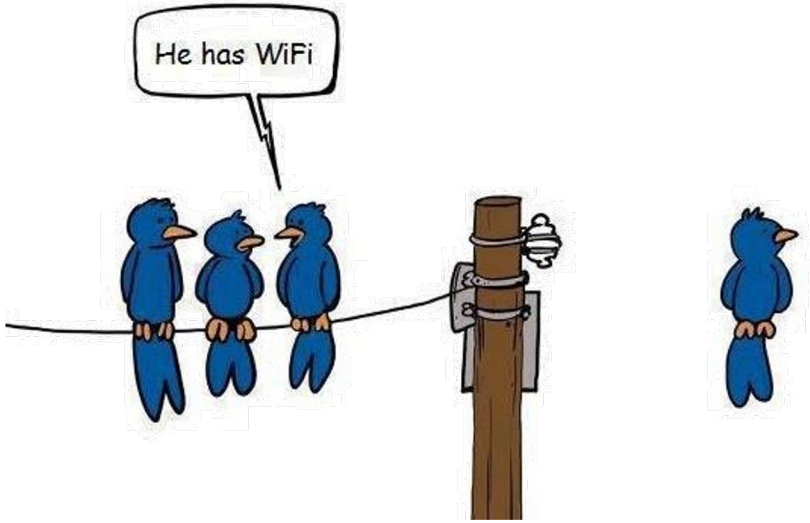
COM-405: Mobile Networks

Lecture 3.0: OFDM II

Haitham Hassanieh



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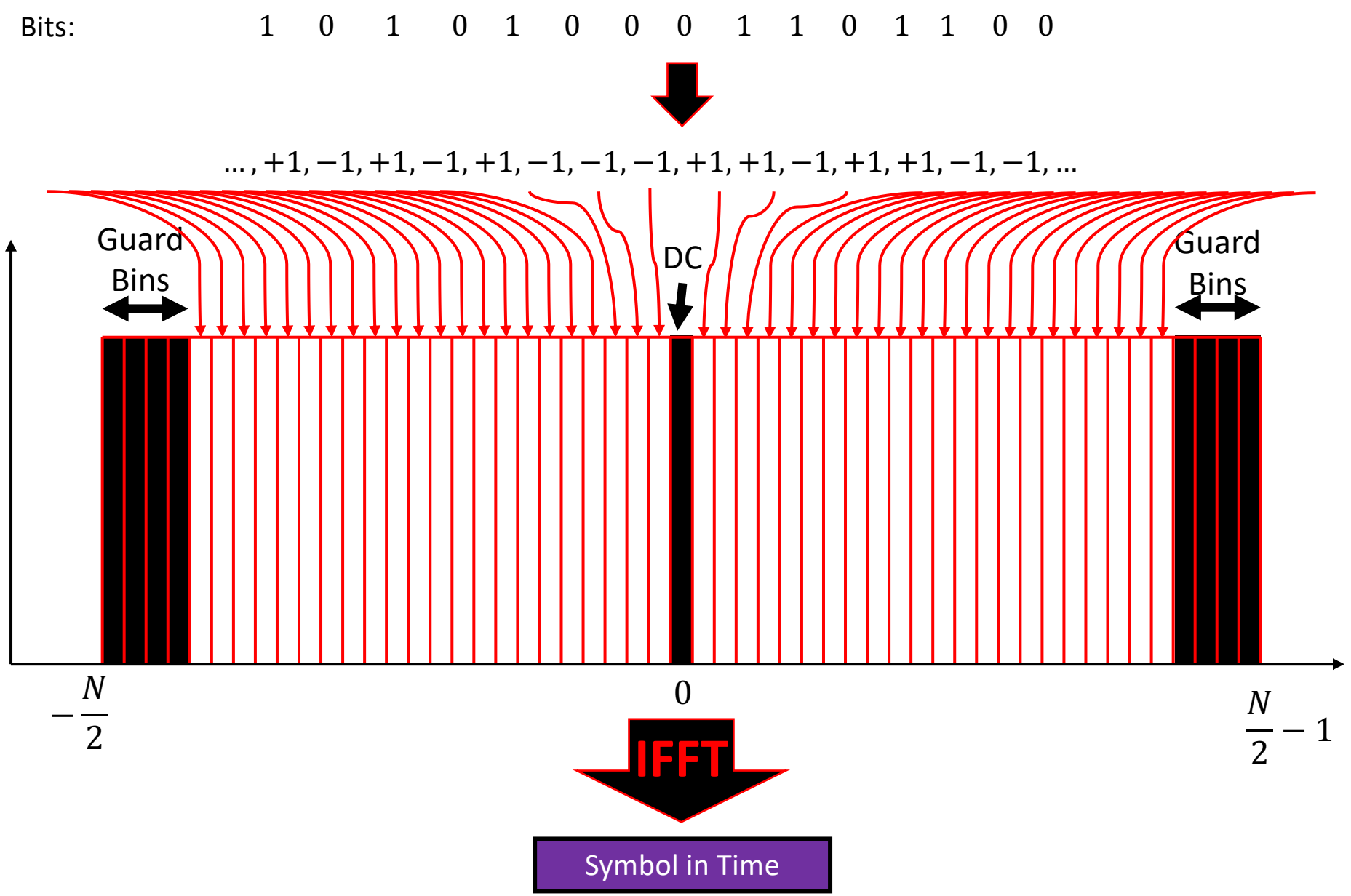
OFDM Symbol


Bits: 1 0 1 0 1 0 0 0 1 1 0 1 1 0 0
…, +1, −1, +1, −1, +1, −1, −1, −1, +1, +1, −1, +1, +1, −1, −1, …
Guard Bins
DC
Guard Bins
$-\frac{N}{2}$
0
$\frac{N}{2}-1$
IFFT
Symbol in Time

OFDM Cyclic Prefix

OFDM Cyclic Prefix

OFDM Cyclic Prefix

Carrier Frequency Offset

$$x(t) \Rightarrow x(t)\times e^{-j2\pi f_c t} \Rightarrow h(t) * x(t)e^{-j2\pi f_c t} \Rightarrow h(t) * x(t)e^{-j2\pi f_c t}\times e^{j2\pi f_c t}$$

$$\Rightarrow h(t) * x(t)$$

$$\Rightarrow y(t) = h(t) * x(t) + v(t)$$

Assumes TX & RX perfectly synched

Carrier Frequency Offset

$$x(t) \Rightarrow x(t)\times e^{-j2\pi f_c t} \Rightarrow h(t) * x(t)e^{-j2\pi f_c t} \Rightarrow h(t) * x(t)e^{-j2\pi f_c t}\times e^{j2\pi f_c' t}$$

TX & RX are not synched

$$\Rightarrow h(t) * x(t)\, e^{-j2\pi\Delta f_c t}$$

CFO: $\Delta f_c = f_c - f_c'$

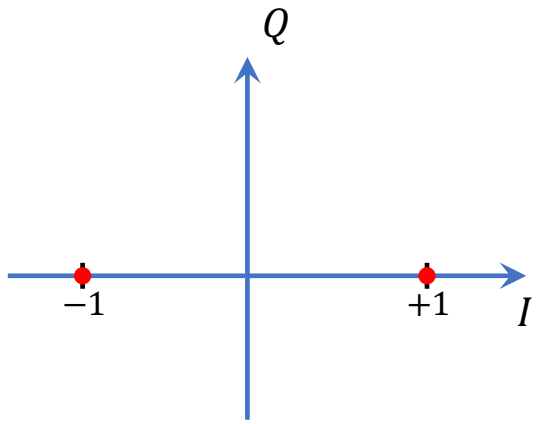
$$\Rightarrow y(t) = h(t) * x(t)e^{-j2\pi\Delta f_c t} + v(t)$$

Phase changes with time!

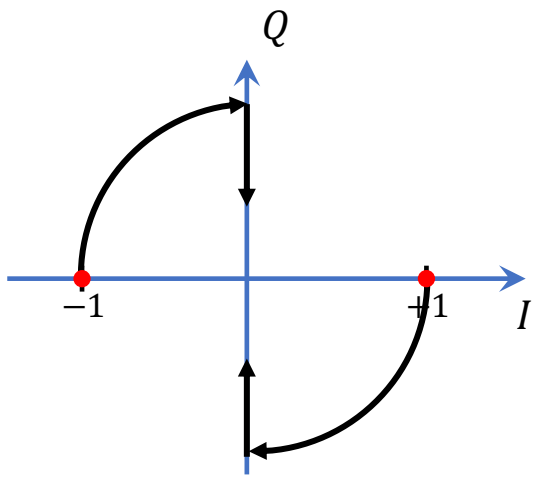
Carrier Frequency Offset

Consider BPSK Modulation.

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

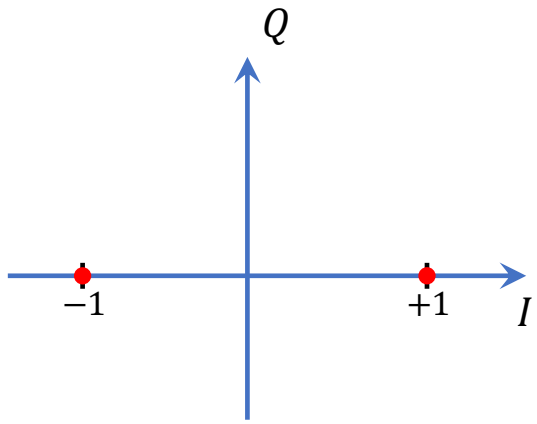


$h\,x(t-\tau)e^{-j2\pi\Delta f_c t} + v(t)$

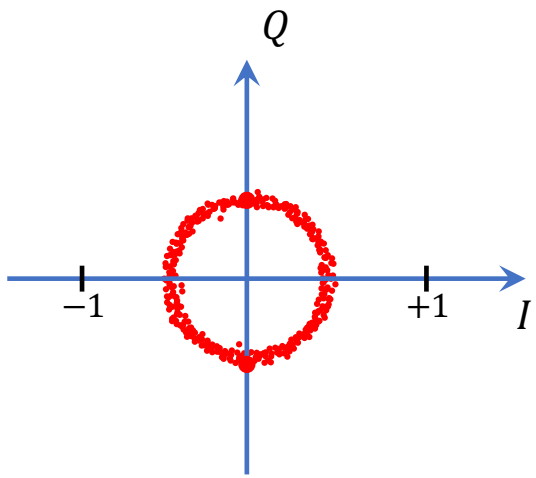
Carrier Frequency Offset

Consider BPSK Modulation.

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

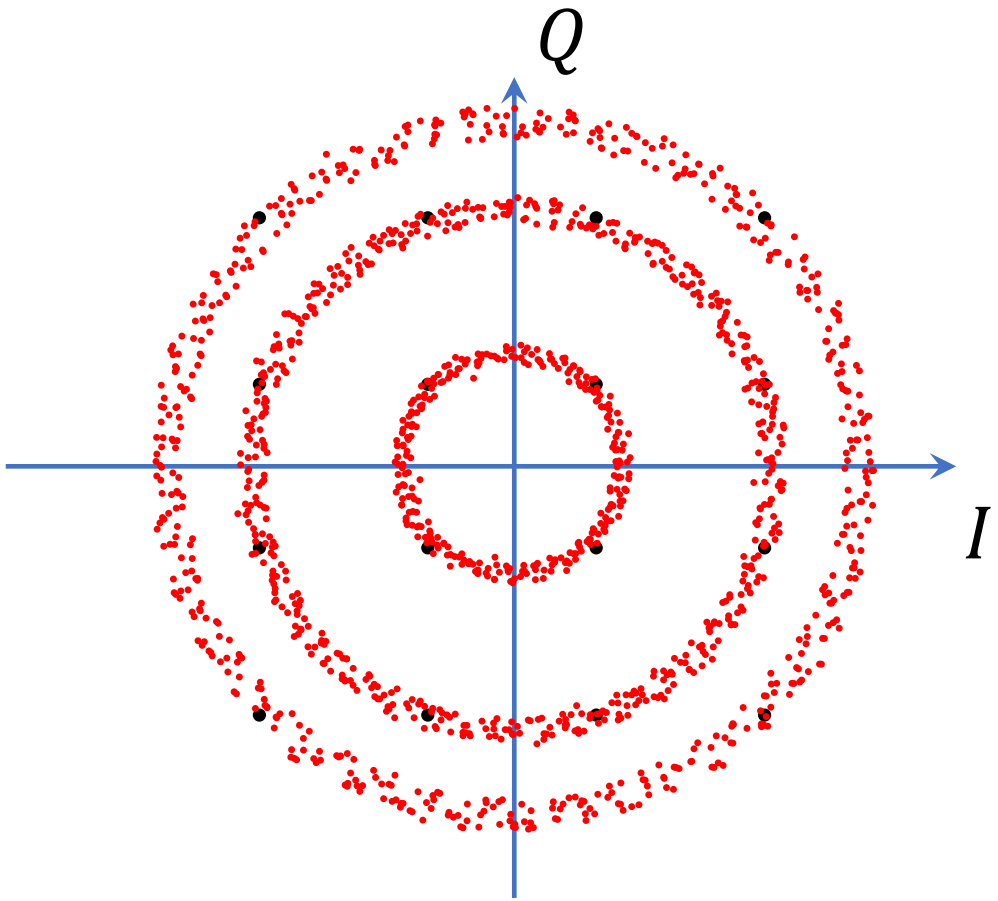


$h\,x(t-\tau)e^{-j2\pi\Delta f_c t} + v(t)$

Impossible to Decode!

Carrier Frequency Offset

Consider 16 QAM Modulation



Need to estimate and correct CFO to decode!

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1(t) = x(t)e^{-j2\pi\Delta f_c t}$$

Sample at $t = nT_s$
Symbol has N samples ➔ $T = NT_s$

$$y_2(t) = x(t)e^{-j2\pi\Delta f_c (t+T)}$$

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c n T_s}$$

Sample at $t = nT_s$
Symbol has N samples ➔ $T = NT_s$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)}$$

- Compute: $A = \sum_{n=1}^{N} y_1[n]y_2^*[n] = \sum_{n=1}^{N} x[n]x^*[n]e^{j2\pi\Delta f_c NT_s}$

$$= e^{j2\pi\Delta f_c NT_s} \sum_{n=1}^{N} |x[n]|^2 \quad \Rightarrow \quad \Delta f_c = \frac{\angle A}{2\pi NT_s}$$

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c n T_s}$$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)}$$

- Compute: $A = \sum_{n=1}^{N} y_1[n] y_2^*[n]$ ➡ $\Delta f_c = \frac{\angle A}{2\pi N T_s}$

- Correct CFO: $y[n] \times e^{j2\pi\Delta f_c n T_s}$

OFDM Channel Estimation

- Use Preamble to estimate the channel

$$y(t) = h(t) * x(t) \leftrightarrow Y(f) = H(f)\,X(f)$$

- Send $X(f)$: $-1, +1, -1, -1, -1, +1, \ldots$
- Receive: $-H(1), H(2), -H(3), -H(4), -H(5), H(6), \ldots$
- Estimate: $\widetilde{H}(f) = \dfrac{Y(f)}{X(f)}$
- Use two preambles to average noise: $\widetilde{H}(f) = \dfrac{Y_1(f) + Y_2(f)}{2\,X(f)}$

Phase Tracking

So Far: Estimated and Corrected For Coarse Value of CFO

- **Residual CFO:**

$$y(t) = h(t)\, x(t) e^{-j2\pi\Delta f_c t} + v(t)$$

$$\Delta f_c = df_c + \delta f_c$$

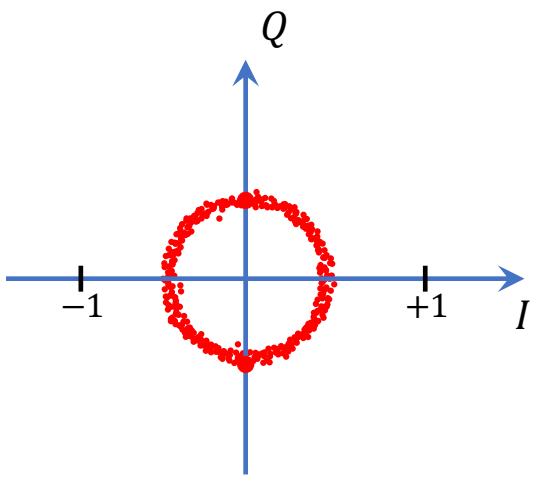
Coarse CFO Residual CFO

We estimated and corrected for coarse CFO!

Even small residual can accumulate over time to create large phase: $e^{-j2\pi\delta f_c t}$

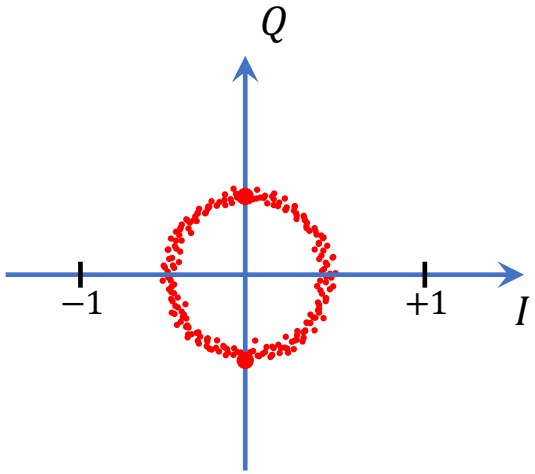
Need to track the phase

Phase Tracking



$$h\,x(t-\tau)e^{-j2\pi\Delta f_c t}+v(t)$$

Without Course CFO Correction



$$h\,x(t-\tau)e^{-j2\pi\delta f_c t}+v(t)$$

Without Residual CFO Correction

Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

When we sample the signal there is a residual sampling offset: $n\delta T_s$

$$y[n] = x[n + n\delta T_s]e^{-j2\pi\delta f_c n T_s}$$

$$= \sum_{f=0}^{N-1} X[f]e^{j\frac{2\pi f(n+n\delta T_s)}{N}} e^{-j2\pi\delta f_c n T_s}$$

$$Y[f] = X[f]e^{j\frac{2\pi f n\delta T_s}{N} - 2\pi\delta f_c n T_s}$$

Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

$$Y_1[f] = X_1[f]e^{j\frac{2\pi f n \delta T_s}{N} - 2\pi\delta f_c n T_s}$$

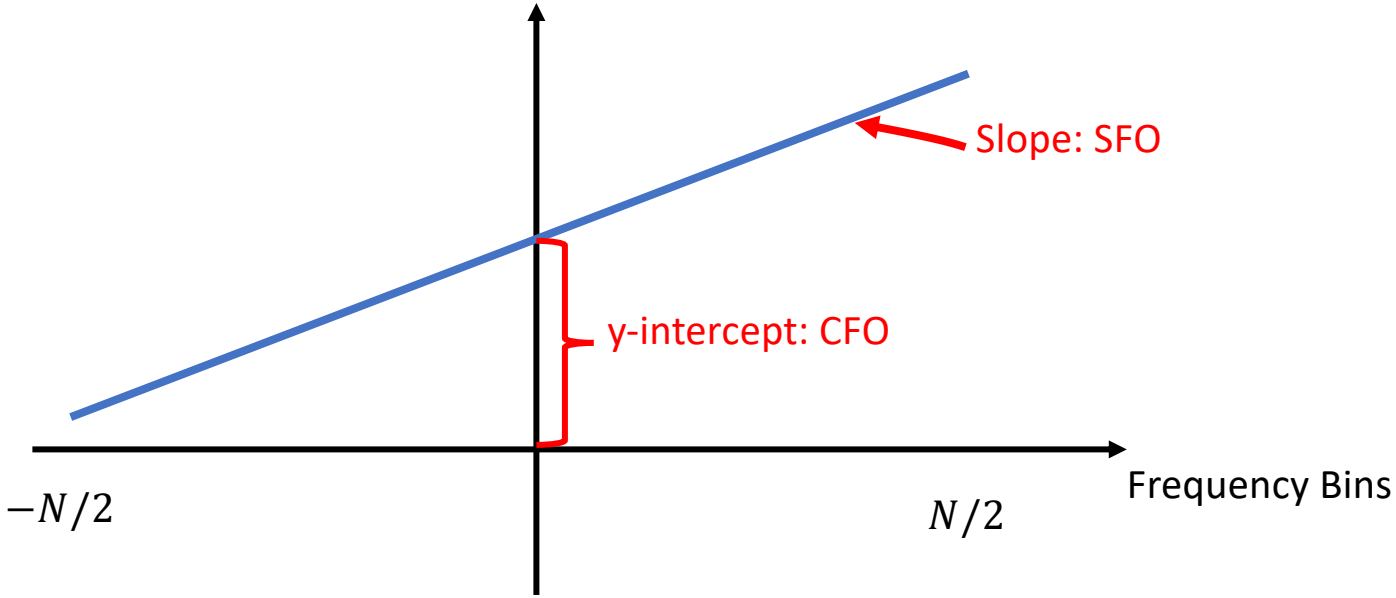
$$Y_2[f] = X_2[f]e^{j\frac{2\pi f (n+N+CP) \delta T_s}{N} - 2\pi\delta f_c (n+N+CP) T_s}$$

Phase accumulation: $\Delta\phi = 2\pi f \frac{(N+CP)\delta T_s}{N} - 2\pi\delta f_c (N+CP) T_s$

Phase Tracking

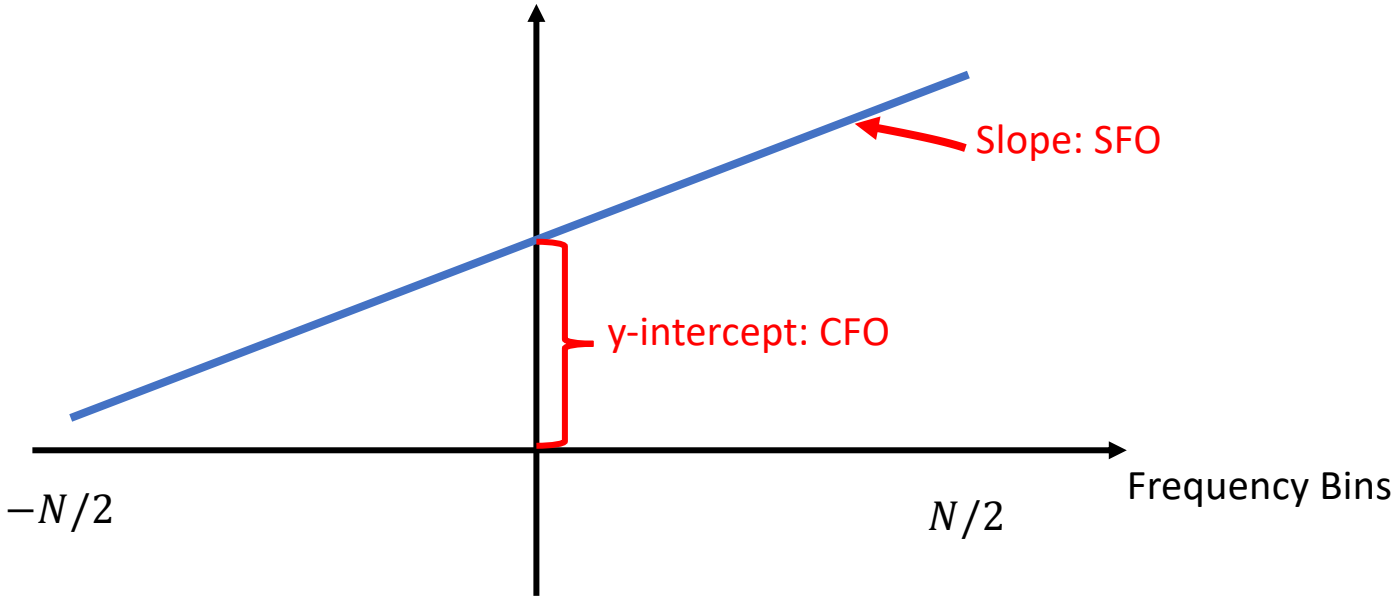
- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

- Phase accumulation: $\Delta\phi = 2\pi f \frac{(N+CP)\delta T_s}{N} - 2\pi\delta f_c (N+CP)T_s$



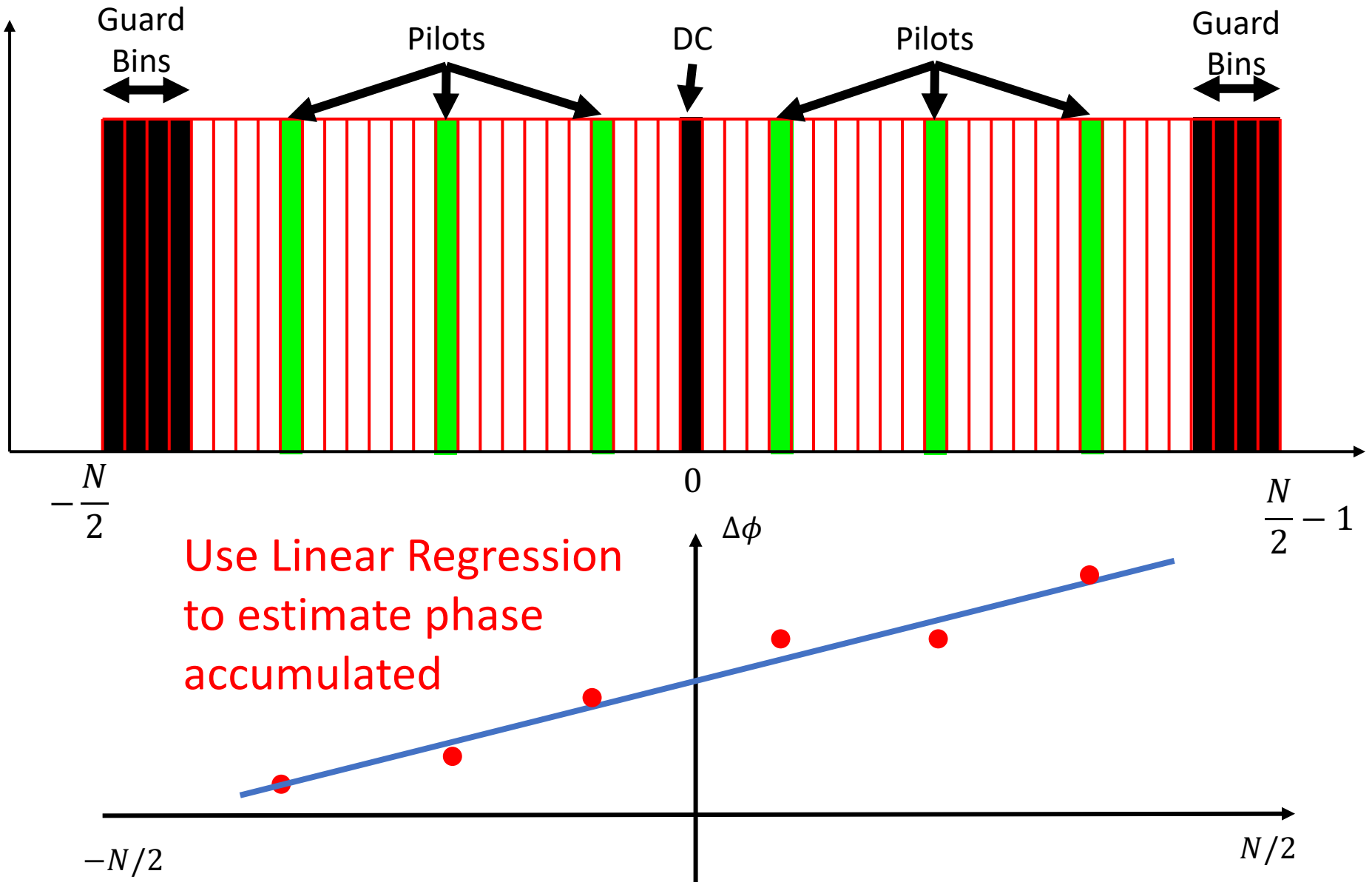
OFDM Phase Tracking

- Phase accumulation: $\Delta\phi = 2\pi f \frac{(N+CP)\delta T_s}{N} - 2\pi\delta f_c (N+CP)T_s$



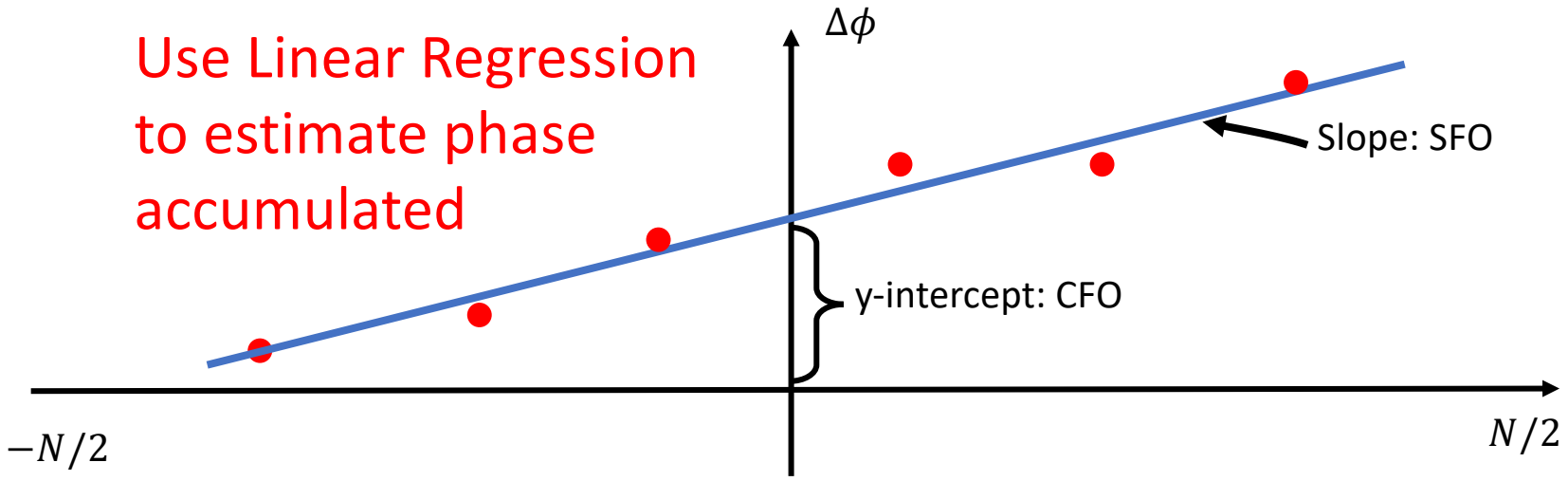
- Sufficient to estimate slope & y-intercept to know the phase accumulated for all subcarriers.
- Use only few subcarriers as pilots & send known bits in them.

OFDM Phase Tracking



OFDM Phase Tracking

Use Linear Regression to estimate phase accumulated



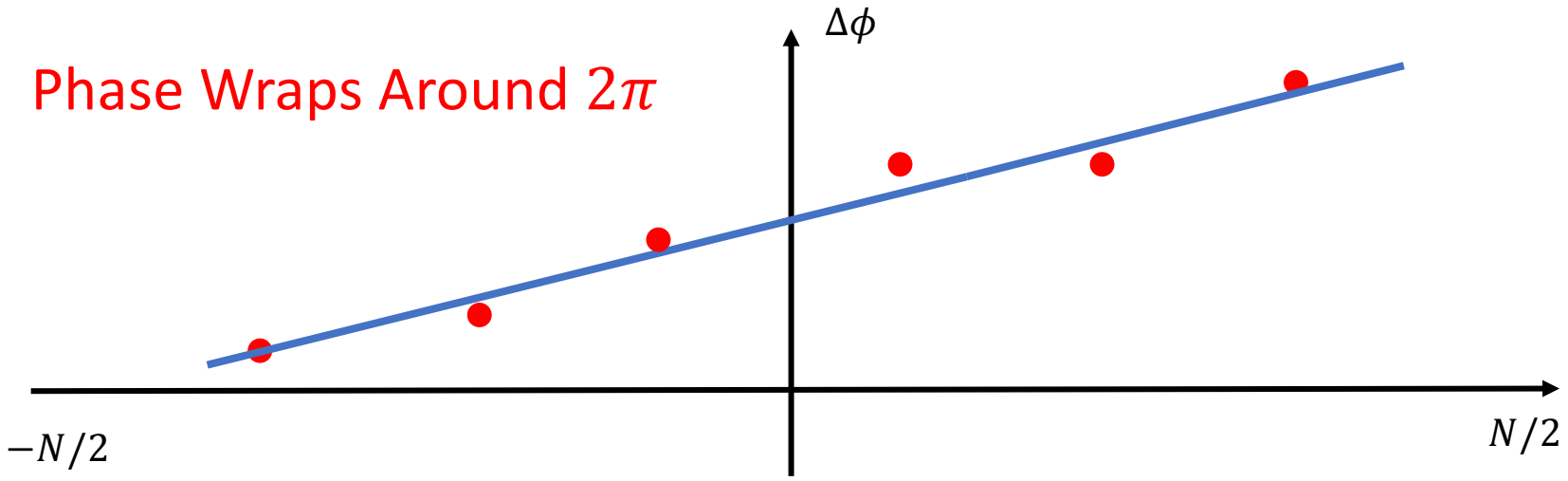
$$Slope = 2\pi \frac{(N + CP)\delta T_s}{N}$$

$$Y - intercept = -2\pi\delta f_c (N + CP)T_s$$

$$\Delta\phi = 2\pi f \frac{(N + CP)\delta T_s}{N} - 2\pi\delta f_c (N + CP)T_s$$

$$\widetilde{H}_{new}(f) = \widetilde{H}_{old}(f) \times e^{j\Delta\phi}$$

Be Careful!



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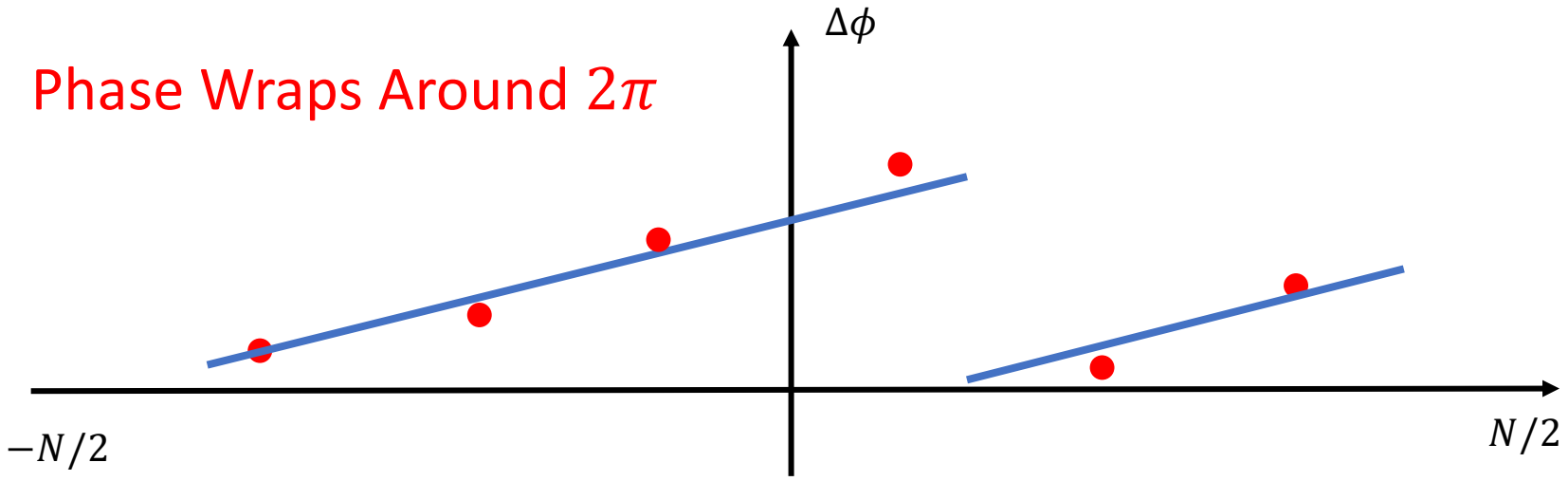

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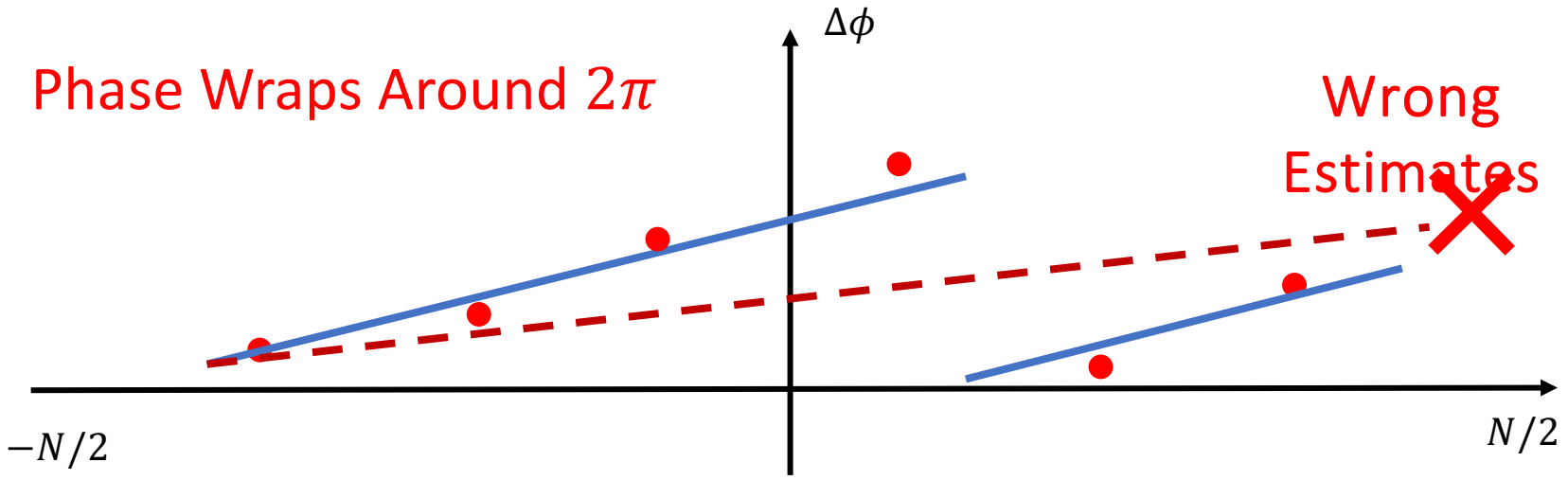

Need to unwrap phase mod 2π first!

When estimating slope and y-intercept, equation assumes subcarriers index f is between $-N/2$ and $N/2-1$

Be Careful!

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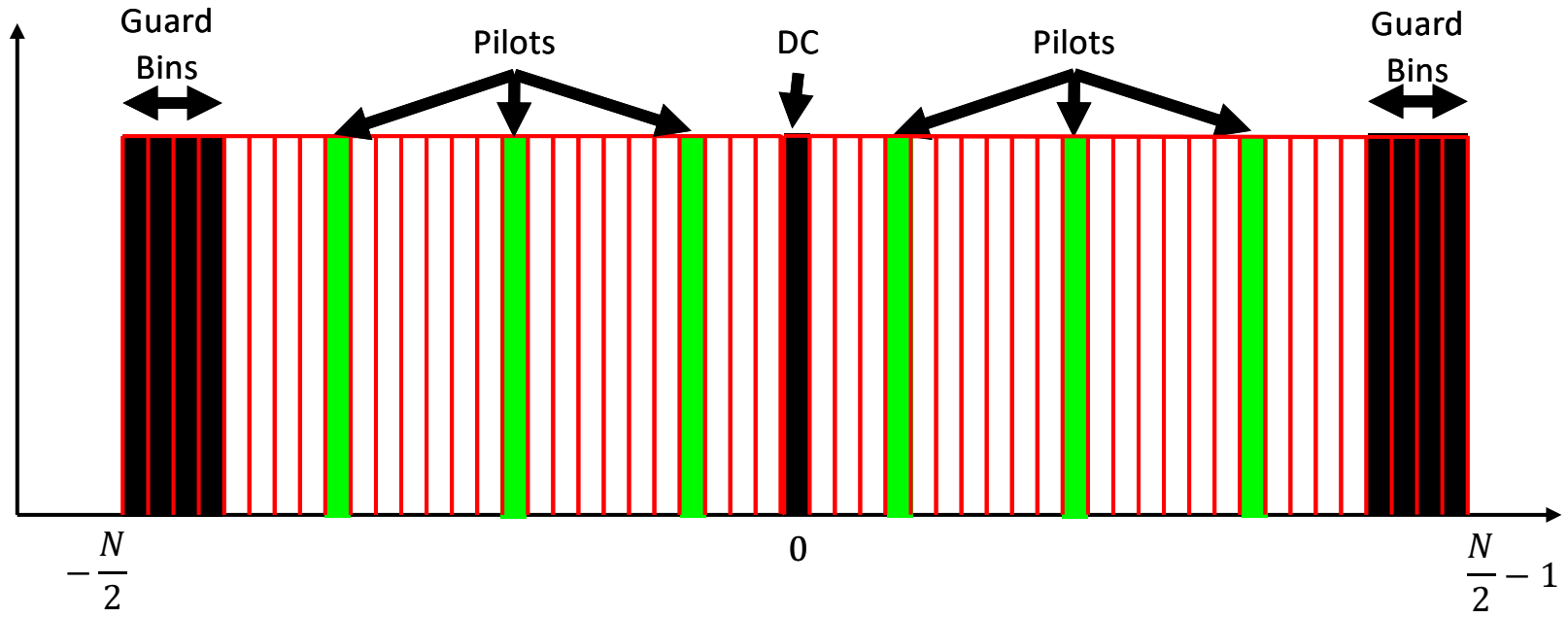

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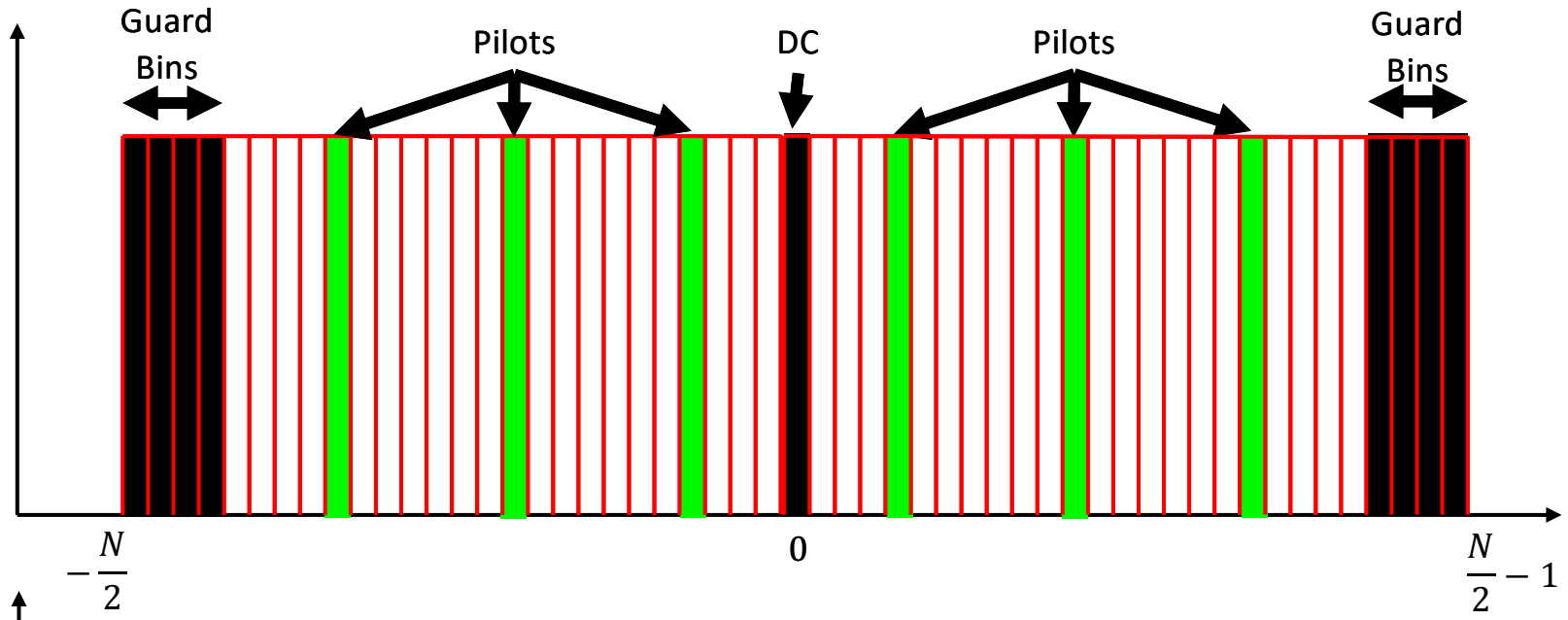

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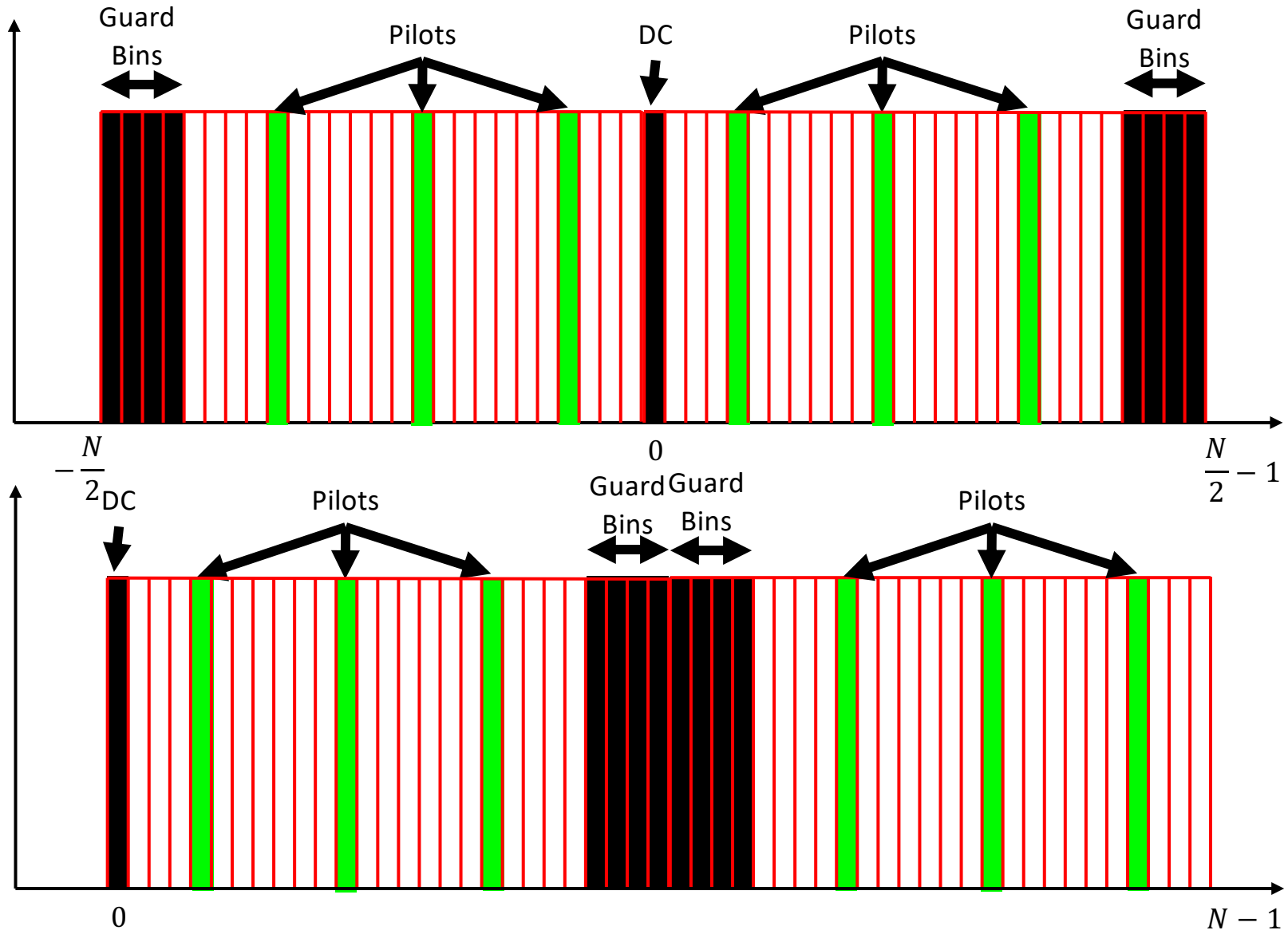

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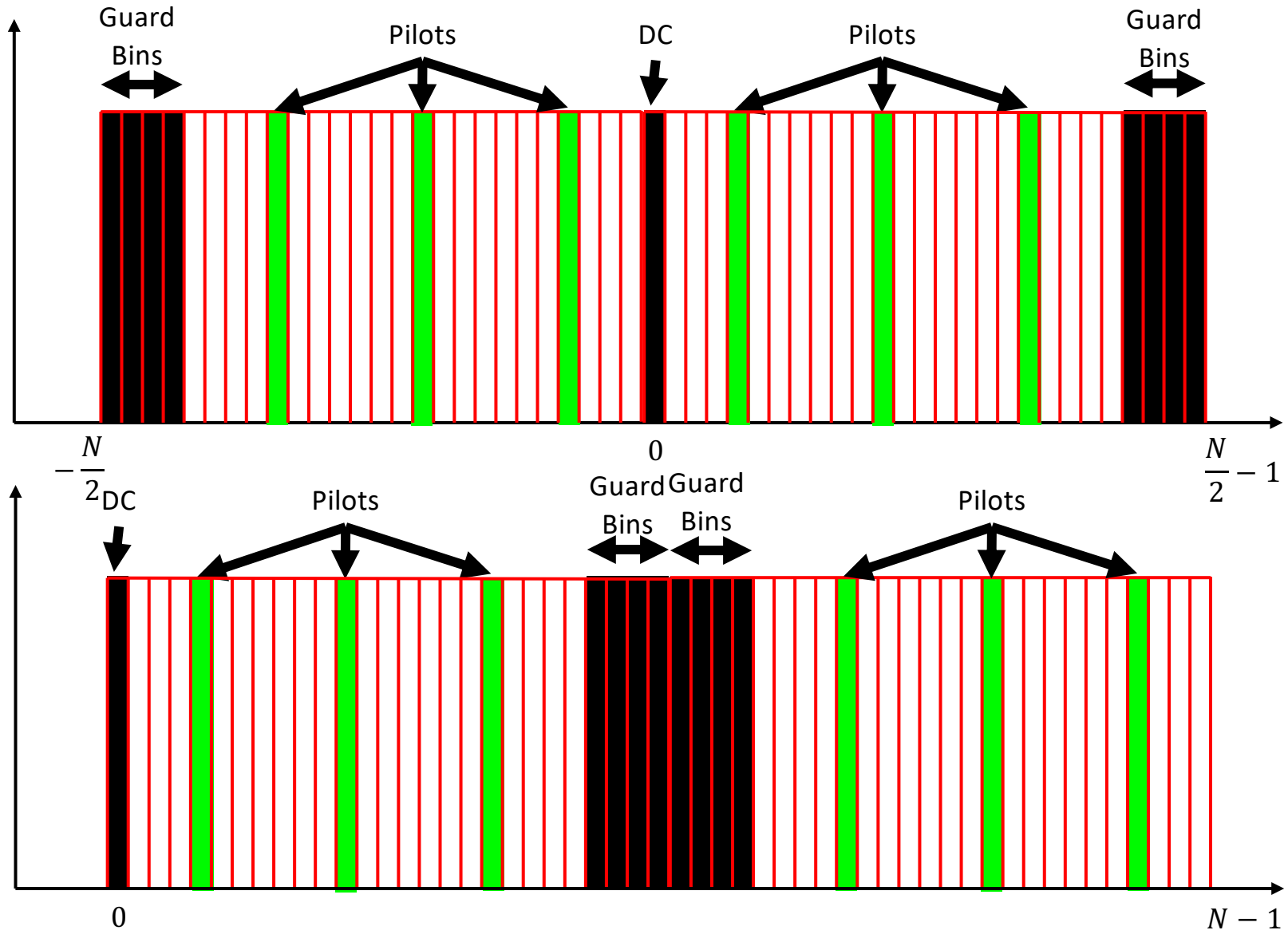

OFDM: Putting it Together

At TX:

- Create preamble symbol from training sequence (Uses BPSK)
- Repeat preamble symbol:
  - 4 times for packet detection
  - 2 times for CFO estimation
  - 2 times for channel estimation
  - Add CP for the last preamble
- Create data symbol from:
  - Data bits (Uses BPSK, QPSK, M-QAM)
  - Pilot bits (Uses BPSK)
- Add cyclic prefix to data symbols.

OFDM: Putting it Together

At RX:

- Detect beginning of packet.
- Estimate & correct for CFO.
- Jump $\approx 0.75\ CP$ samples into symbol to avoid ISI
- Estimate the channel.
- For each subsequent data symbol:
  - Remove CP
  - Take FFT of Size N
  - Correct for channel by dividing with $\widetilde{H}(f)$
  - Use linear regression to estimate residual CFO and SFO
  - Estimate accumulated phase $\Delta\phi(f)$ for each frequency bin
  - Add $\Delta\phi(f)$ to channel estimate $\widetilde{H}(f)$
  - Decode Bits