Exercises 5

Exercise 1.
The *annihilating filter* (AF) is a method to estimate the parameters of a discrete signal with *sparse* DTFT representation. In other words, given a signal $x : \mathbb{Z} \to \mathbb{C}$ with

$$X(e^{j\omega}) = \sum_{k=1}^{K} \alpha_k \delta(\omega - \omega_k),$$

where the max degree of sparsity K is known, the AF method lets one estimate $\{\alpha_k, \omega_k\}_{k=1,\dots,K}$. For each parametric signal below, devise a strategy using the AF method to estimate the signal parameters.

(a) Observable: $x[n] = \sum_{k=1}^{K} \alpha_k^d e^{j\omega_k^d n}$.
Goal: estimate $\{\alpha_k^d, \omega_k^d\}_{k=1,\dots,K}$.

(b) Observable: samples of $x(t) = \sum_{k=1}^{K} \alpha_k^c e^{j\omega_k^c t}$.
Goal: estimate $\{\alpha_k^c, \omega_k^c\}_{k=1,\dots,K}$, where we know beforehand that $\max\{\omega_k\}_{k=1,\dots,K} < \Omega$.
Specify what sampling rate should be used.

(c) Observable: samples of the T_p-periodic function $x(t) = \sum_q \sum_{k=1}^{K} \alpha_k^c h(t - qT_p - t_k^c)$, where h is a known bandlimited function.
Goal: estimate $\{\alpha_k^c, t_k^c\}_{k=1,\dots,K}$.
Specify what sampling rate should be used.

(d) Signal of interest: T_p-periodic function $x(t) = \sum_q \sum_{k=1}^{K} \alpha_k^c \delta(t - qT_p - t_k^c)$.
Goal: estimate $\{\alpha_k^c, t_k^c\}_{k=1,\dots,K}$.
Notice that $x(t)$ cannot be sampled in practice. How would you therefore estimate the signal parameters?