Exercises 4

Exercise 1.
The process $X[n]$ is a real AR process:

$$X[n] = 0.3X[n-1] - 0.4X[n-2] + 0.5X[n-3] + W[n],$$

where $W[n]$ is a white noise.

(a) What is the order of the above AR process?.

(b) Using Yule Walker equations, give the best linear predictor of order 2 of $X[n]$, i.e. find a and b in $\hat{X}[n] = aX[n-1] + bX[n-2]$ such that the residual $\|\epsilon\|_2^2 = \mathbb{E}[|X[n] - \hat{X}[n]|^2]$ is minimized.

Exercise 2. Line Spectrum Estimation: the Dual Problem
Let $x(t)$ be a continuous periodic signal of period T,

$$x(t) = \sum_{n\in\mathbb{Z}} \sum_{k=0}^{M-1} a_k \delta(t - nT - t_k)$$

where $\delta(t)$ is a Dirac delta function. Assume that you want to use the annihilating filter method to estimate parameters $t_k,\ k = 0 \ldots M-1$ from an appropriate set of the Fourier series coefficients.

(a) Compute the Fourier series coefficients $\widehat{x}[n]$ of $x(t)$.

(b) Write a system of equations that allows you to find t_k for $M = 3$. What is the minimum number of Fourier series coefficients required for a unique solution?

(c) How does the noisy case differ from the previous case, where the presence of noise was not considered?

Exercise 3. Spectral factorization and estimation
Let $\{X[n]\}_{n\in\mathbb{Z}}$ be a centered AR process with power spectral density of the form

$$S_X(\omega) = \frac{b}{(1 + a_1^2 - 2a_1\cos\omega)(1 + a_2^2 - 2a_2\cos\omega)}\,, \quad |a_1| < 1,\ |a_2| < 1,\ b > 0\,,$$

where a_1, a_2 and b are unknown real-valued parameters.

(a) Give the canonical representation of the process $X[n]$

$$P(z)X[n] = W[n]$$

Give the whitening filter $P(z)$ and the variance σ^2 of the noise process $\{W[n]\}_{n\in\mathbb{Z}}$.

(b) Give the procedure to determine the parameters a_1, a_2 and b of the AR process $X[n]$, and to estimate the power spectral density $S_X(\omega)$.