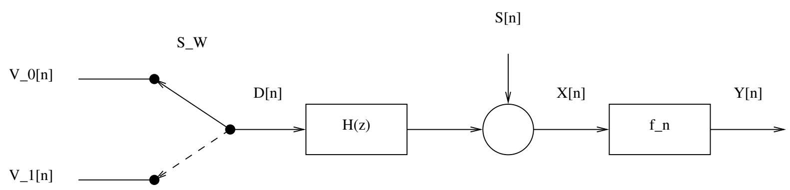
Exercises 11

Exercise 1.
Consider the following schematic diagram:



The two processes $V_0[n]$ and $V_1[n]$ are zero mean jointly Gaussian, they are mutually uncorrelated and their self correlation functions are

$$R_{V_0}[n] = \mathbb{E}[V_0[n+m]V_0[m]] = \rho_0^{|n|},$$

$$R_{V_1}[n] = \mathbb{E}[V_1[n+m]V_1[m]] = \rho_1^{|n|}.$$

In the following we will take $\rho_0 = 1/2$, $\rho_1 = 1/3$. The switch S_W selects one of the two processes to generate the "desired" process $D[n]$ that has to be estimated. The measurements are obtained by filtering $D[n]$ with the filter $H(z) = 1 + z^{-1}$ and adding the noise process $S[n]$, which is i.i.d, zero mean, jointly Gaussian with V_0 and V_1 and with variance $\sigma_S^2 = 1$. The measurement process $X[n]$ is filtered by the time-varying filter f_n of length $L = 3$ to obtain the estimate $Y[n]$. The goal is to minimize the variance of the estimation error $E[n] = D[n] - Y[n]$.

(a) Assume first that the switch is in position "0" (i.e. $D[n] = V_0[n]$). Write the normal equations for the filter f_n and find the optimal linear filter. Is this a Wiener filter? Compute the estimation error variance, $\mathbb{E}[E[n]^2]$. Do the same with the switch in the position "1".

(b) Assume now that the switch is in position "0" for the even samples and "1" for the odd samples. Do the following steps:

  (a) Compute the correlation function $R_D[n,m] = \mathbb{E}[D[n]D[m]]$. Is the process $D[n]$ stationary?

  (b) Compute the correlation functions $R_X[n,m] = \mathbb{E}[X[n]X[m]]$ and $R_{DX}[n,m] = \mathbb{E}[D[n]X[m]]$.

  (c) Write the normal equations for the even and odd time indexes. Find the optimal linear filter and the error variance for the two cases (even and odd time indexes) and compare them with the result of question a).

(c) Assume that the position of the switch is chosen randomly and independently for each sample. The probability of position "0" is $p_0 = 1/2$. Compute again the correlation $R_D[n,m] = \mathbb{E}[D[n]D[m]$ and check if the process is stationary. Compute the optimal linear filter in this case and compare the answer with the results of question b).

Exercise 2. TRANSMITTING PULSES

A transmitted signal is composed of a sequence of spikes. The interval between each spike codes the information to be transmitted. In particular, the interval is a when bit 0 is transmitted, and $2a$ when bit 1 is transmitted.

We observe the spikes over an interval [0,T].

1) Knowing that in such an interval there are at most 100 spikes, precisely describe a method to estimate the spike positions and the interval between the spikes.

Markov Coding

Using the method you have described, we have been able to decode the information and we have obtained the following sequence of bits: 00011010110110111011, where $x[1] = 0$ and $x[20] = 1$. The coding is a Markovian one, that is, the process $X[n]$ that generates the bits $\{0, 1\}$ is a Markov chain.

2) Write the likelihood function associated to the observations $x[1], \ldots, x[20]$.
3) Using the maximum likelihood approach and Lagrangian multipliers, estimate the parameters of the Markov chain based on the observations $x[1], \ldots, x[20]$.

The channel corrupts the transmission with an additive Gaussian white noise $W[n]$ (centered, with variance σ_W^2). The decoded signal can be the written as $Y[n] = X[n] + W[n]$.

4) Write the probability distribution and then the probability density function of $Y[n]$.
5) Using Baye's rule, write the joint probability density function of $Y[1], \ldots, Y[10]$.

Transforming Spikes into Pulses

Engineers are not sure if transmitting spikes with the information 0 1 coded into the spike distance is an optimal method. They decide to transmit pulses and to increase the coding symbols: instead of only 0 and 1, the information is coded into different symbols and for each symbol there is a different pulse shape.

At the receiver end each shape is received as a sequence of 30 samples, and a total of 20000 shapes are received. The problem now is that at the receiver end the number of possible different shapes (and therefore of coding symbols) is unknown and has to be determined. That is, the 20000 shapes received need to be analyzed so to understand of how many different shapes the transmission is composed.

6) Given the received data (20000 shapes of 30 samples each) precisely describe a method to simply understand how many different shapes the transmission is composed of.