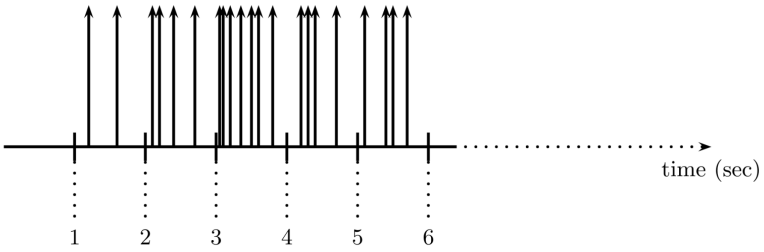
Exercises 8

Exercise 1. Firing Neuron

Researchers in a neurobiological laboratory have been recording the activity of a single neuron over 20 seconds.



In particular, they have counted the number of spikes that fall into disjoint intervals of 1 second (therefore 20 intervals) obtaining the following data:

$$2, 4, 7, 4, 4, 7, 2, 9, 7, 2, 2, 9, 9, 4, 2, 4, 9, 2, 7, 4.$$

They have denoted with N_i, $i = 1, \ldots, 20$, the random variable describing the number of spikes in the i-th interval (that is the interval $(i-1, i]$ seconds), and with N the random variable describing the number of spikes over the 20 seconds. They decide to adopt a Poisson process to model the spikes the neuron has fired and, based on the data recorded, they would like to estimate the parameter(s) of the process using the maximum likelihood method. Initially, they suppose that the neuron has a constant firing rate, that is, the average number of spikes per unit of time is constant.

1 Provide the probability law of each random variable N_i, $i = 1, \ldots, 20$; the **joint** probability law of the random variables N_i, $i = 1, \ldots, 20$; the probability law of the random variable N. **Justify precisely your answer**.

2 Write the Poisson process likelihood function associated to the 20 seconds observation. **Justify precisely your answer**.

3 Using the maximum likelihood method, estimate the parameter(s) of the Poisson process.

The researchers realize that assuming the average number of spikes per unit of time to be constant is not a good idea. They decide to model the firing rate as a two state Markov chain. After analyzing the observed 20 seconds data, they estimate the two states to have the values 3 and 8, and, from the data, they estimate the realizations of the Markov chain, that is, the realization of the firing rate, to be

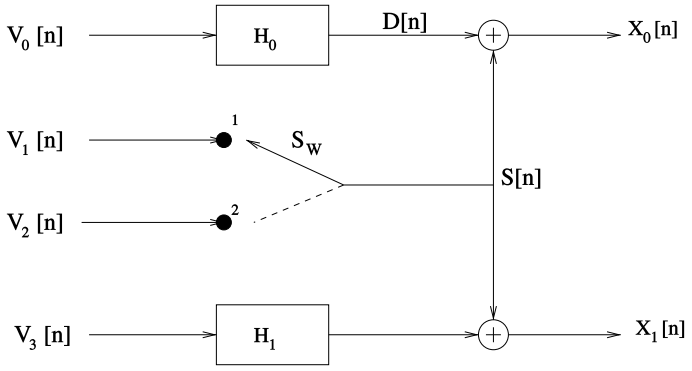
$$3, 3, 8, 3, 3, 8, 3, 8, 8, 3, 3, 8, 8, 3, 3, 3, 8, 3, 8, 3$$

4 How many parameters do you have to estimate to characterize the Markov chain? **Justify precisely your answer**.

5 Write the Markov chain likelihood function associated to the observed realization of the firing rate. **Justify precisely your answer**.

6 Using the maximum likelihood method and the **Lagrange multipliers**, estimate the parameters of the Markov chain. **Justify precisely your answer**.

Exercise 2.
Consider the following diagram:



where H_0 and H_1 are causal filters with transfer functions:

$$H_0(z) = 1 + z^{-1},$$

$$H_1(z) = 1 - z^{-1}.$$

V_0, V_1, V_2 and V_3 are white stationary processes, uncorrelated, jointly gaussian with zero mean and the variances

$$\sigma^2_{V_0} = 1 \quad \sigma^1_{V_1} = 1 \quad \sigma^2_{V_2} = 2 \quad \sigma^2_{V_3} = 1.$$

The switch S_W is in position 1 when the time index n is even and 2 when n is odd.

(a) Is $X_0[n]$ a Gaussian process? Is $X_0[n]$ a wide sense stationary process? Compute the correlation of $X_0[n]$.

(b) Determine the optimal filter that estimates the process $D[n]$, i.e. $\hat{D}[n]$, given the observation of $X_0[n]$ and $X_0[n-1]$.

(c) Determine the optimal filter that estimates the process $S[n]$, i.e. $\hat{S}[n]$, given the observation of $X_1[n]$ and $X_1[n-1]$.

How can we use such an estimator to determine an estimate of $D[n]$?

(d) Determine the optimal filter that estimates the process $D[n]$ given the observation $X_0[n]$, $X_1[n]$, $X_0[n-1]$ and $X_1[n-1]$.