-2(i)})}$$

Conclusion

- MUSIC :
    - Needs more data, based on assumptions that need to be right
    - One of the first algorithm created but becomes obsolete

- ESPRIT :
    - Fastest algorithm
    - Also based on assumptions, better results than MUSIC for same type of algo
    - Go-to algorithm for line-spectra estimation

- SAMV :
    - Take a long time to run
    - Able to give very (more) precise results with few data and difficulties like insufficient snapshot
    - With the same amount of samples will be a lot more informative than MUSIC and ESPRIT

Questions ?

Thank you !