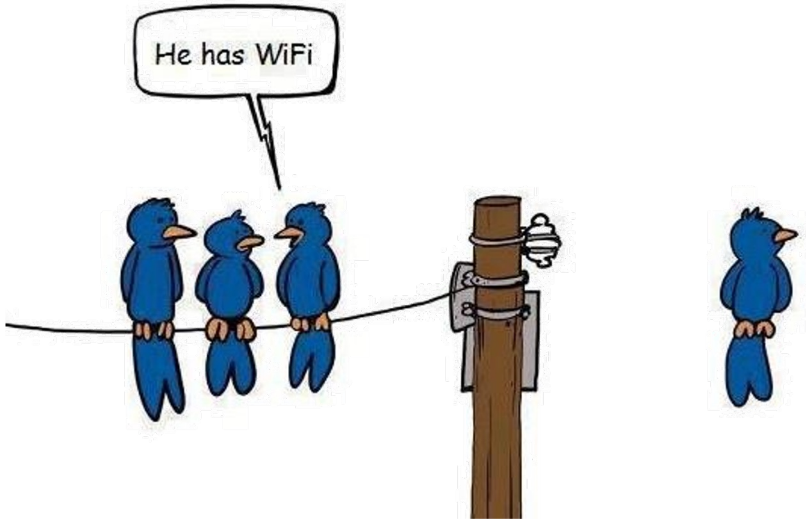
COM-405: Mobile Networks

Lecture 2.1: OFDM I

Haitham Hassanieh



EPFL

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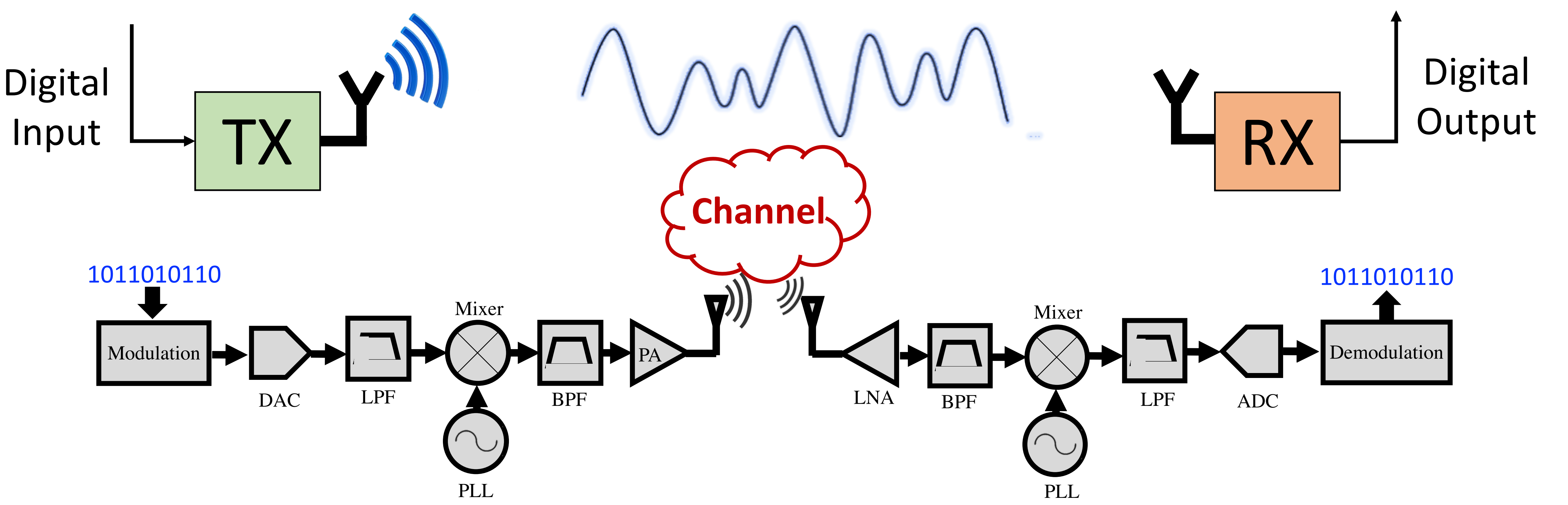
Why OFDM?

- Makes life easy → channel estimation and correction is straightforward
- Used in most high throughput mobile networks:
  - WiFi 4, 5, 6
  - 4G, 5G, … 6G
- Higher layer resource allocation schemes use OFDMA (WiFi 6, 4G, 5G)

Wireless Communication



1011010110 $\rightarrow s[n] \rightarrow s(t) \rightarrow s(t)\cos 2\pi f_c t \rightarrow h(t) * s(t)\cos 2\pi f_c t \times \cos 2\pi f_c t \rightarrow h(t) * s(t) \rightarrow h[n] * s[n]$

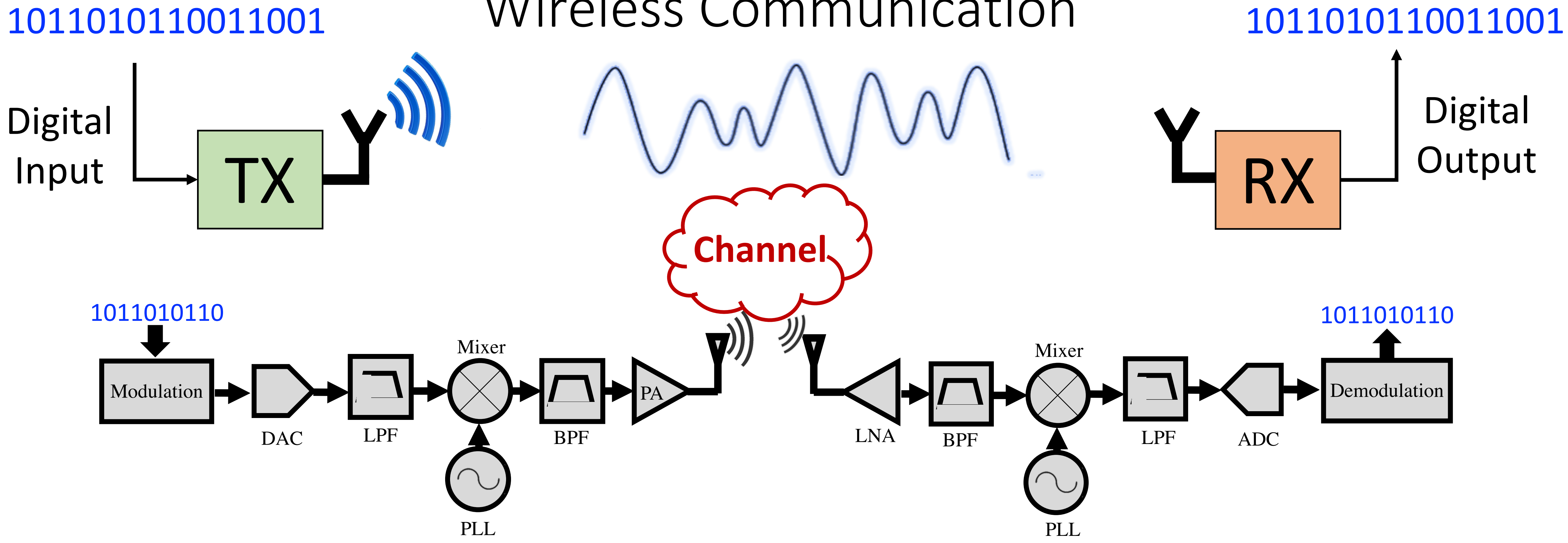
$\downarrow$

$s[n]$

$\downarrow$

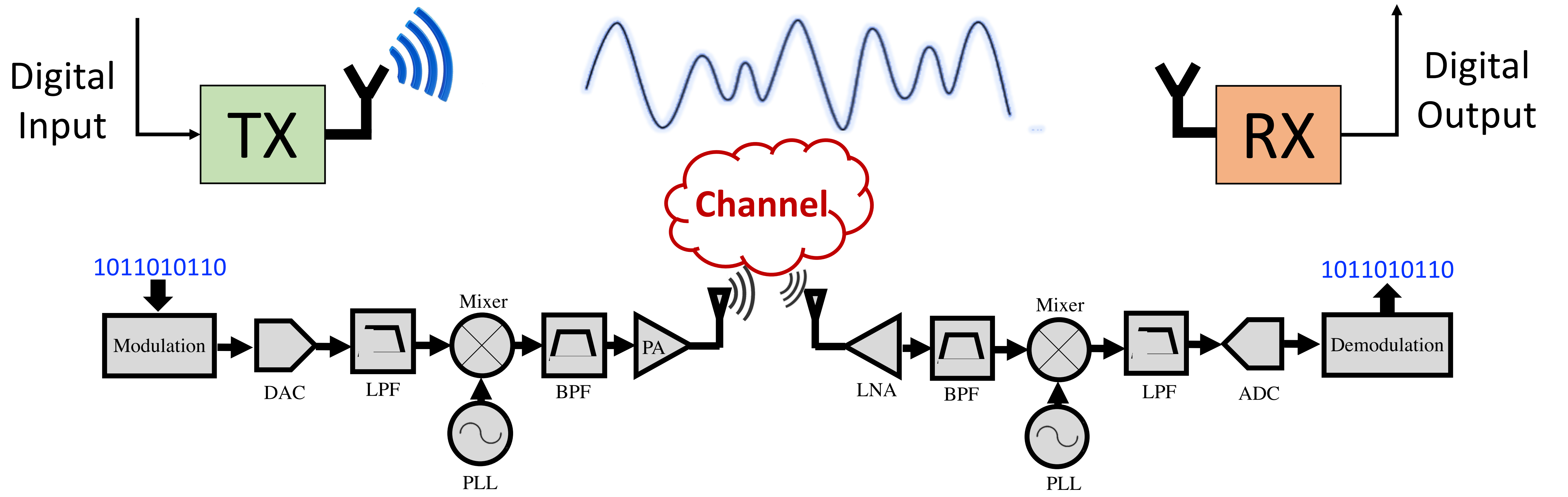
1011010110

Wireless Communication



1011010110 → $s[n]$ → $s(t)$ → $s(t)e^{-j2\pi f_c t}$ → $h(t) * s(t)e^{-j2\pi f_c t} \times e^{+j2\pi f_c t}$ → $h(t) * s(t)$ → $h[n] * s[n]$ → $s[n]$ → 1011010110

Wireless Communication



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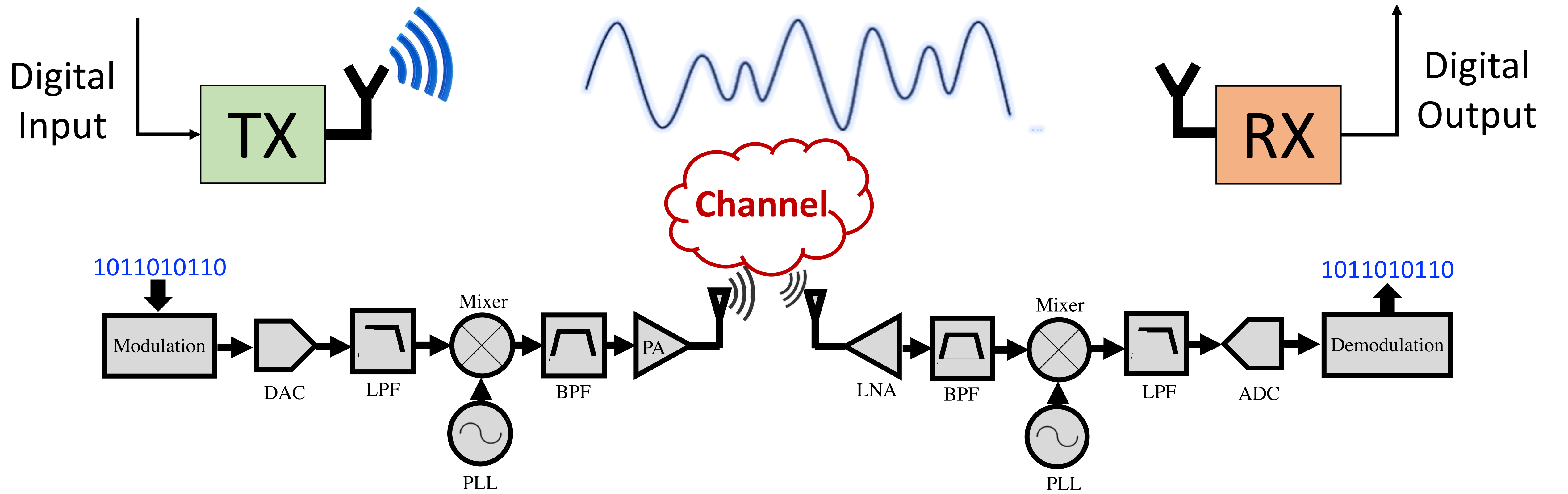

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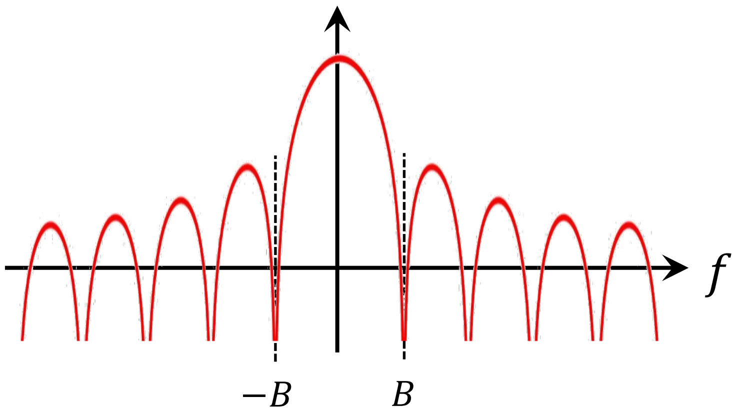
Single Carrier Modulation

Symbols modulated on a single carrier frequency: $s[n]e^{-j2\pi f_c t}$

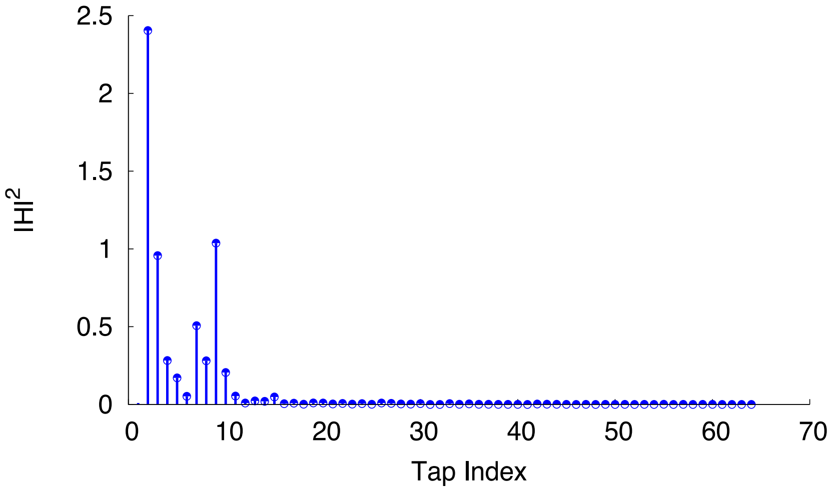
Single Carrier Modulation

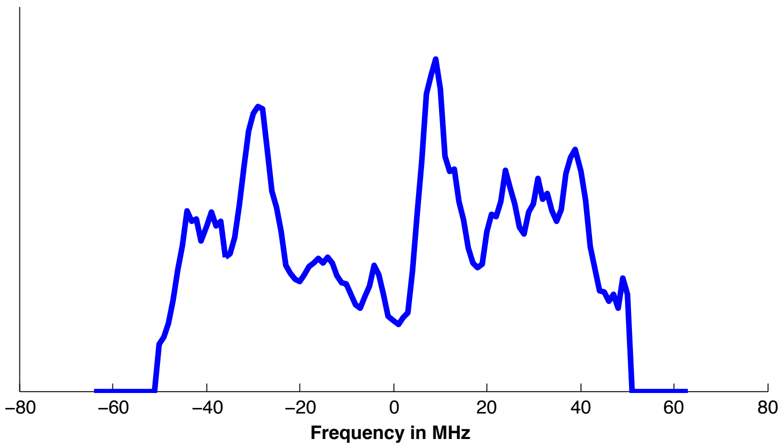
Symbols modulated on a single carrier frequency

- Low Spectral Efficiency: leakage



- ISI: Inter-Symbol-Interference limits performance

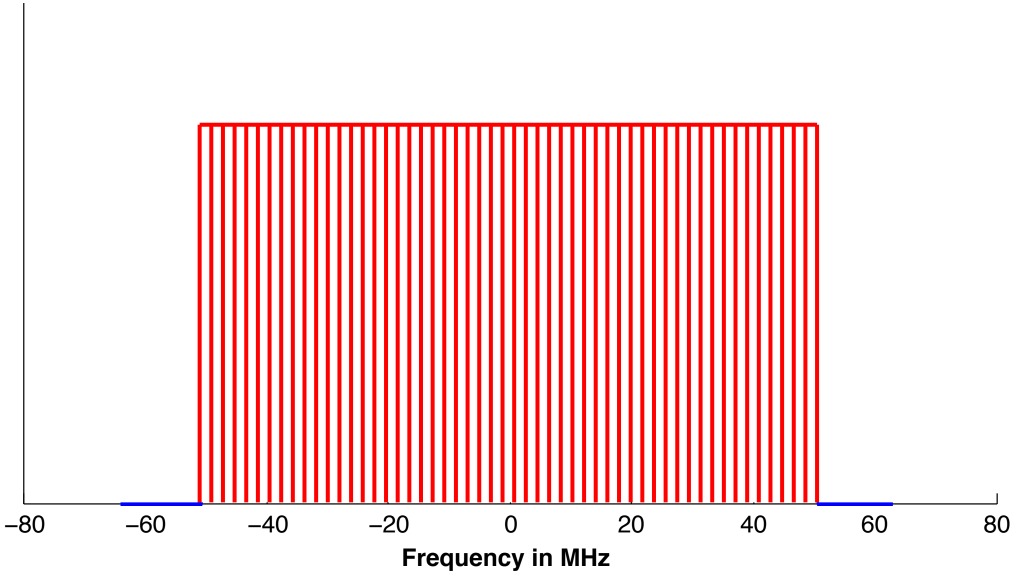




Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

- Divide spectrum into many narrow bands



$$x(t) = \sum_i s_i[n] \mathrm{e}^{-j2\pi f_i t}$$

- Transmit symbols on different carriers in narrow bands

- Channel is Flat → No need to worry about ISI

Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

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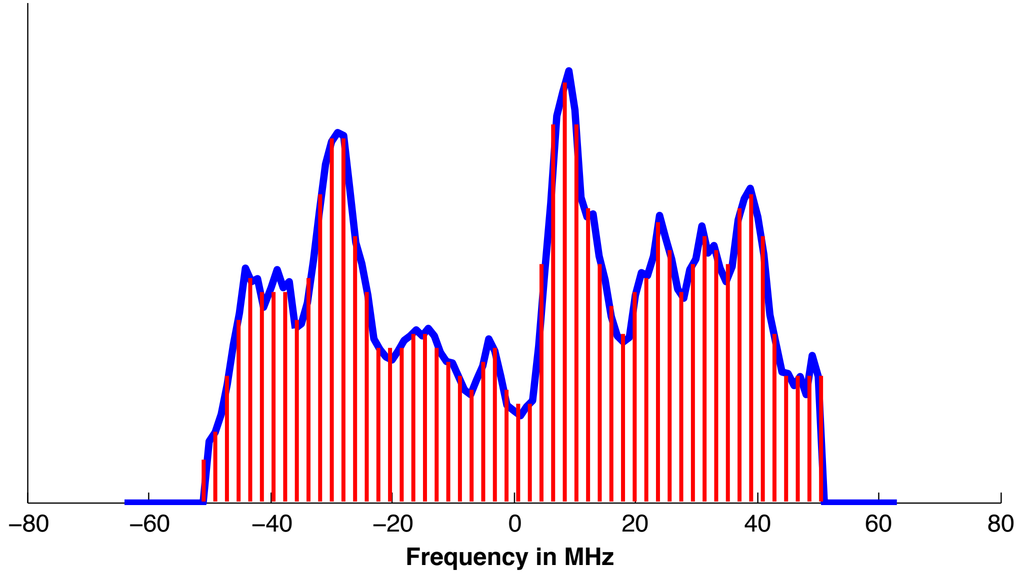

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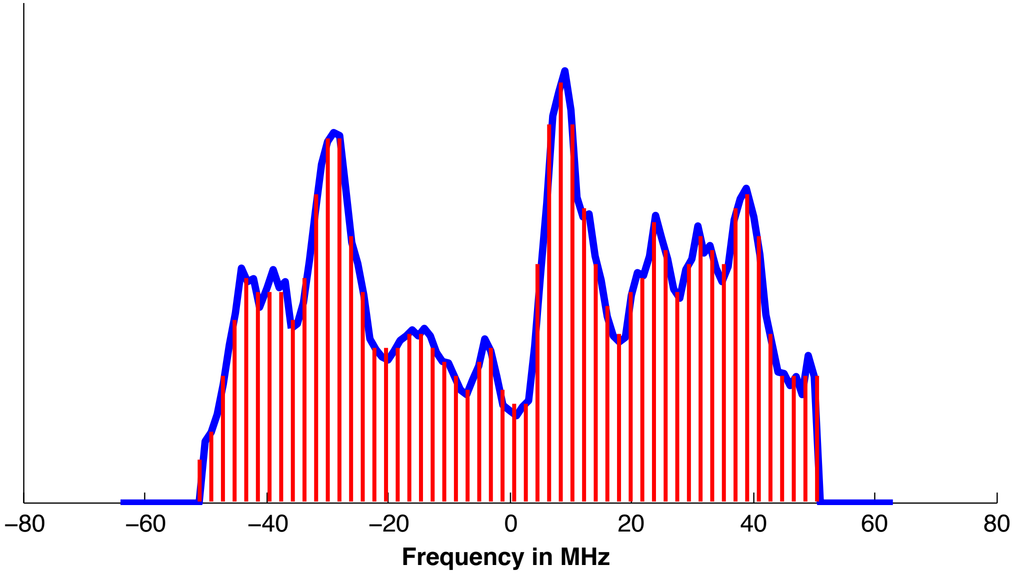

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Not That Simple!

Multi-Carrier Modulation

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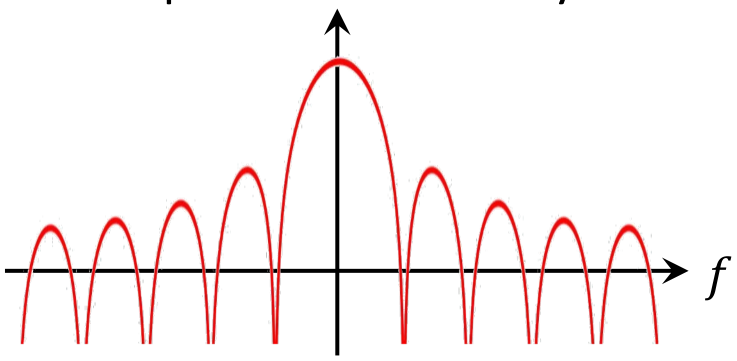
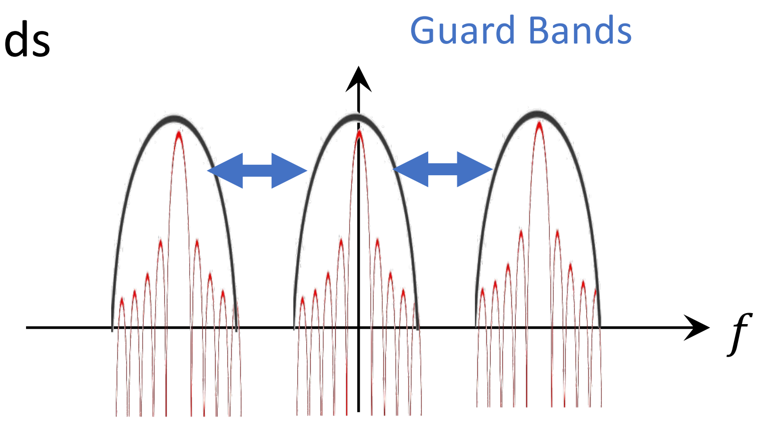




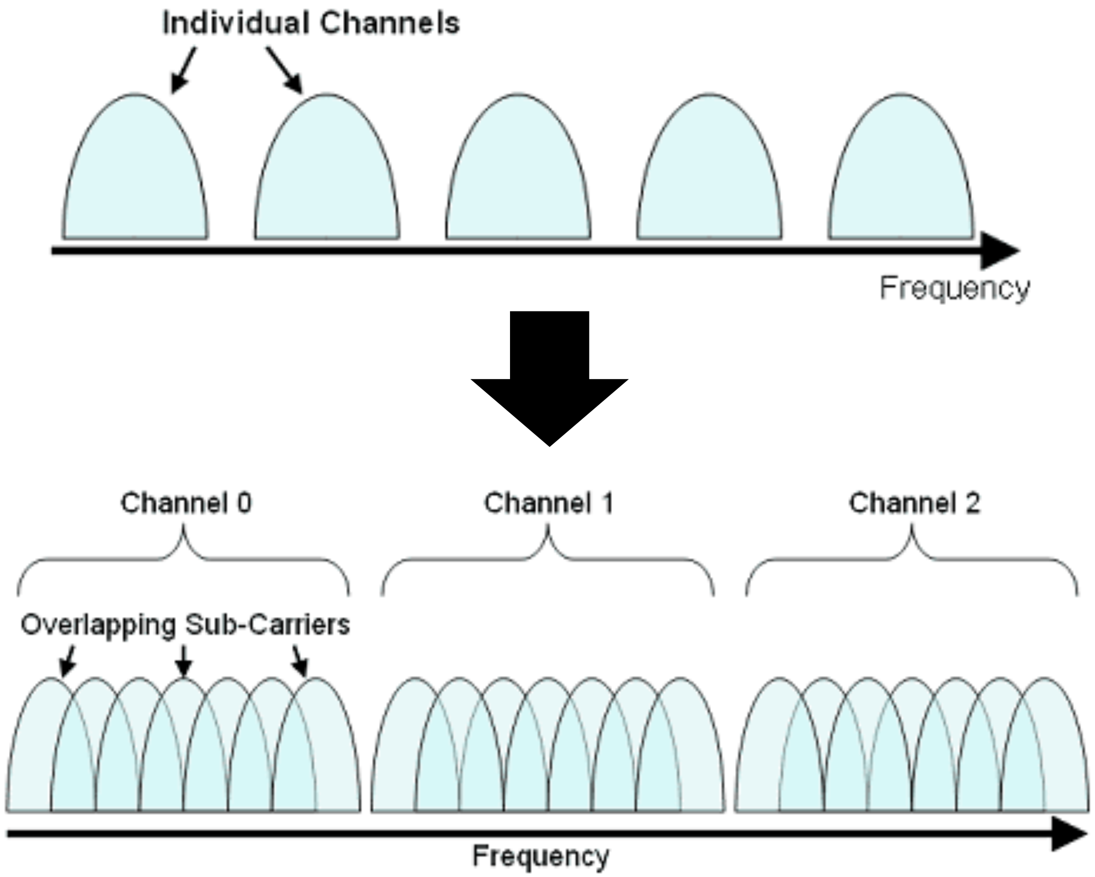
- Significant Leakage between adjacent subcarriers
- Need Guard Bands → Very inefficient!

Solution: Make the Sub-Carriers Orthogonal

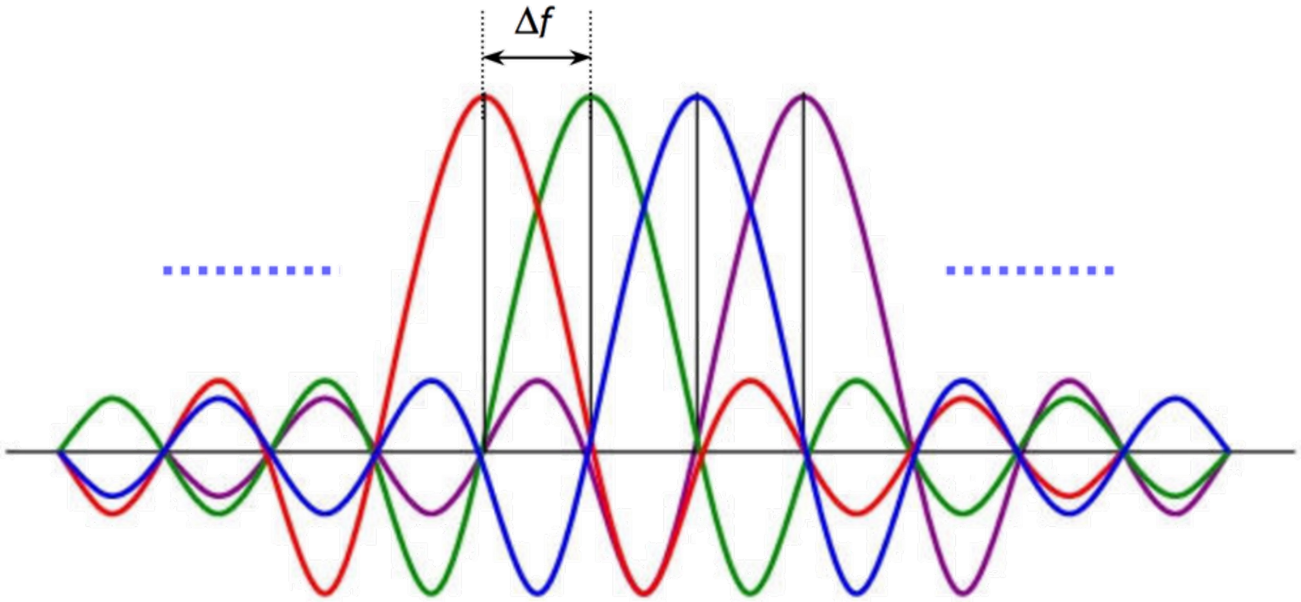
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

Make the Sub-Carriers Orthogonal



OFDM: Orthogonal Frequency Division Multiplexing

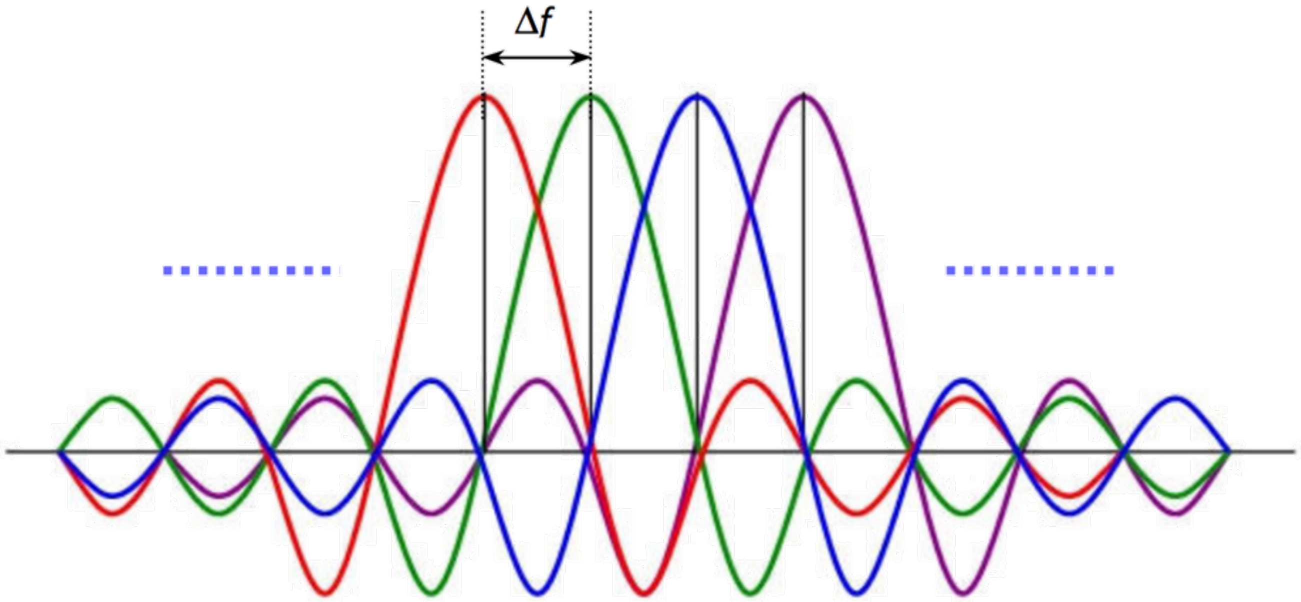


- Subcarriers are orthogonal: At the sub-carrier frequency, the sampled value has zero leakage from other subcarriers.

- Subcarrier separation can be very small, for N subcarriers and bandwidth B:

$$\Delta f = \frac{B}{N}$$

OFDM: Orthogonal Frequency Division Multiplexing



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How to Achieve This?

OFDM: Orthogonal Frequency Division Multiplexing

Use DFT: Discrete Fourier Transform

N-Point DFT: $X(f_i) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-j\frac{2\pi f_i t}{N}}$

N-Point IDFT: $x(t) = \sum_{f_i=0}^{N-1} X(f_i)e^{j\frac{2\pi f_i t}{N}}$

Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

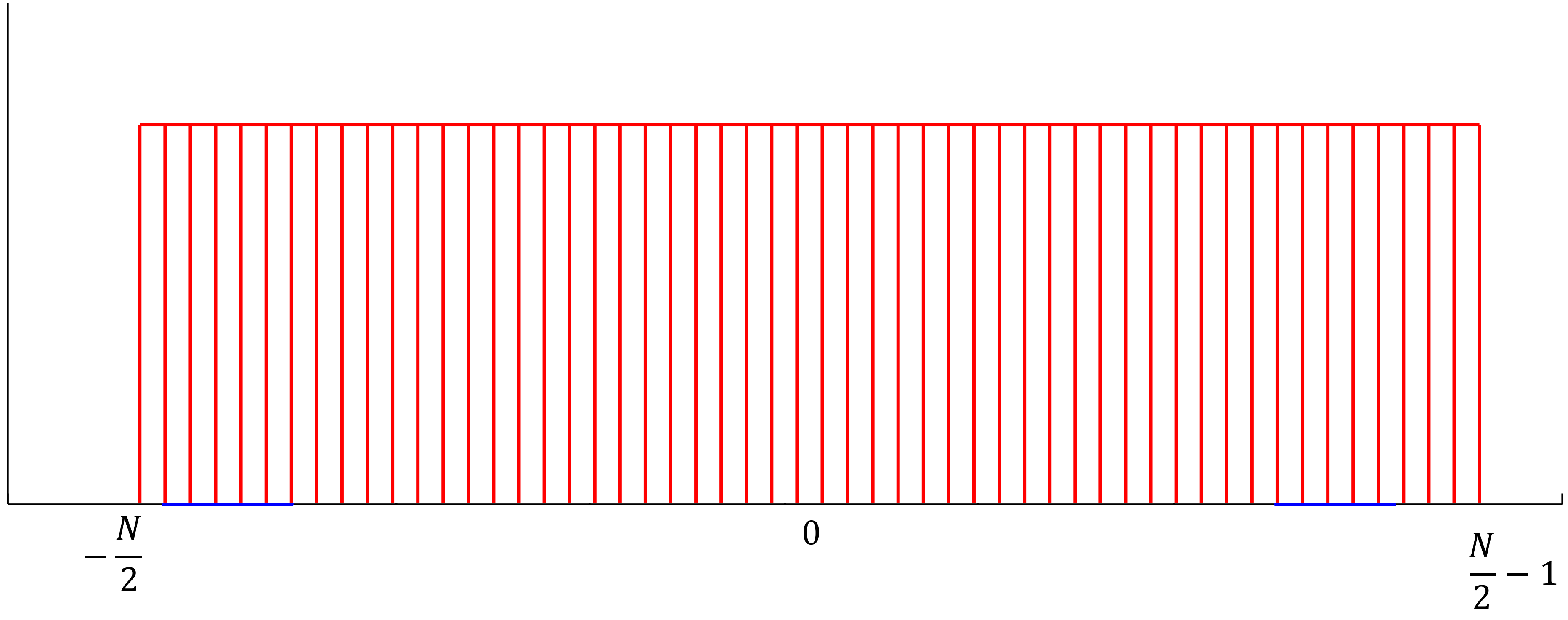
OFDM: Orthogonal Frequency Division Multiplexing

Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

- N subcarrier → IDFT of length N
- Symbols $s[n]$ can come from any modulation: BPSK, QPSK, QAM…
- $x(t)$ is complex → need I & Q → No point using PAM or ASK …
- OFDM Symbol: N samples of $x(t)$ generated from the same modulated symbols using IDFT.
- OFDM Symbol Time: $T = N/B$ where B is the bandwidth.
- OFDM Frequency Bin Width: $\Delta f = 1/T = B/N$

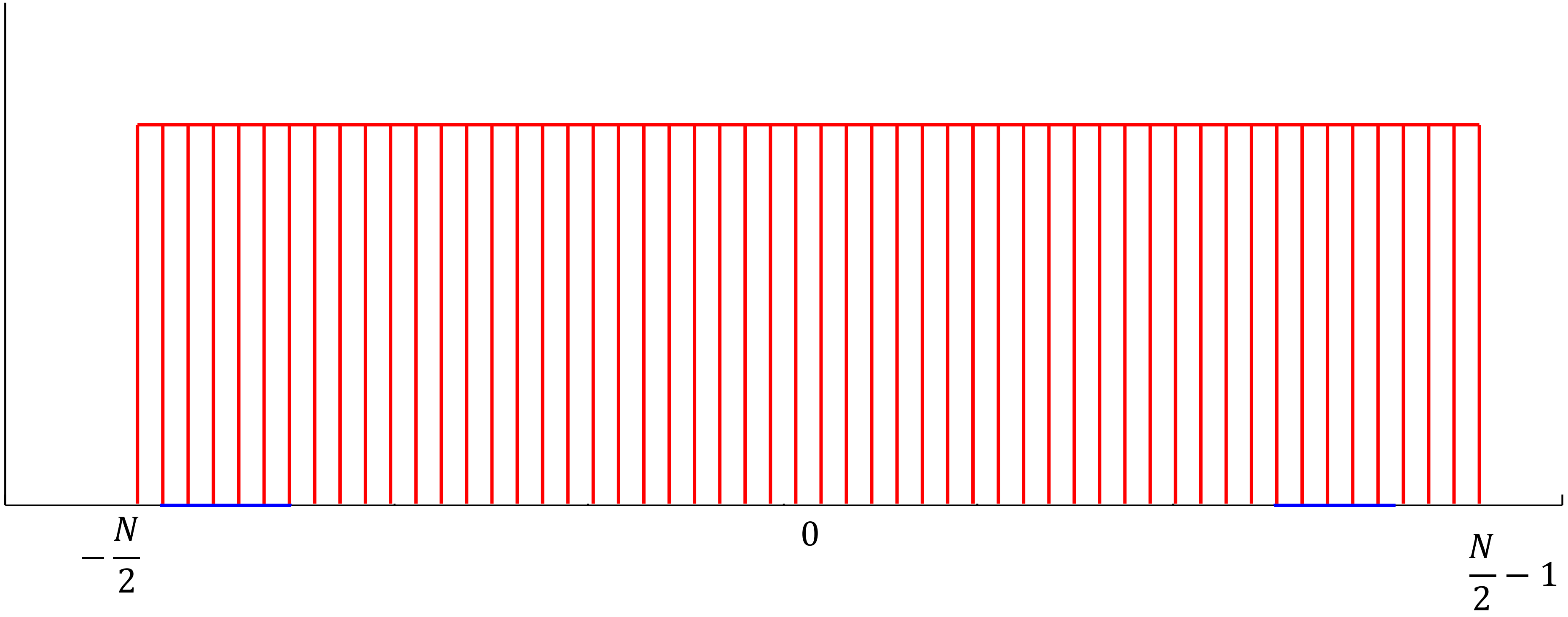
OFDM Symbol in Frequency Domain



- FFT can be represented 0 to $N-1$ or $N/2$ to $N/2\ -1$.
- OFDM Symbol created in digital baseband → 0 bin corresponds to DC

$$X(0)=\frac{1}{N}\sum_{t=0}^{N-1}x(t)e^{-j\frac{2\pi 0t}{N}}\ =\frac{1}{N}\sum_{t=0}^{N-1}x(t)\ =DC$$

OFDM Symbol in Frequency Domain



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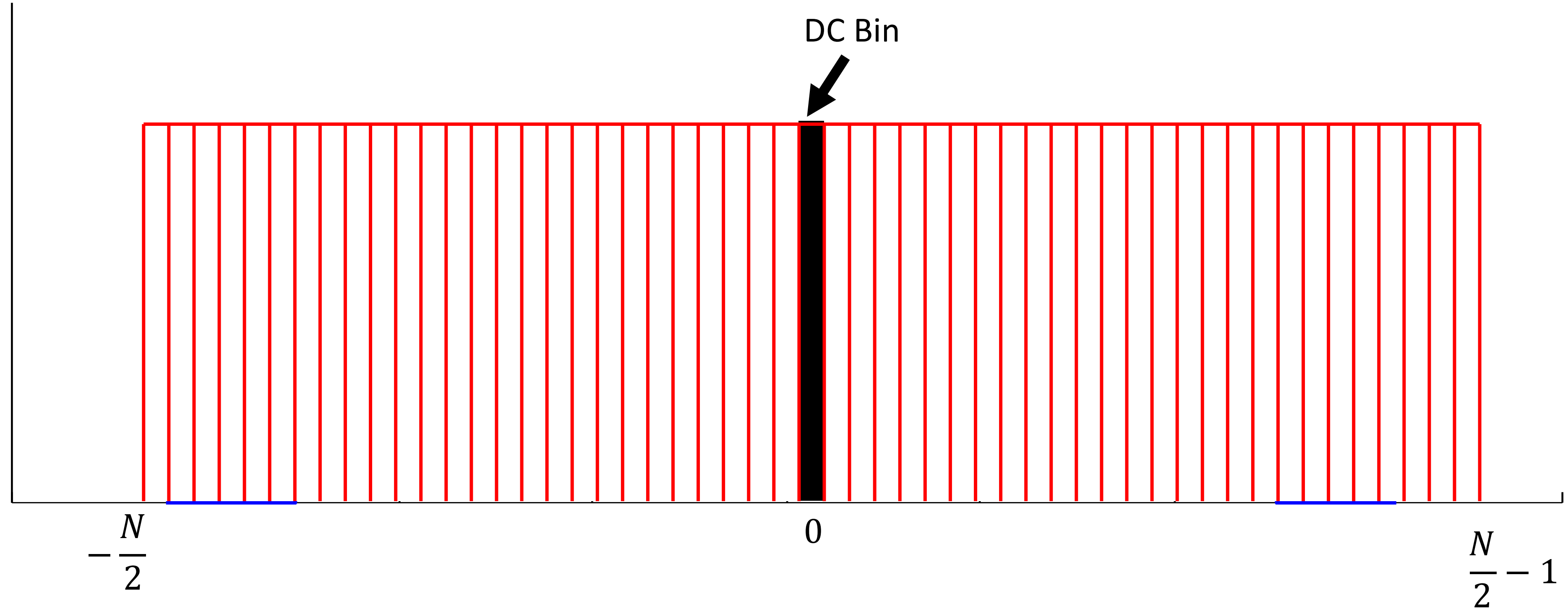
- DC of the circuits corrupts bits sent on the 0 bin → Do not use 0 bin

OFDM Symbol in Frequency Domain



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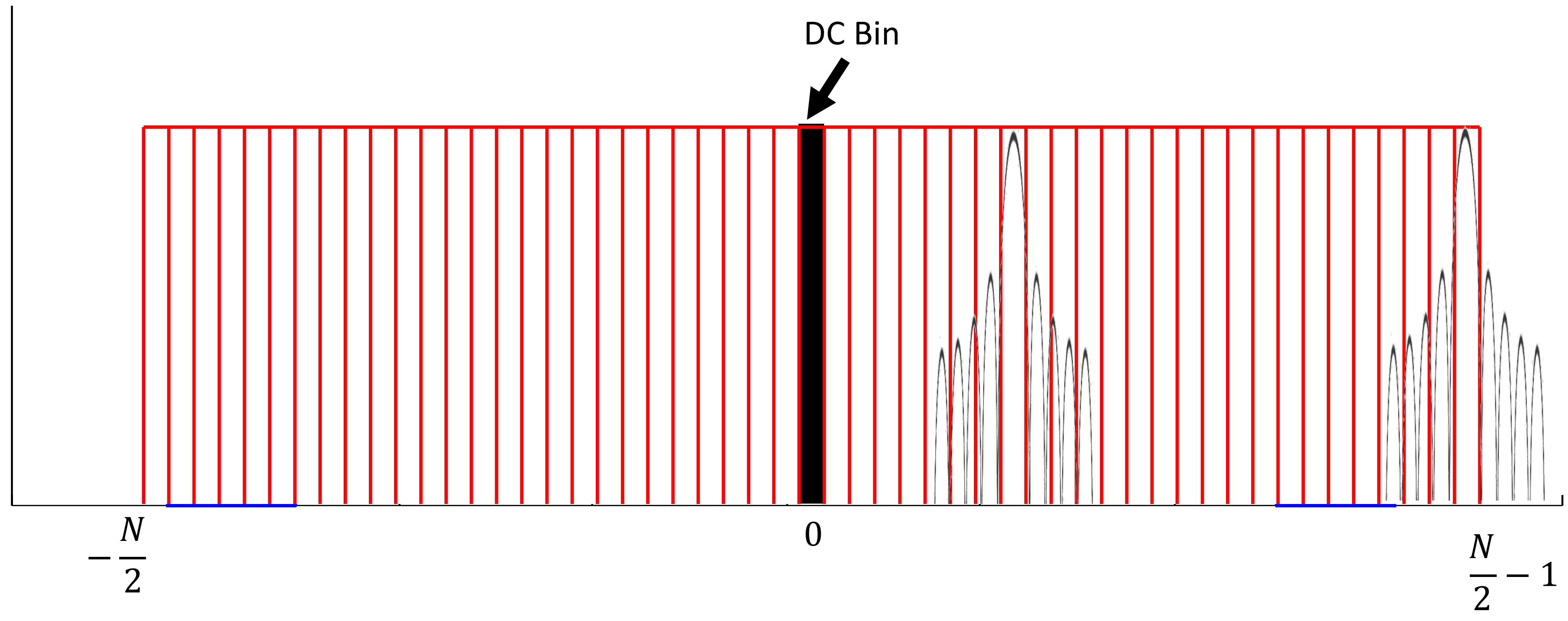
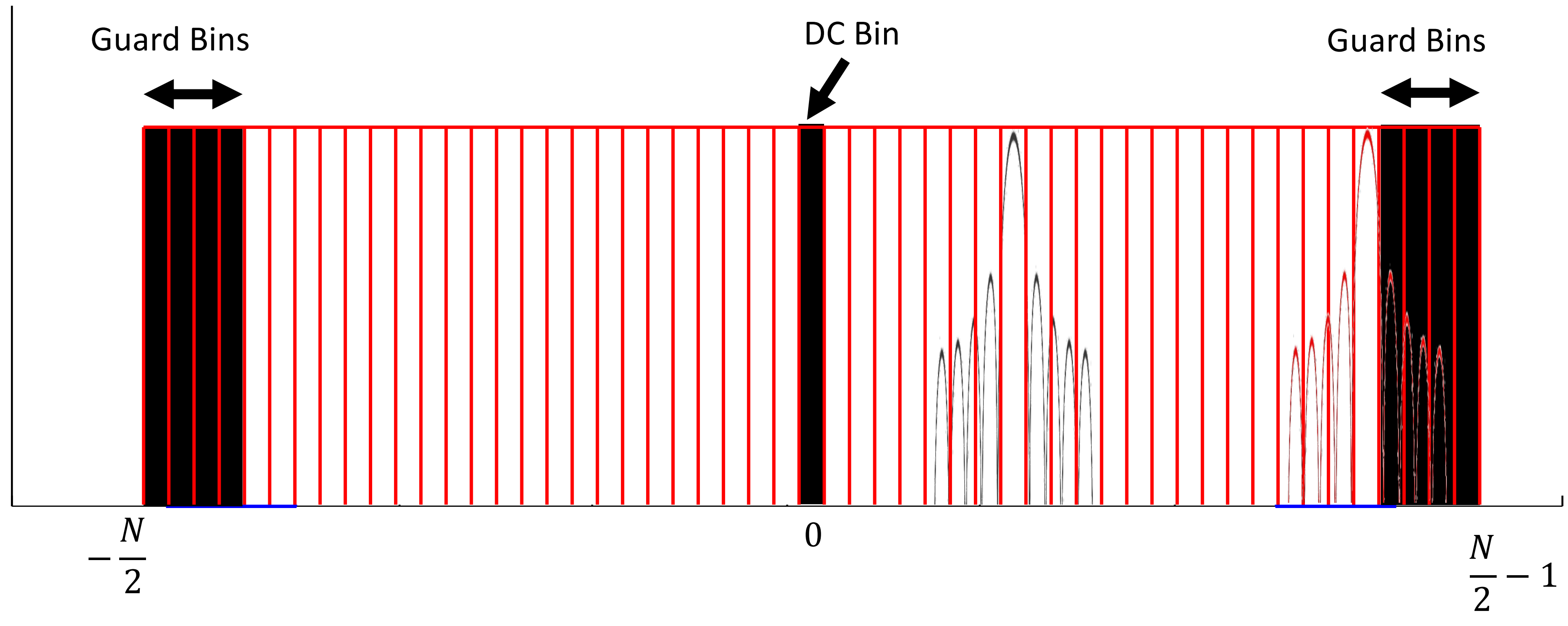


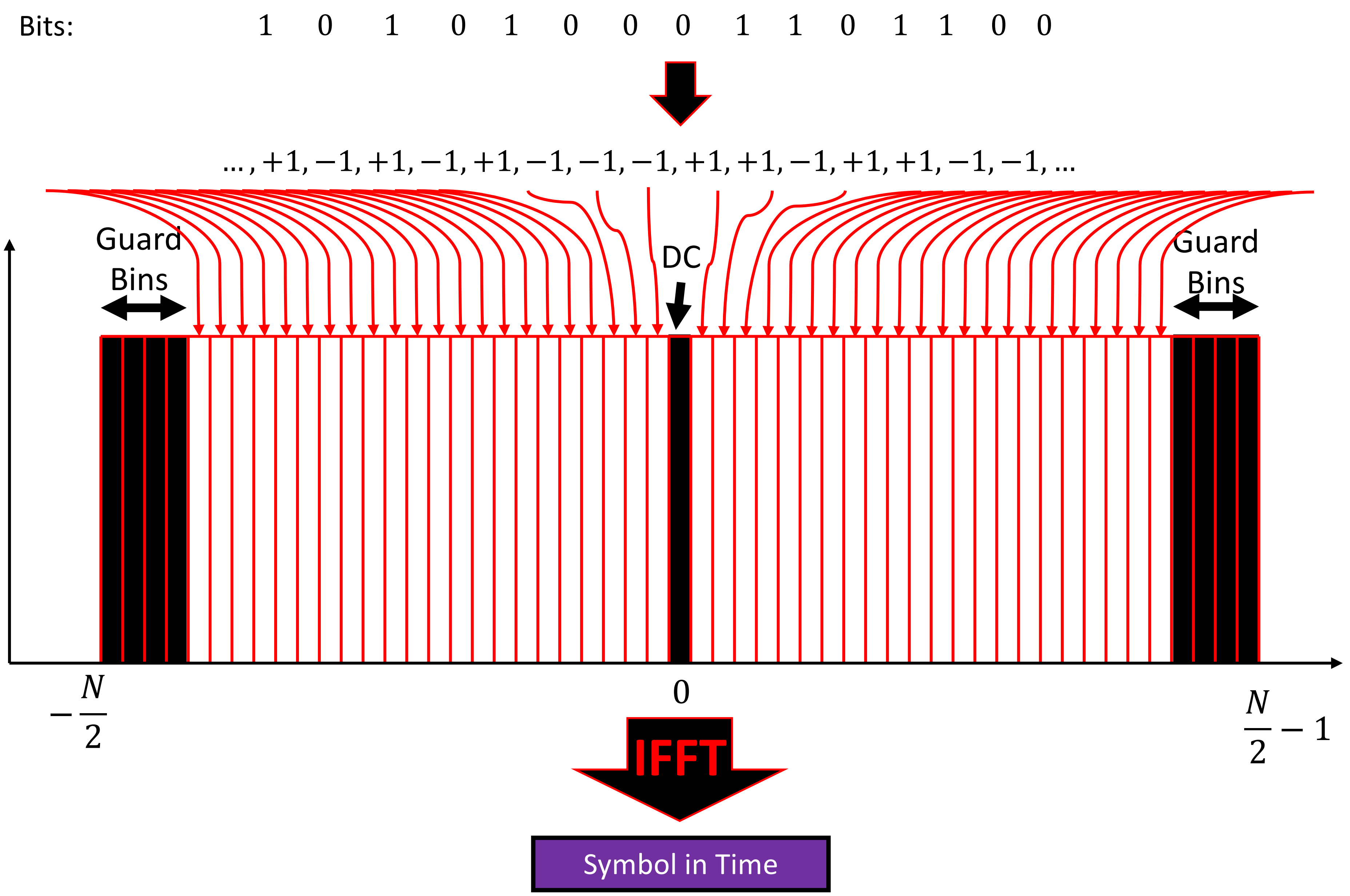
- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there

OFDM Symbol in Frequency Domain

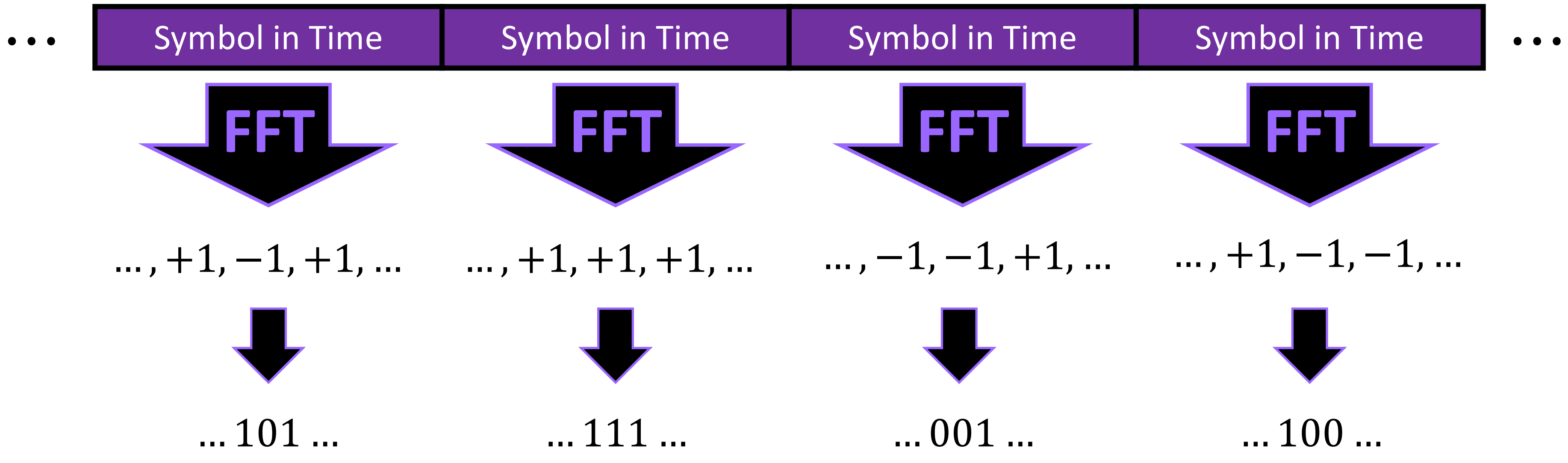


- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there
- Reduce Number of Guard band from N to 2 → Very Spectrally Efficient

OFDM Symbol

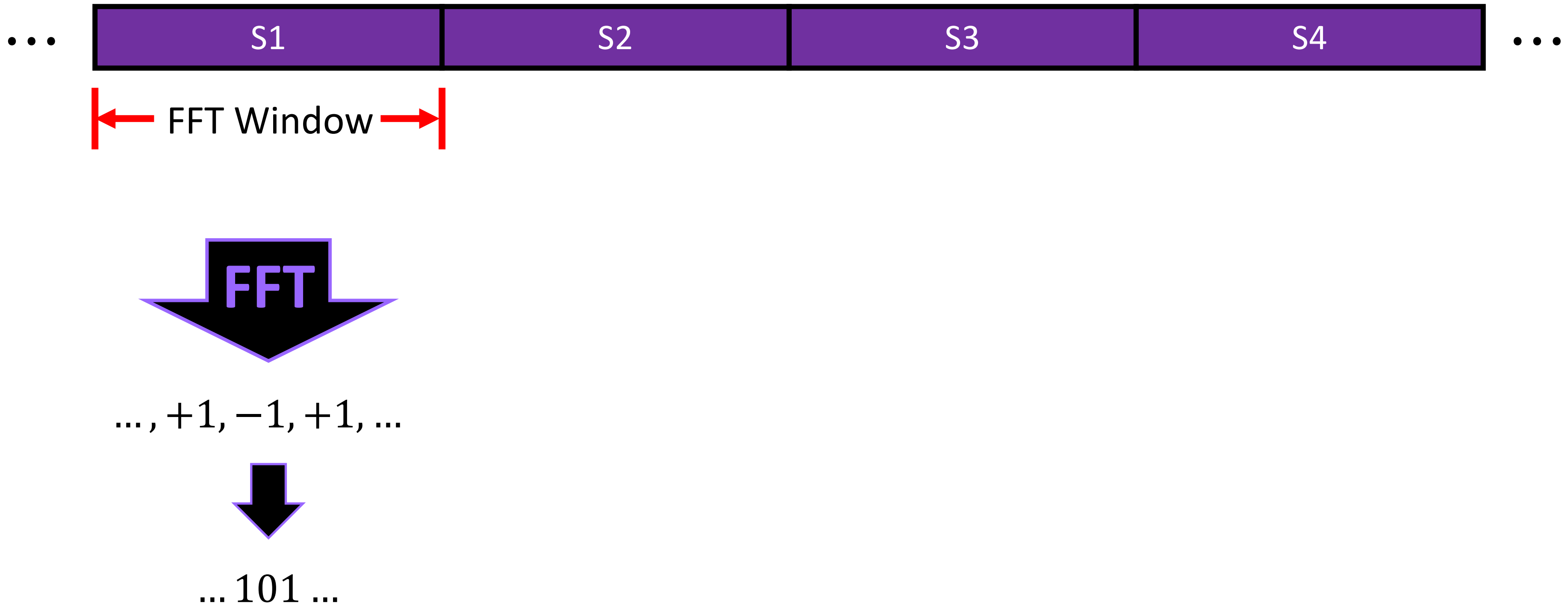

Bits:
1 0 1 0 1 0 0 0 1 1 0 1 1 0 0
…, +1, −1, +1, −1, +1, −1, −1, −1, +1, +1, −1, +1, +1, −1, −1, …
Guard Bins
DC
Guard Bins
$-\frac{N}{2}$
0
$\frac{N}{2}-1$
IFFT
Symbol in Time

OFDM Symbol



Not That Simple

OFDM Symbol



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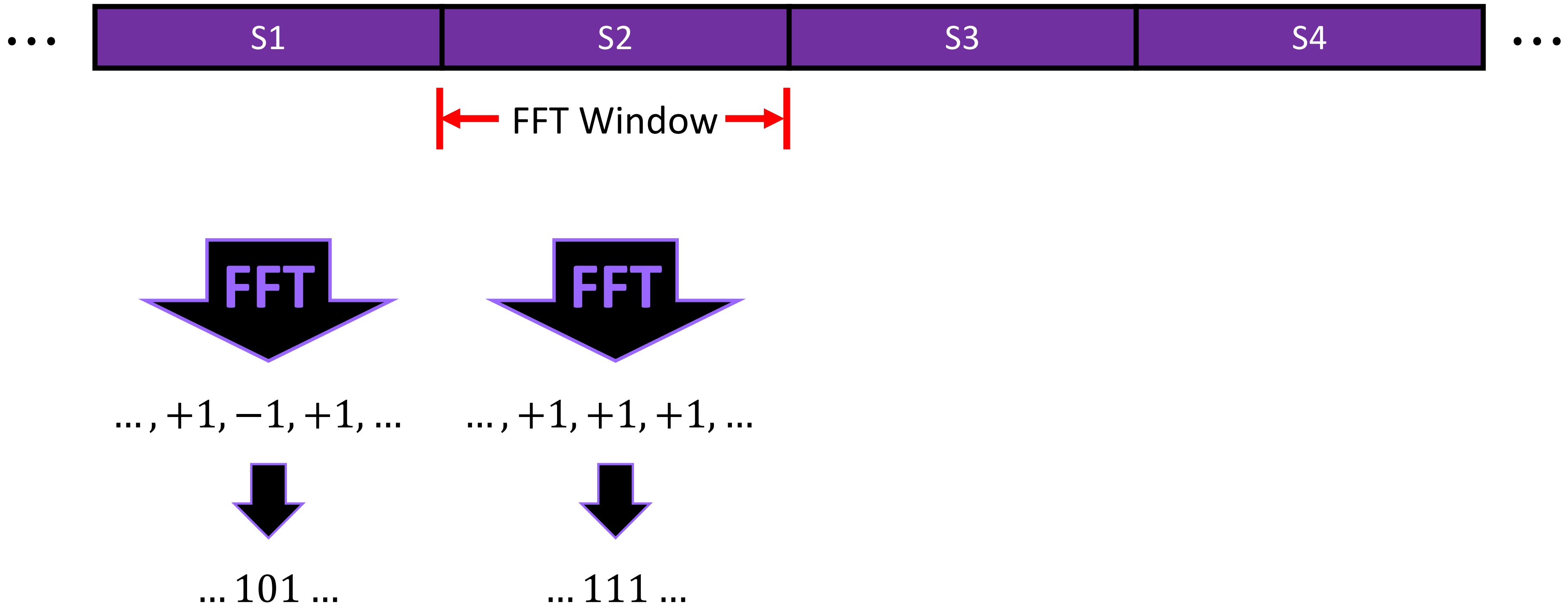
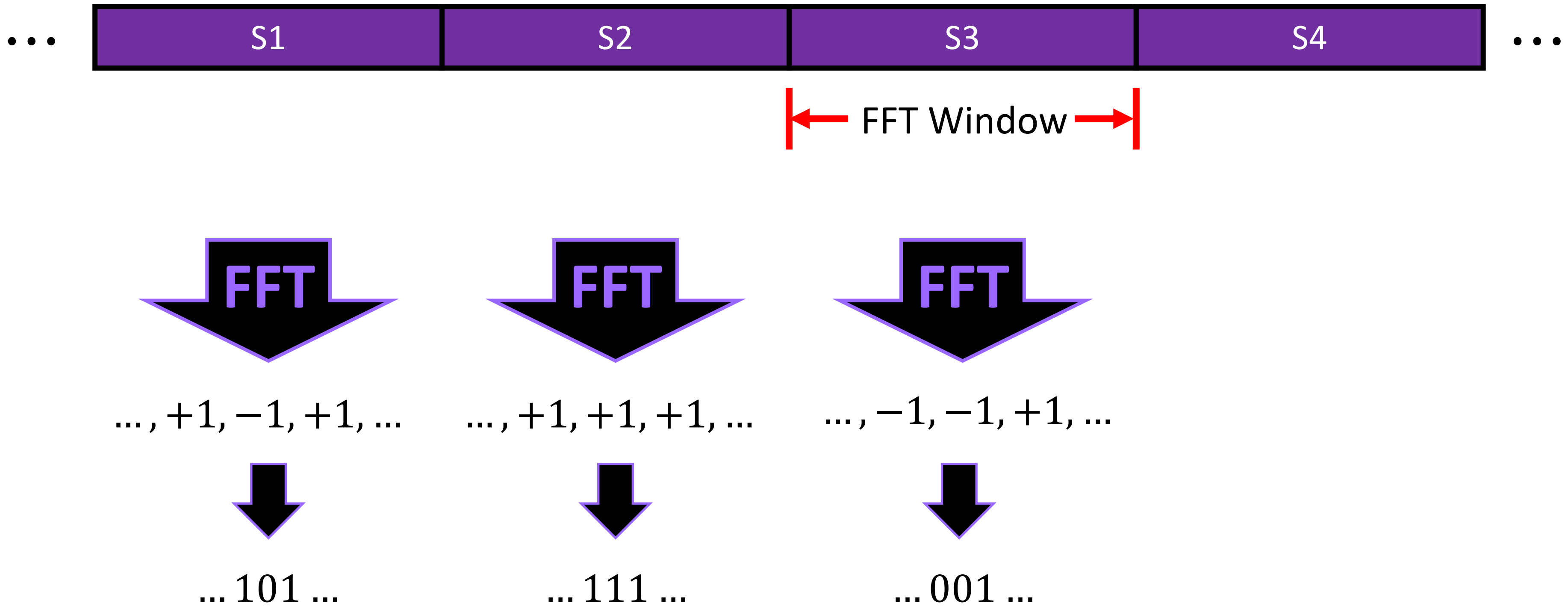
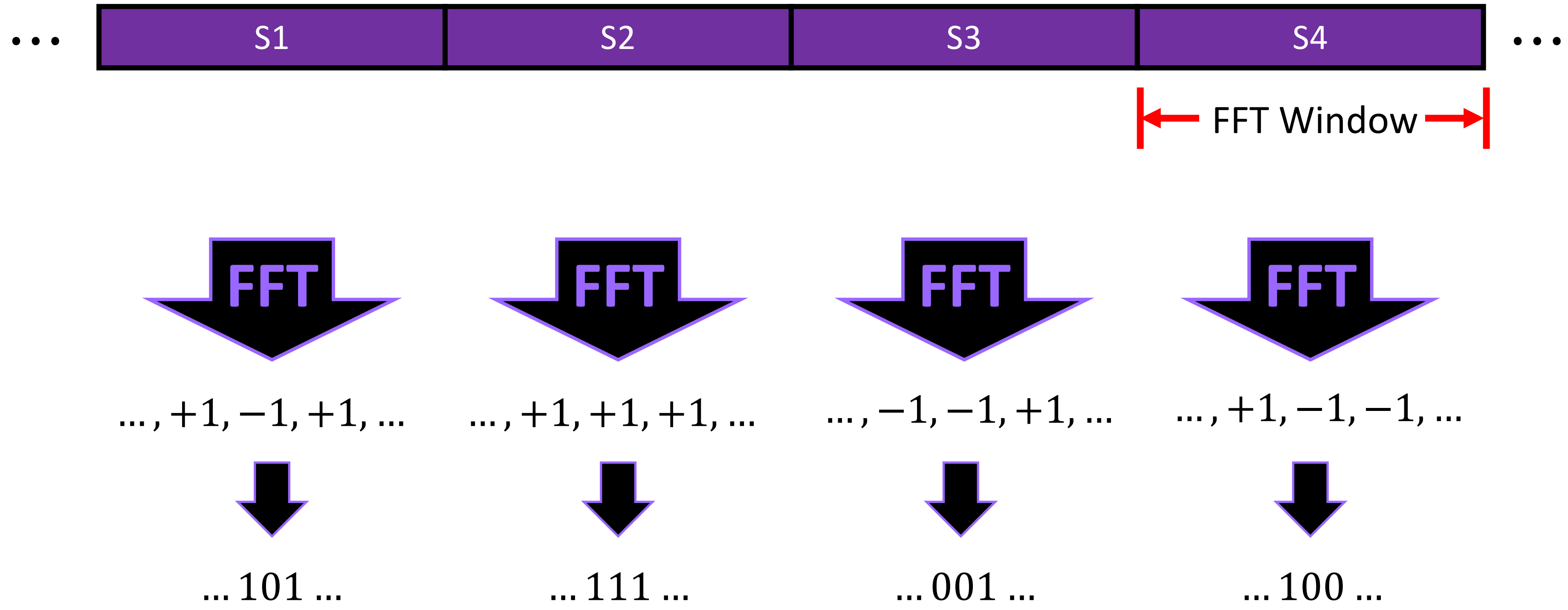


OFDM Symbol

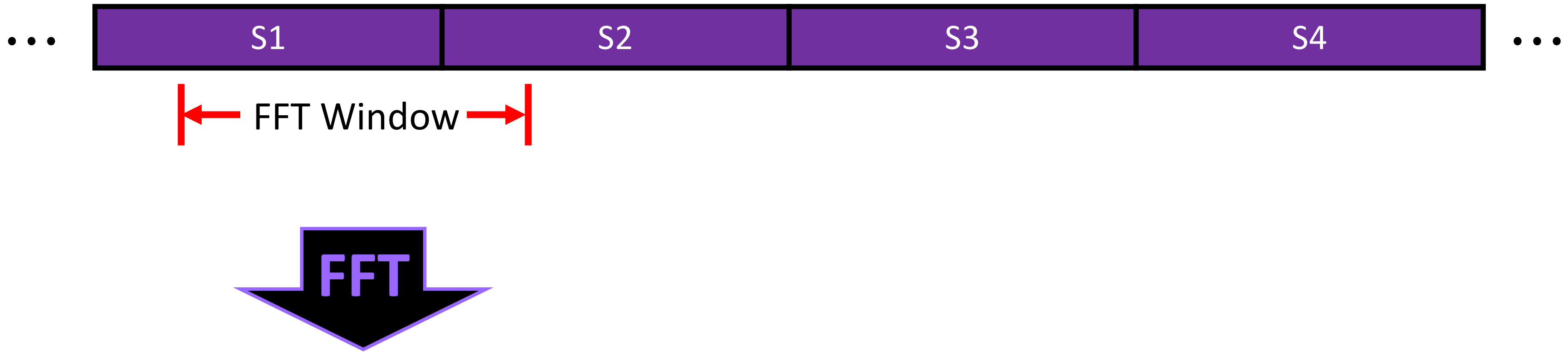


OFDM Symbol



Assumes FFT window is perfectly aligned with symbol boundaries

OFDM Symbol



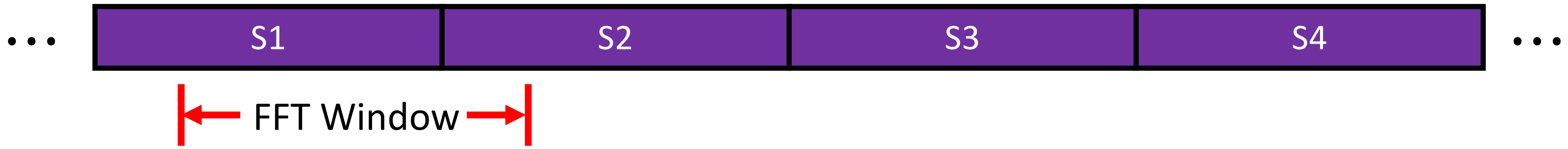
$\ldots, +0.5 + 1i, -0.7 + 0.3i, \ldots$

❌ Cannot decode!

FFT window is misaligned with symbol

Subcarriers are no longer orthogonal.

OFDM Cyclic Prefix



- DFT (FFT) assumes time samples are periodic of period N

- Circular Shift before taking FFT:

$$x[t] \rightarrow X[f]$$

$$x[t - \tau \bmod N] \rightarrow X[f]e^{-j\frac{2\pi f \tau}{N}}$$

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OFDM Cyclic Prefix

- Even if FFT window is misaligned, CP ensures that all samples come from the same symbol → Orthogonality is preserved!

- Cyclic Prefix can be created by:
  - Take first few samples and append them to end of symbol.
  - Take last few samples and prefix them to beginning of symbol.

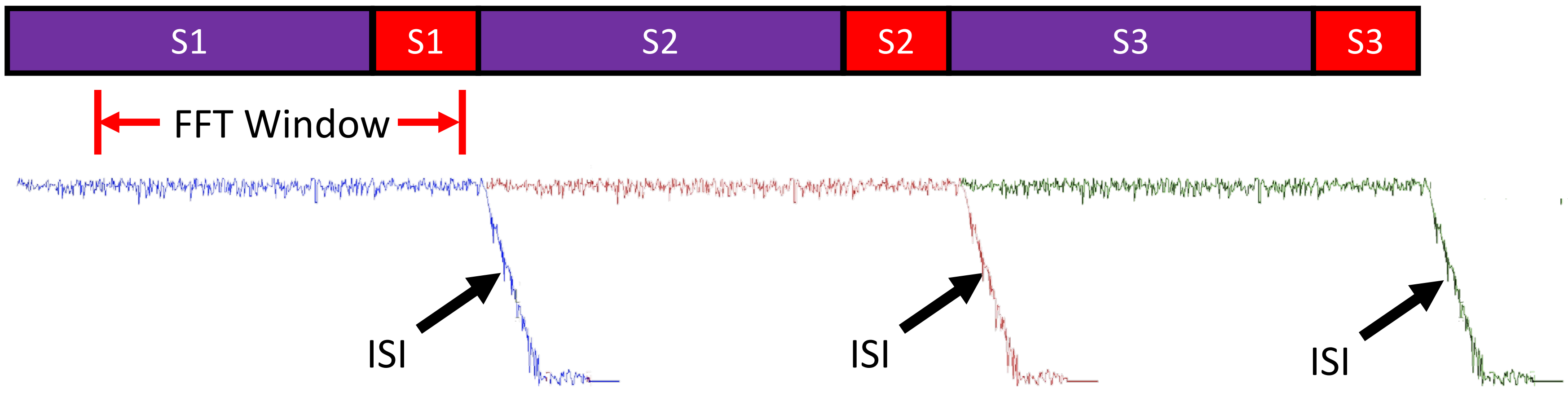
- Simple Phase Shift → Can be corrected by lumping with channel $H[f]$

OFDM Cyclic Prefix

Cyclic Prefix:

- Preserves orthogonality by allowing some misalignment in FFT Window

- Deals with Inter-Symbol-Interference



OFDM Cyclic Prefix

Cyclic Prefix:

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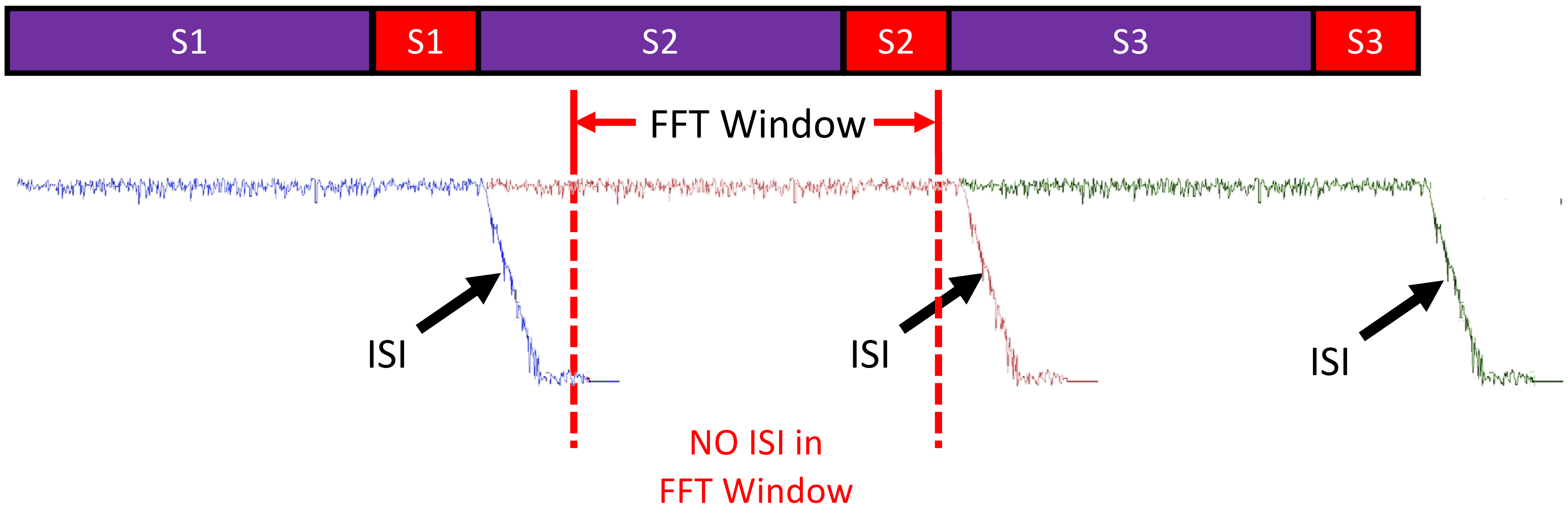

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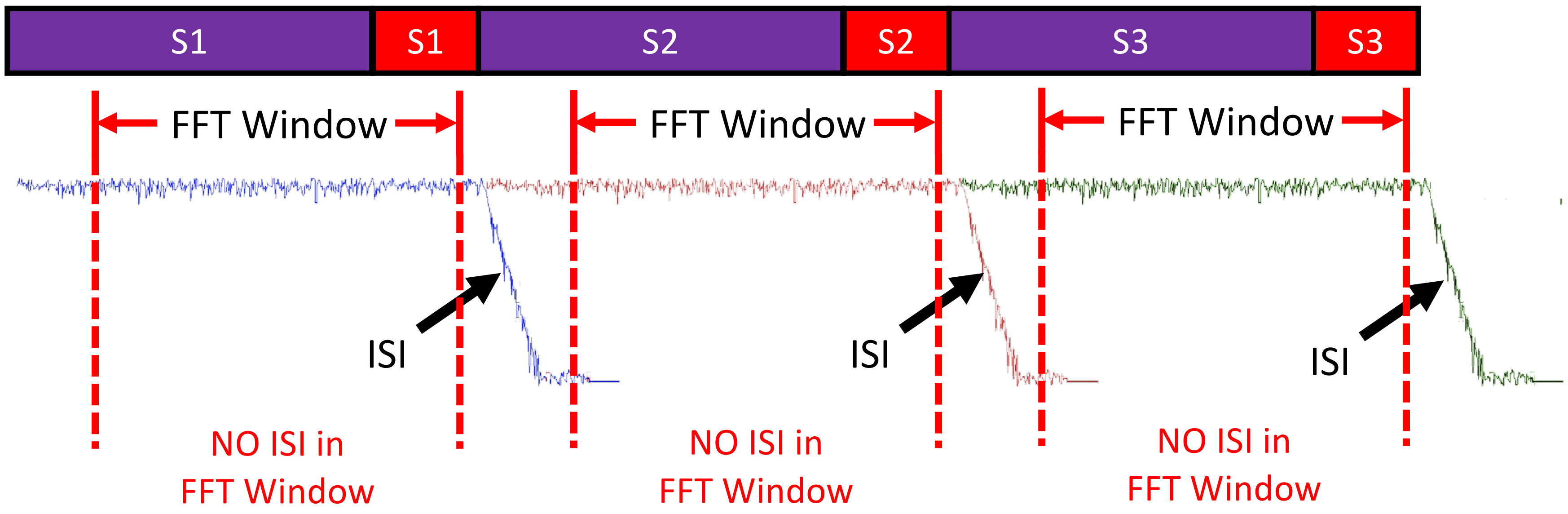


OFDM Cyclic Prefix

Cyclic Prefix:

\+ Preserves orthogonality by allowing some misalignment in FFT Window

\+ Deals with Inter-Symbol-Interference

− Overhead: Send $CP + N$ samples for every N samples

$$\text{Overhead} = \frac{CP}{CP + N}$$

$$\text{e.g. WiFi 802.11n: } N = 64, \text{CP} = 16 \rightarrow \text{Overhead} = 20\%$$

$$\text{e.g. LTE: } N = 1024, \text{CP} = 72 \rightarrow \text{Overhead} = 6.5\%$$

OFDM Cyclic Prefix

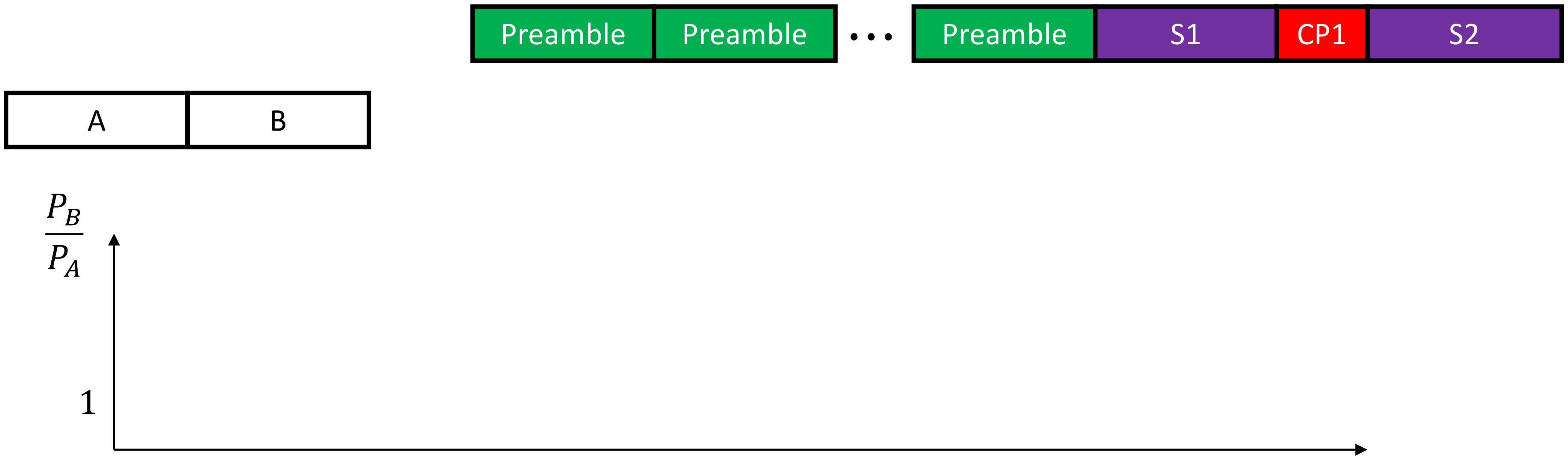
- Cyclic prefix is a not a bullet proof solution.
- Can still end up misaligned!
- Need a way to ensure we detect the beginning of the packet correctly.
- If we do, CP will ensure that even if we are not accurate, we can still decode.

OFDM Packet Detection

- Detect Beginning of packet to make sure we are within the CP
- Send Training Sequence: Preamble Symbols
- Preamble Symbols: Known Symbol Repeated at the beginning of packet

- No need for CP with preamble symbols

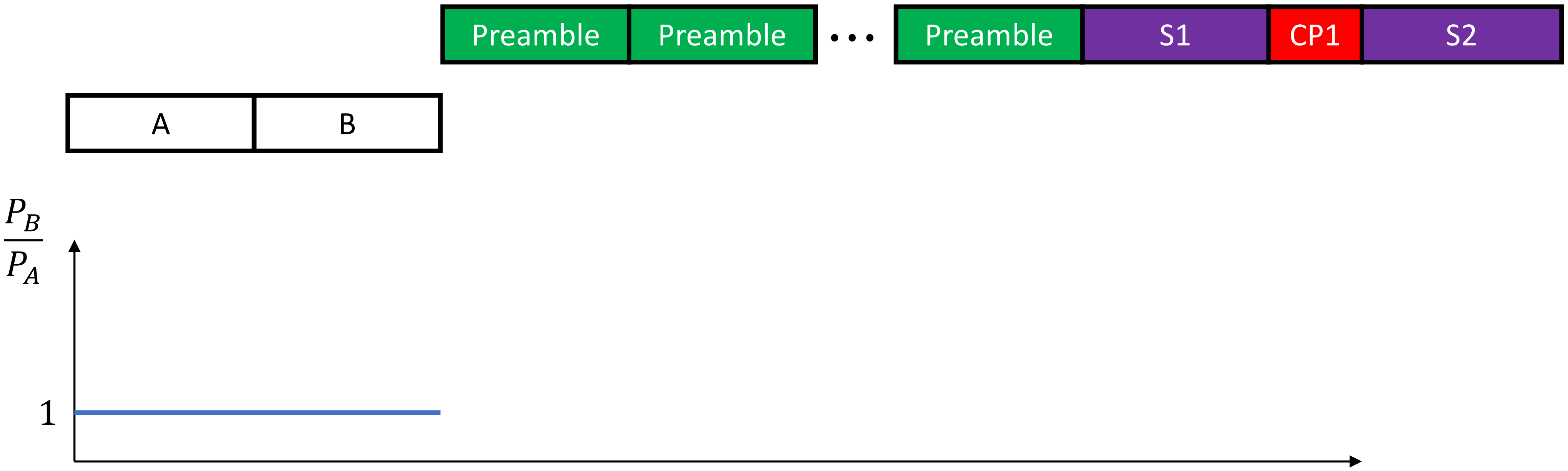
OFDM Packet Detection: Sliding Window



- Two windows of L $(2N)$ samples each.

- Compute: $\frac{P_B}{P_A} = \frac{\sum_{k=t+L}^{t+2L} |y[k]|^2}{\sum_{k=t}^{t+L} |y[k]|^2}$

OFDM Packet Detection: Sliding Window



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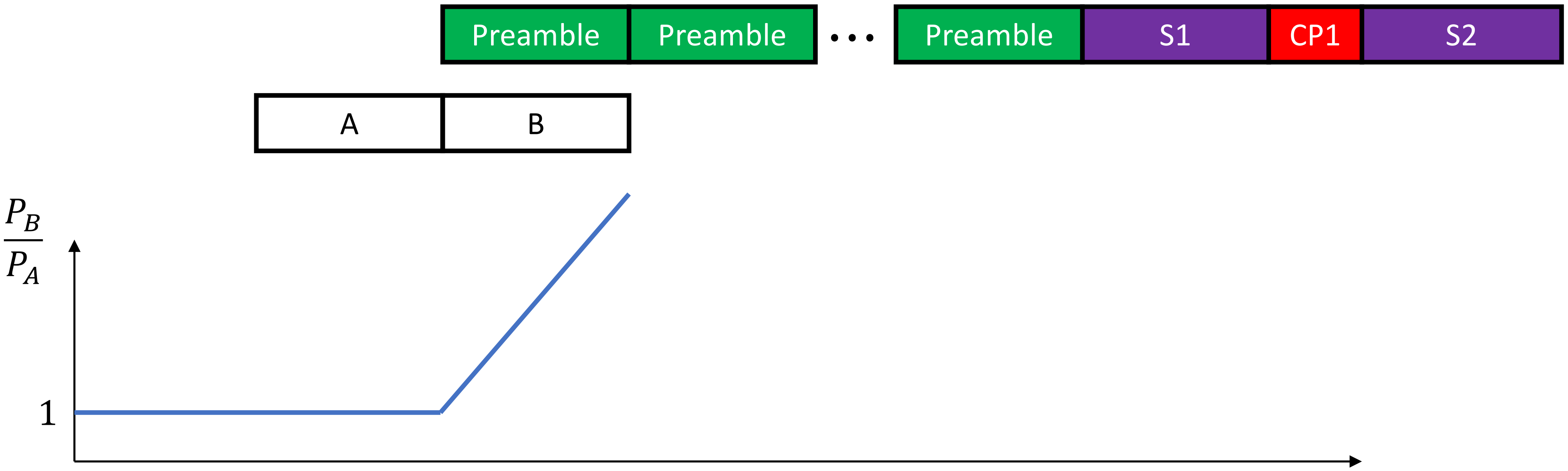

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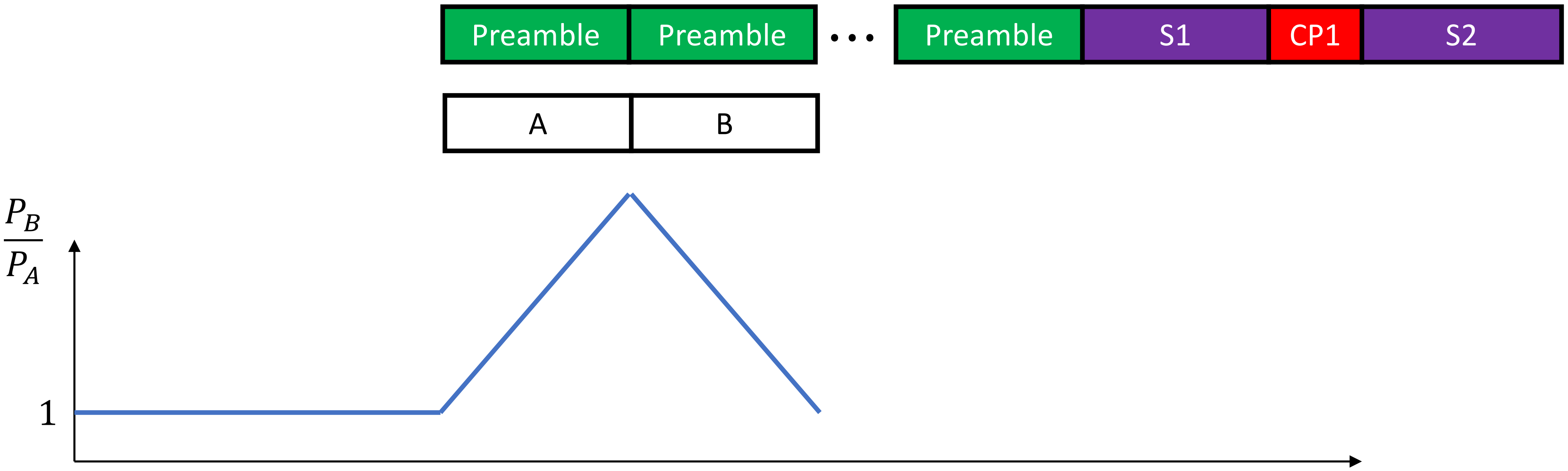

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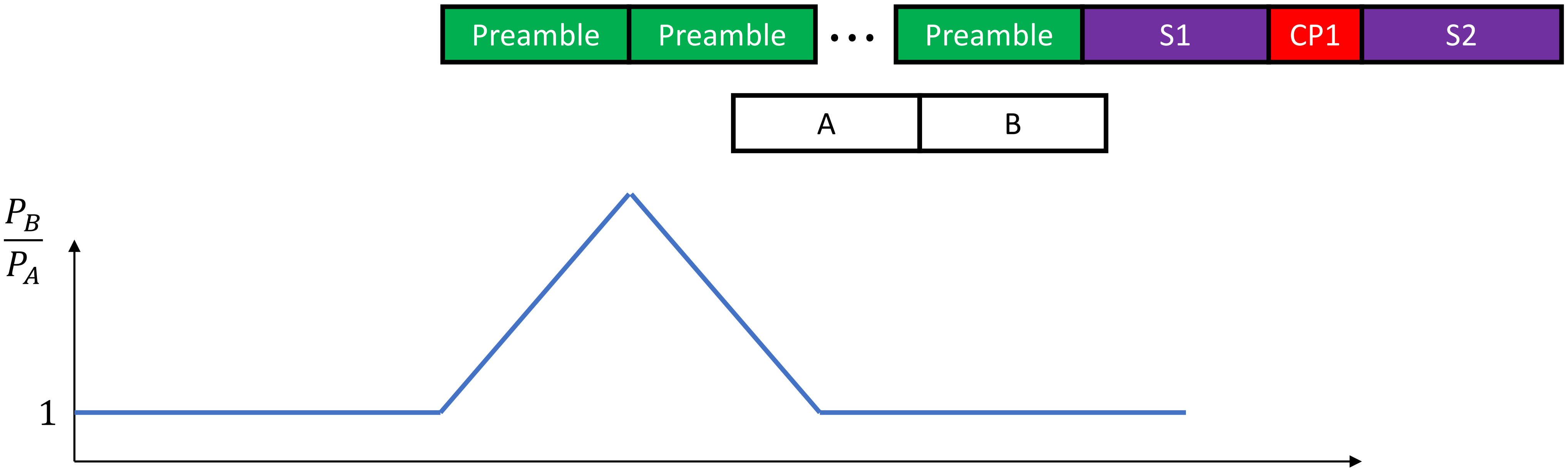

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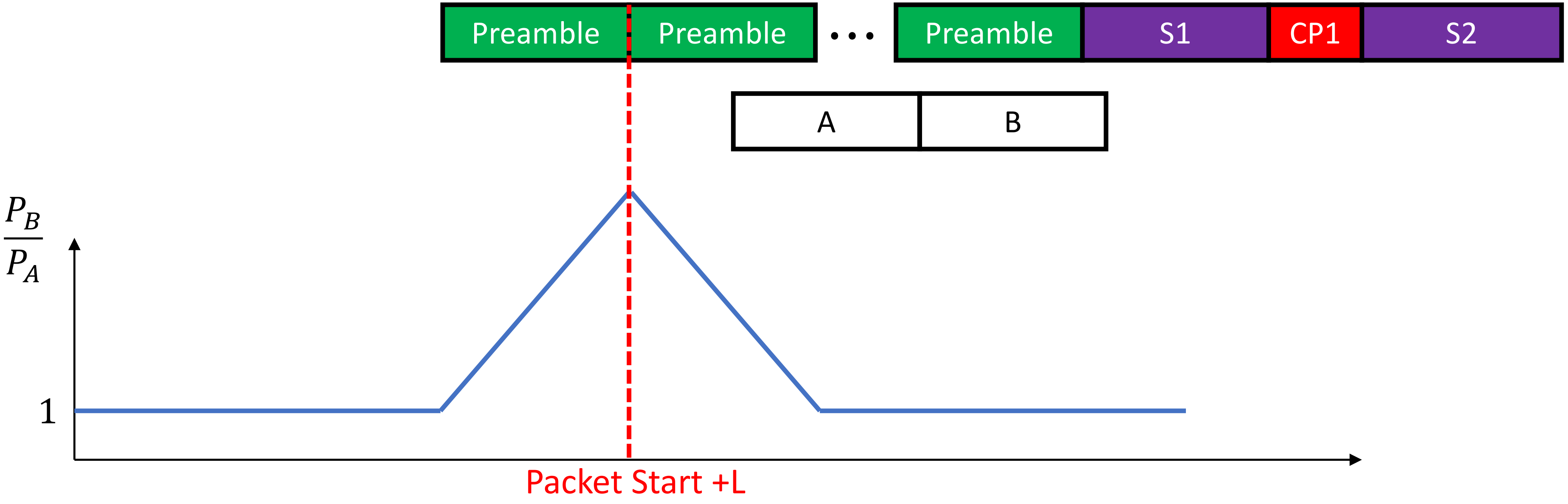

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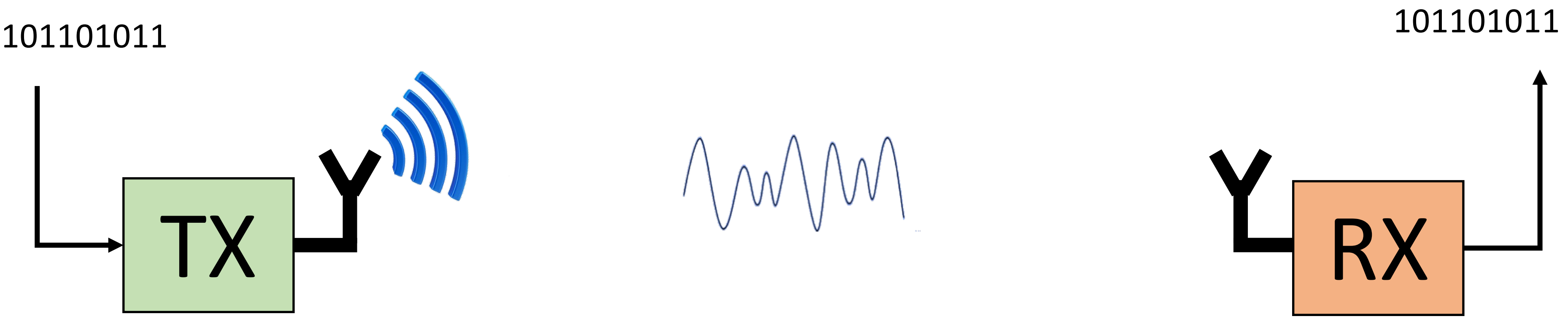
Carrier Frequency Offset



$x(t)$ ➡ $x(t)\times e^{-j2\pi f_c t}$ ➡ $h(t) * x(t)e^{-j2\pi f_c t}$ ➡ $h(t) * x(t)e^{-j2\pi f_c t}\times e^{j2\pi f_c t}$

➡ $h(t) * x(t)$

➡ $y(t) = h(t) * x(t) + v(t)$

Assumes TX & RX perfectly synched

Carrier Frequency Offset

$x(t)$ ➡ $x(t)\times e^{-j2\pi f_c t}$ ➡ $h(t) * \; x(t)e^{-j2\pi f_c t}$ ➡ $h(t) * \; x(t)e^{-j2\pi f_c t} \times e^{j2\pi f_c' t}$

TX & RX are not synched ➡ $h(t) * \; x(t) \; e^{-j2\pi \Delta f_c t}$

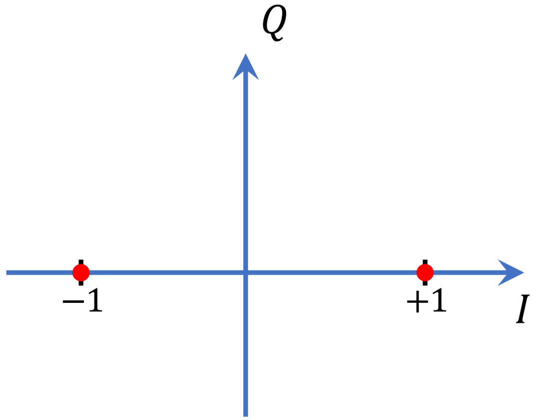
CFO: $\Delta f_c = f_c - f_c'$ ➡ $y(t) = h(t) * \; x(t)e^{-j2\pi \Delta f_c t} + v(t)$

Phase changes with time!

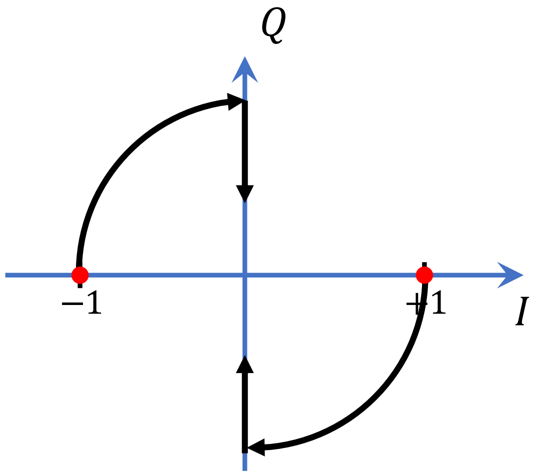
Carrier Frequency Offset

Consider BPSK Modulation.

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$



$h\, x(t-\tau) e^{-j2\pi\Delta f_c t} + v(t)$

Carrier Frequency Offset

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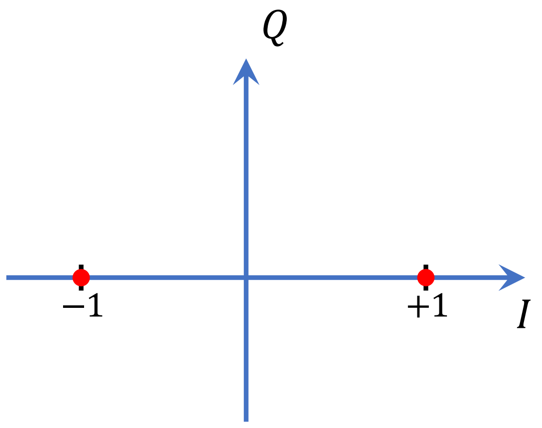
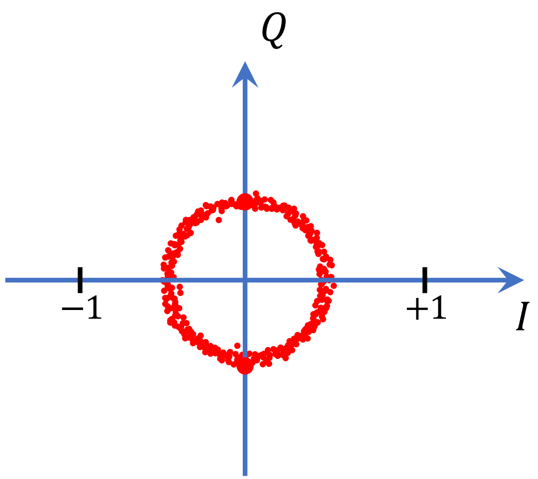


$x(t)$

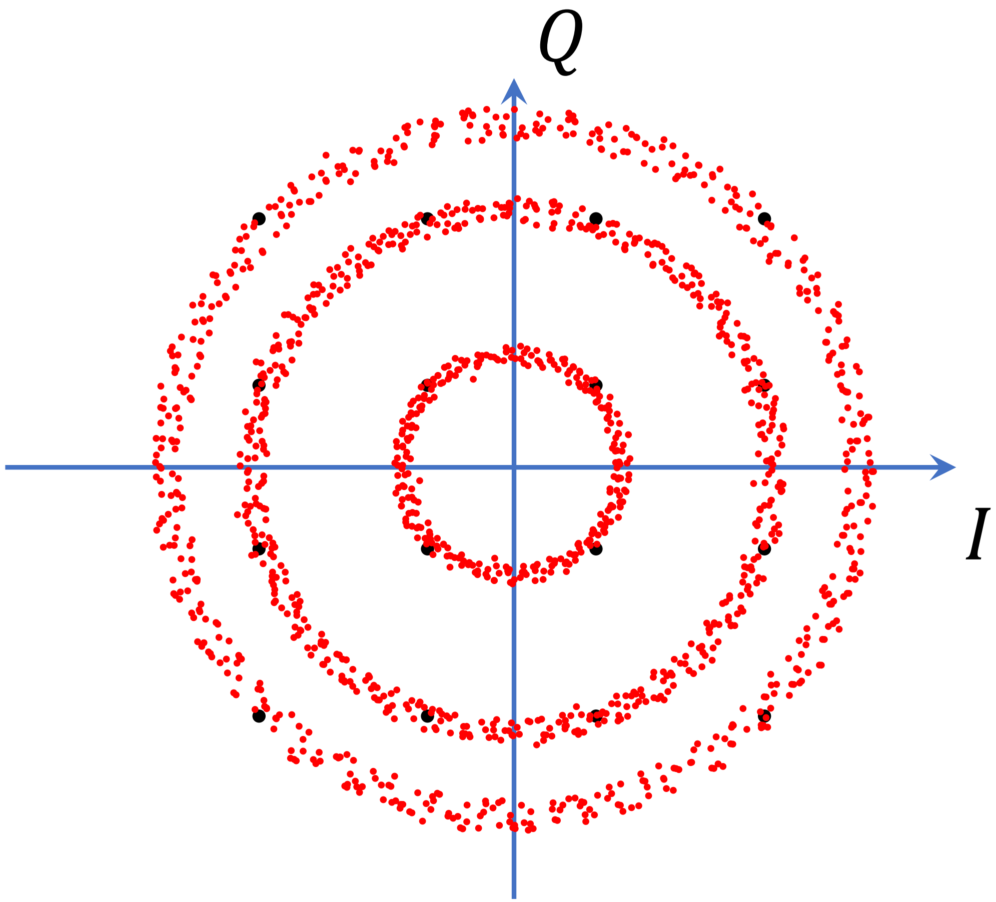


$h\, x(t-\tau)e^{-j2\pi\Delta f_c t} + v(t)$

Impossible to Decode!

Carrier Frequency Offset

Consider 16 QAM Modulation



Need to estimate and correct CFO to decode!

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1(t) = x(t)e^{-j2\pi\Delta f_c t}$$

Sample at $t = nT_s$
Symbol has N samples ➔ $T = NT_s$

$$y_2(t) = x(t)e^{-j2\pi\Delta f_c (t+T)}$$

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c nT_s}$$

Sample at $t = nT_s$
Symbol has N samples ➔ $T = NT_s$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)}$$

- Compute: $A = \sum_{n=1}^{N} y_1[n]y_2^*[n] = \sum_{n=1}^{N} x[n]x^*[n]e^{j2\pi\Delta f_c NT_s}$

$$= e^{j2\pi\Delta f_c NT_s}\sum_{n=1}^{N}|x[n]|^2 \quad \Rightarrow \quad \Delta f_c = \frac{\angle A}{2\pi NT_s}$$

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c n T_s}$$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)}$$

- Compute: $A = \sum_{n=1}^{N} y_1[n] y_2^*[n]$ $\Rightarrow$ $\Delta f_c = \frac{\angle A}{2\pi N T_s}$

- Correct CFO: $y[n] \times e^{j2\pi\Delta f_c n T_s}$