Exercises 3

Exercise 1. Power Spectrum Density
Consider the stochastic process defined as

$$Y[n] = X[n] + \beta X[n-1]$$

where $\beta \in \mathbb{R}$ and $X[n]$ is a zero-mean wide-sense stationary process with autocorrelation function given by

$$R_X[k] = \sigma^2 \alpha^{|k|}$$

for $|\alpha| < 1$.

(a) Compute the power spectrum density $S_Y(e^{j\omega})$ of $Y[n]$.

(b) For which values of β does $Y[n]$ corresponds to a white noise? Explain.

Exercise 2. Projection of AR process
Consider the autoregressive process defined as:

$$X[n] = X[n-1] - X[n-2] + 2X[n-3] + W[n], \tag{1}$$

where $W[n]$ is white noise. Your task is to find the best *linear* predictor $Y[n-1]$ of order 3 of $X[n+2]$ given the past $(X[n-1], X[n-2], X[n-3])$, using two methods:

(a) by using the projection theorem and solving a system of linear equations,

(b) by recursively expanding the definition of $X[n+2]$ and using an intuitive property.

Exercise 3.
The process $X[n]$ is a real AR process of order M.

1) Write the recursion that allows to synthesize the process $X[n]$ from a white noise process $W[n]$.

2) Write the correlation structure of X.

   Suppose that the order M is unknown but an estimate of the correlation function $R_X[n]$ is available for $n \geq 0$.

3) Describe precisely a procedure to determine the order M, the parameters of the AR model and the variance of the input noise W.

4) What is the expression of the power spectral density of $X[n]$, $S_X(\omega)$ (as a function of the parameters and the order M)?