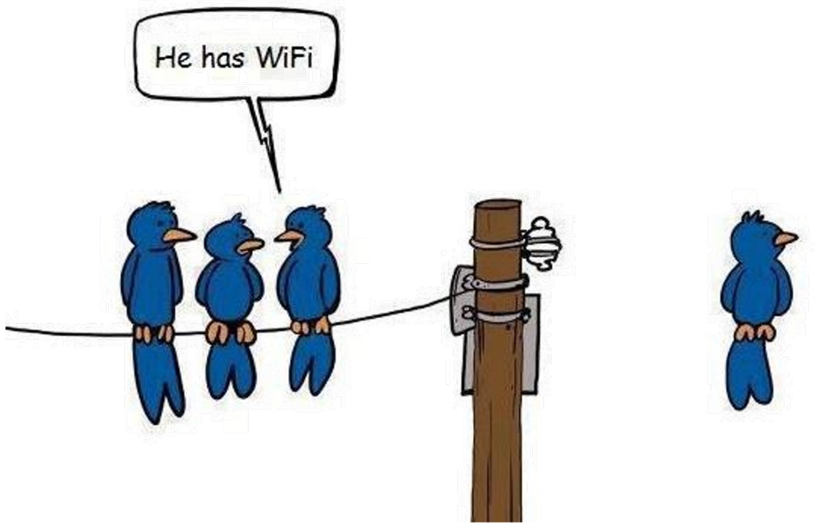
COM-405: Mobile Networks

Lecture 10.0: Wireless Localization

Haitham Hassanieh

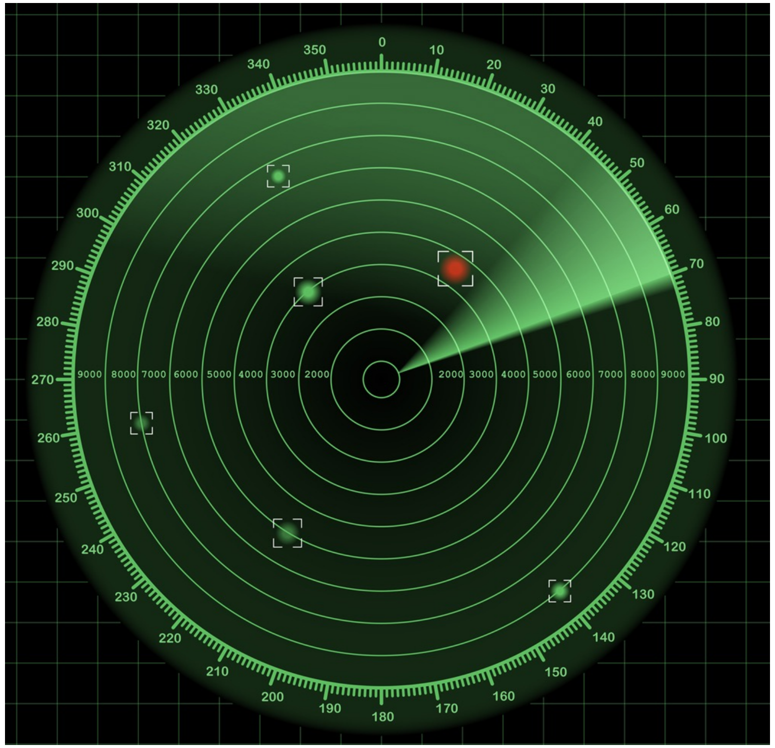


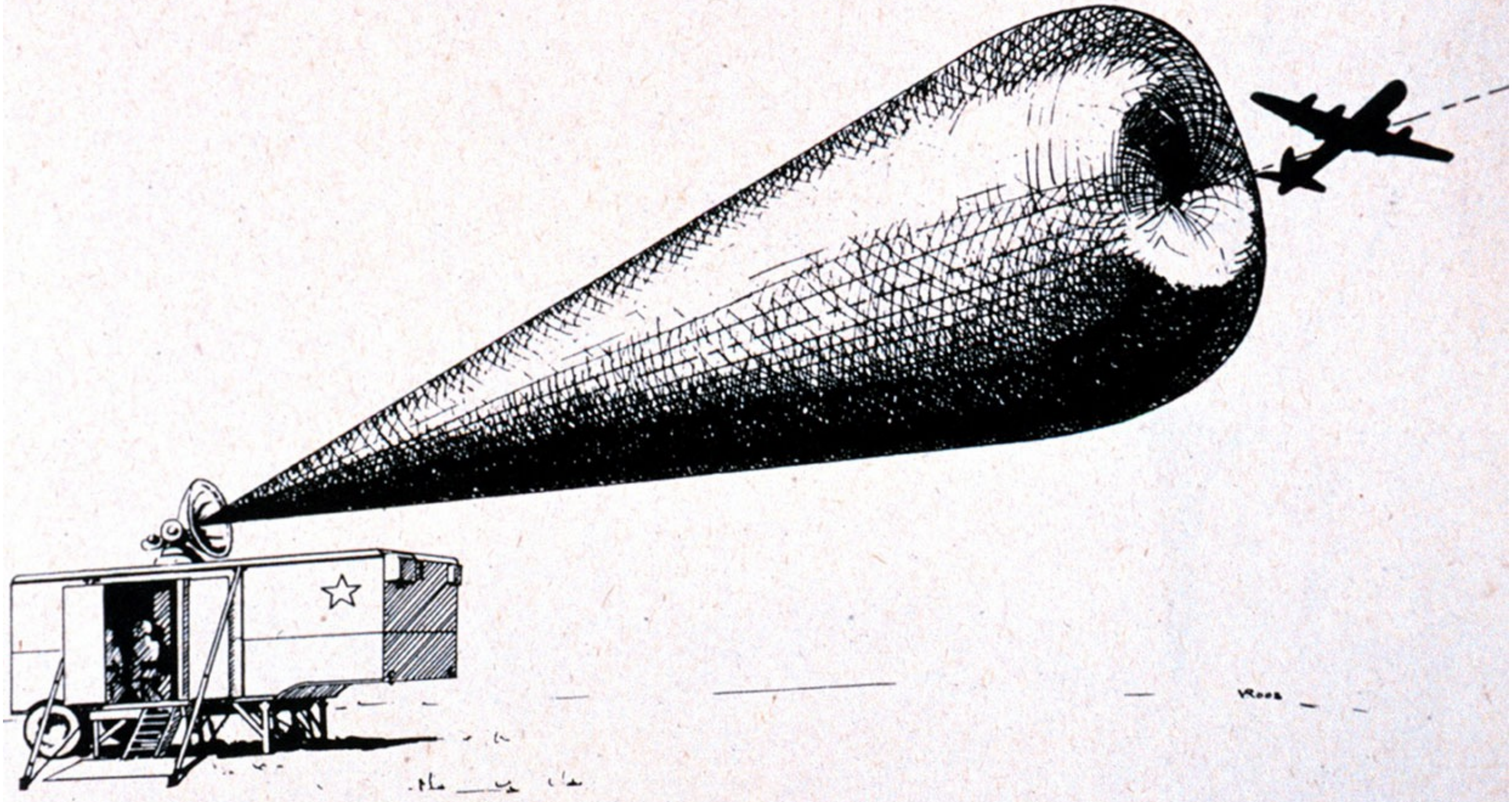
EPFL

Laboratory of SEnsing & Networking Systems

RADAR: RAdio Detection And Ranging

- Invented during world war 2 to detect planes
- Changed the course of the war
- Transmit wireless signals, detect the reflections



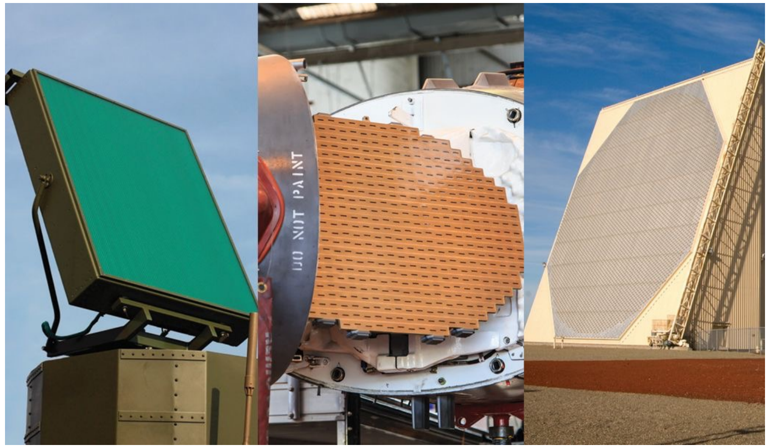


Radar: Interested in Detecting

- Range: Distance from the radar
  - UWB: Ultra Wide Band
  - FMCW: Frequency Modulated Carrier Wave
- Direction: Angle from which the object is coming

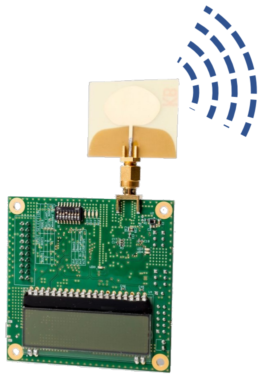
Directional antenna that is rotating.

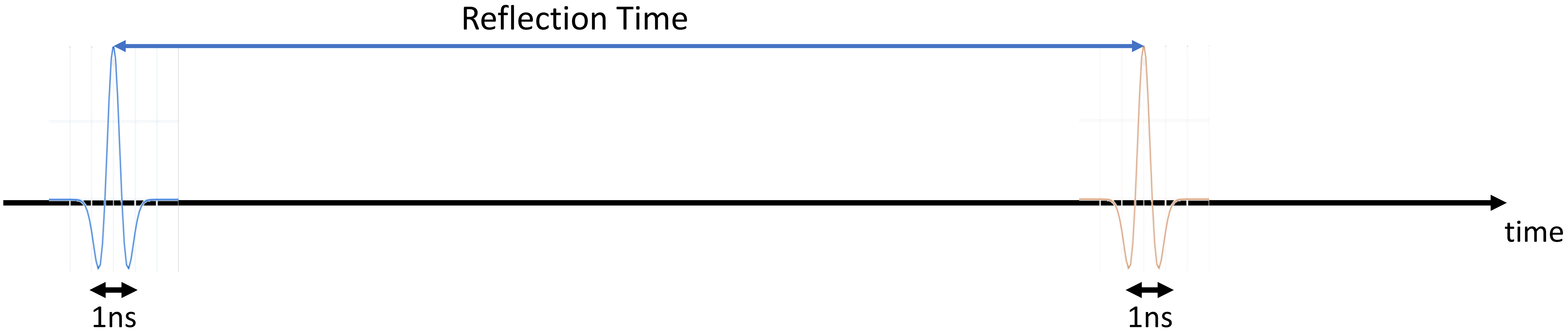
Phased Antenna Array that is always Scanning.



UWB: Ultra Wide Band

- Transmit a Narrow Pulse in Time (1 ns)





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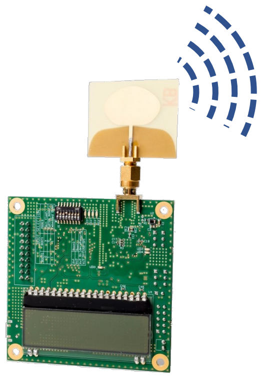
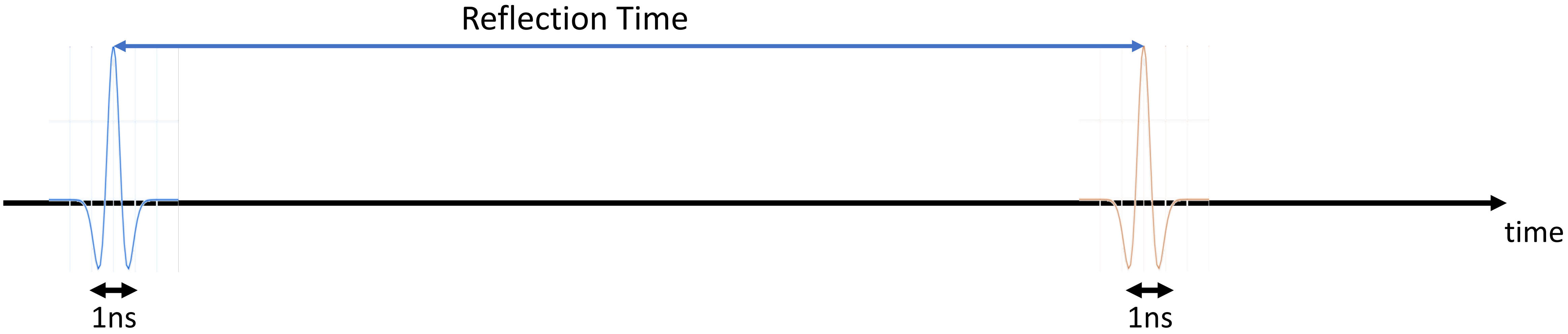


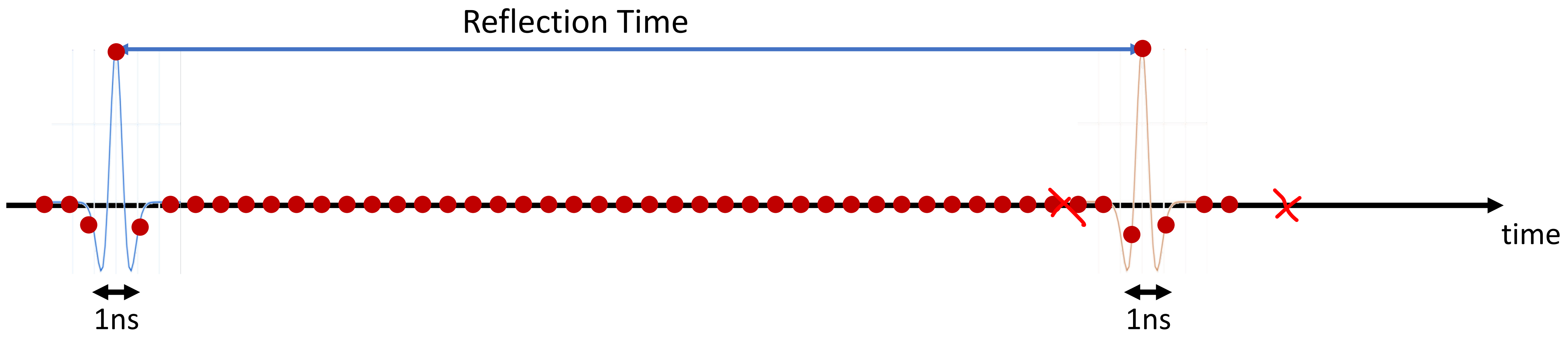
Distance = (Reflection Time /2) × Speed of Light



UWB: Ultra Wide Band

- Transmit a Narrow Pulse in Time (1 ns)
- Bandwidth = 1/pulse time = 1 GHz → UWB
- To detect pulse need to sample at Multi-GigaSample/sec

Distance = $(\text{Reflection Time } /2) \times$ Speed of Light



UWB: Ultra Wide Band

Pros:

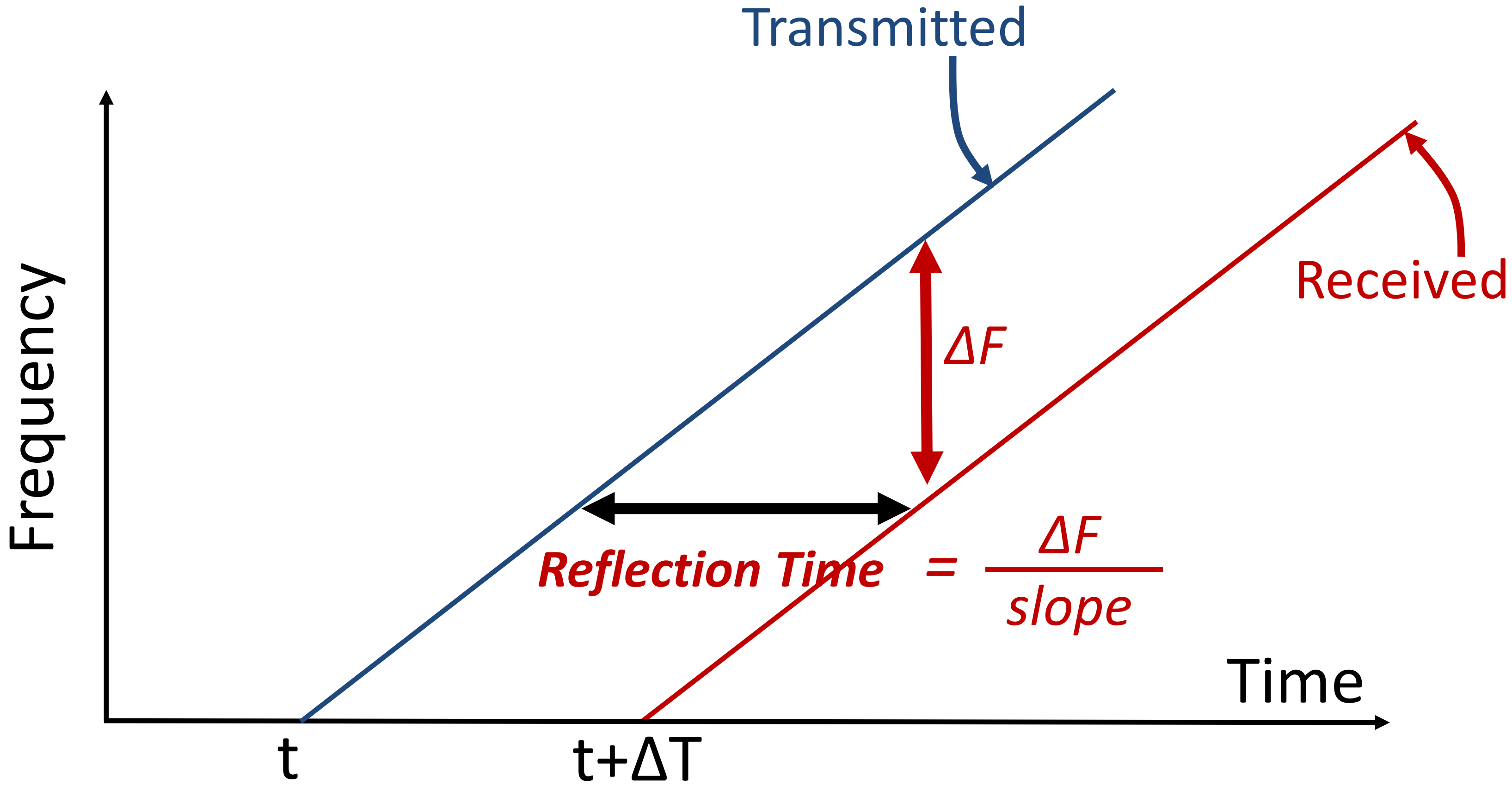
- High resolution
- Simple idea
- Used also for communication (UWB 802.15.3a, 3 - 10 GHz)

Cons:

- Very High Speed Sampling Rate!

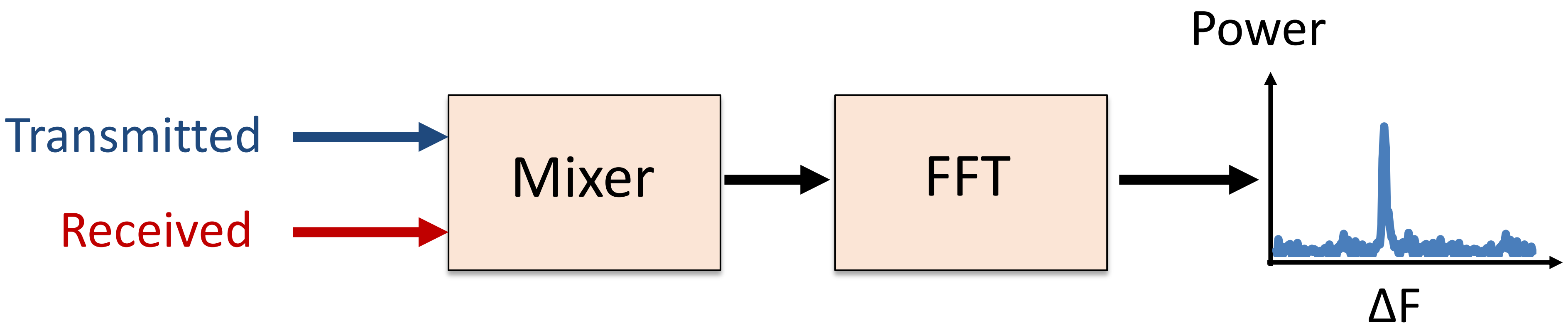
FMCW: Frequency Modulated Carrier Wave

- Use chirps: signals where the frequency changes with time
- Measure time using a frequency change



How do we measure ΔF ?

- Subtracting frequencies is easy (e.g., removing carrier in WiFi)
- Done using a mixer (low-power; cheap)



Signal whose frequency is ΔF

ΔF → Reflection Time → Distance

FMCW: Frequency Modulated Carrier Wave

Slope

- FMCW Transmitted Signal: $x(t) = e^{j2\pi\left(\frac{\alpha}{2}t^2 + f_0 t + \phi_0\right)}$

Reflection Time

- FMCW Received Signal: $y(t) = Ae^{j2\pi\left(\frac{\alpha}{2}(t-\tau)^2 + f_0(t-\tau) + \phi_0\right)}$
  (Single reflector)
  $= Ae^{j2\pi\left(\frac{\alpha}{2}t^2 - \alpha\tau t + \frac{\alpha}{2}\tau^2 + f_0 t - f_0\tau + \phi_0\right)}$

Phase

- Mix TX and RX Signal: $y_b(t) = y(t) \times x^*(t) = Ae^{-j2\pi(\alpha\tau t + f_0\tau)}$

Frequency at $f = \alpha\tau$

FMCW: Frequency Modulated Carrier Wave

- FMCW Transmitted Signal: $x(t) = e^{j2\pi\left(\underbrace{\frac{\alpha}{2}}_{\text{Slope}} t^2 + f_0 t + \phi_0\right)}$

- FMCW Received Signal: (Multiple reflectors) $y(t) = \sum_i A_i e^{j2\pi\left(\frac{\alpha}{2}(t-\tau_i)^2 + f_0(t-\tau_i) + \phi_0\right)}$

Reflection Time

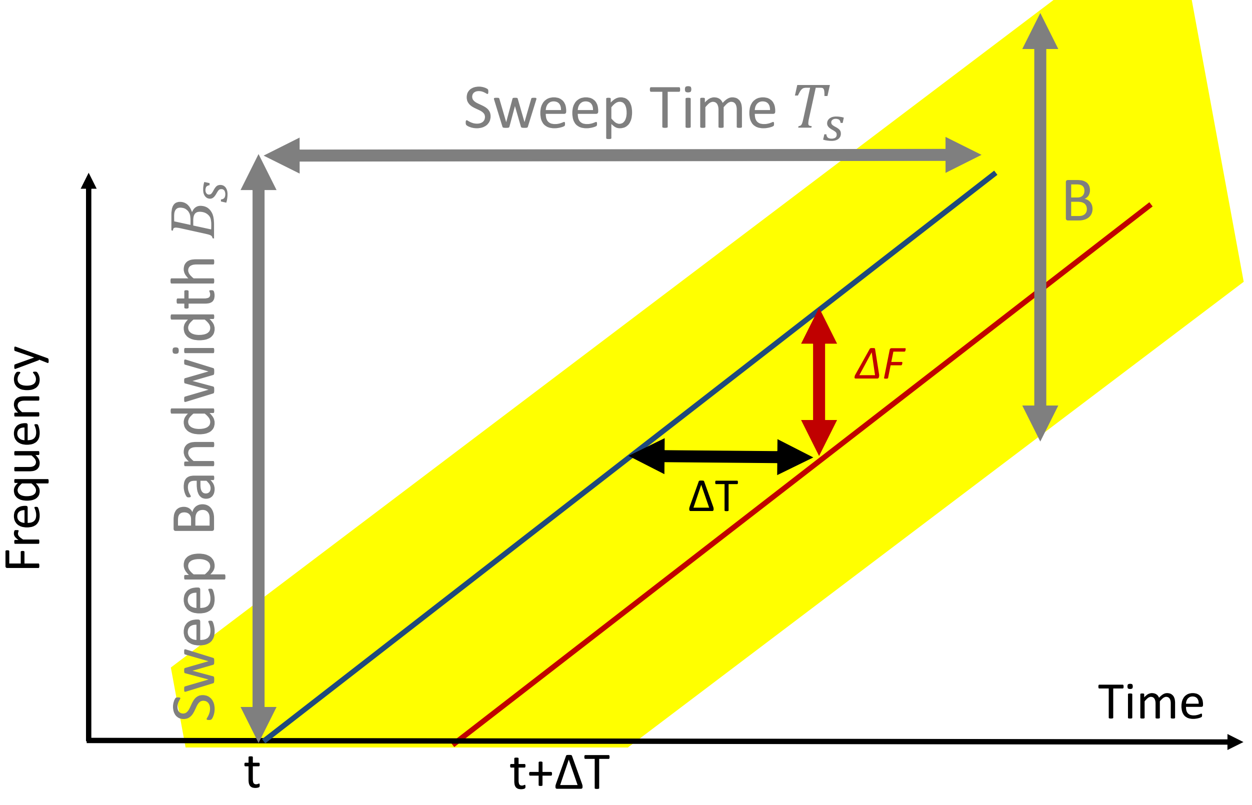
- Mix TX and RX Signal: $y_b(t) = y(t) \times x^*(t) = \sum_i A_i e^{-j2\pi(\alpha\tau_i t + f_0\tau_i)}$

Phase

Frequencies at $f = \alpha\tau_i$

FMCW: Frequency Modulated Carrier Wave

- FMCW Transmitted Signal: $x(t) = e^{j2\pi\left(\frac{\alpha}{2}t^2+f_0t+\phi_0\right)}$
- Mix TX and RX Signal: $y_b(t) = y(t) \times x^*(t) = \sum_i A_i e^{-j2\pi(\alpha\tau_i t + f_0\tau_i)}$



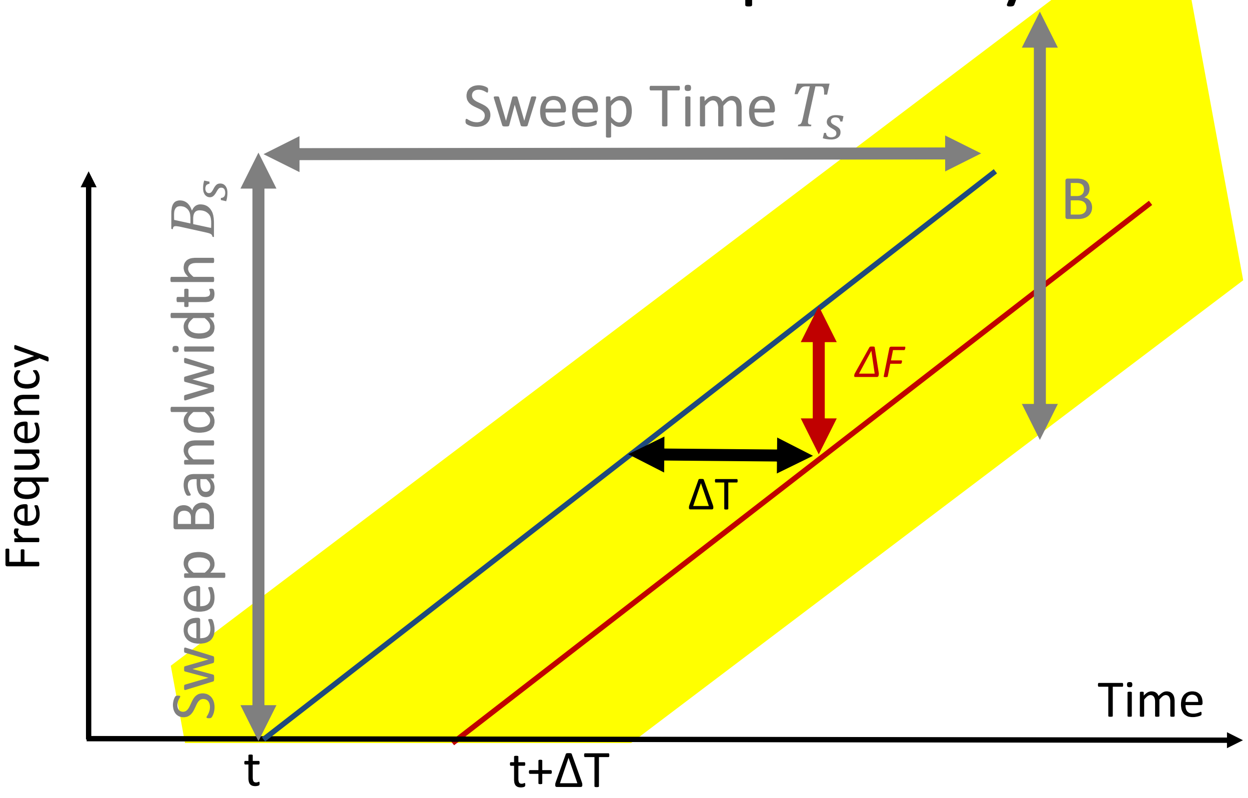
Slope: $\alpha = \frac{B_s}{T_s}$

$\Delta F = \alpha \Delta T = \alpha \tau_i$

Sampling Rate: $B \ll B_s$

FFT Window: $T = T_s$

FMCW: Frequency Modulated Carrier Wave



Slope: $\alpha = \dfrac{B_s}{T_s}$

$\Delta F = \alpha \Delta T = \alpha \tau_i$

Sampling Rate: $B \ll B_s$

FFT Window: $T = T_s$

Max Range: $\Delta F < B \quad \Rightarrow \tau_{max} < B/\alpha \quad = B\ T_s/B_s \quad \Rightarrow d_{max} < c\ B\ T_s/2B_s$

Resolution: $\delta F = 1/T = 1/T_s \ \Rightarrow \tau_{min} = 1/\alpha T_s = 1/B_s \ \Rightarrow d_{min} = c/2B_s$

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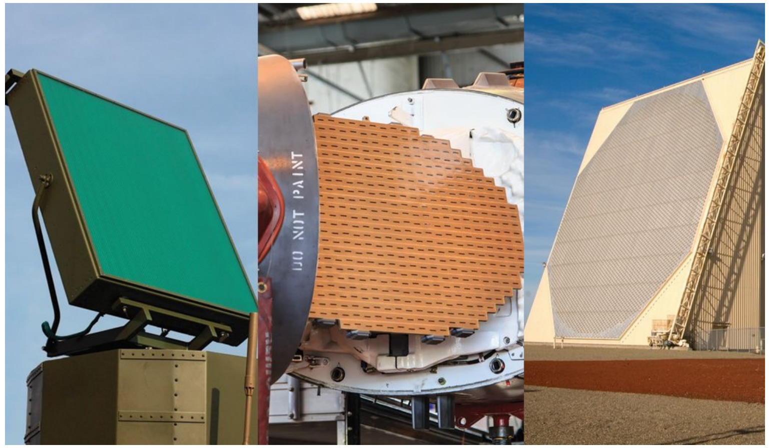
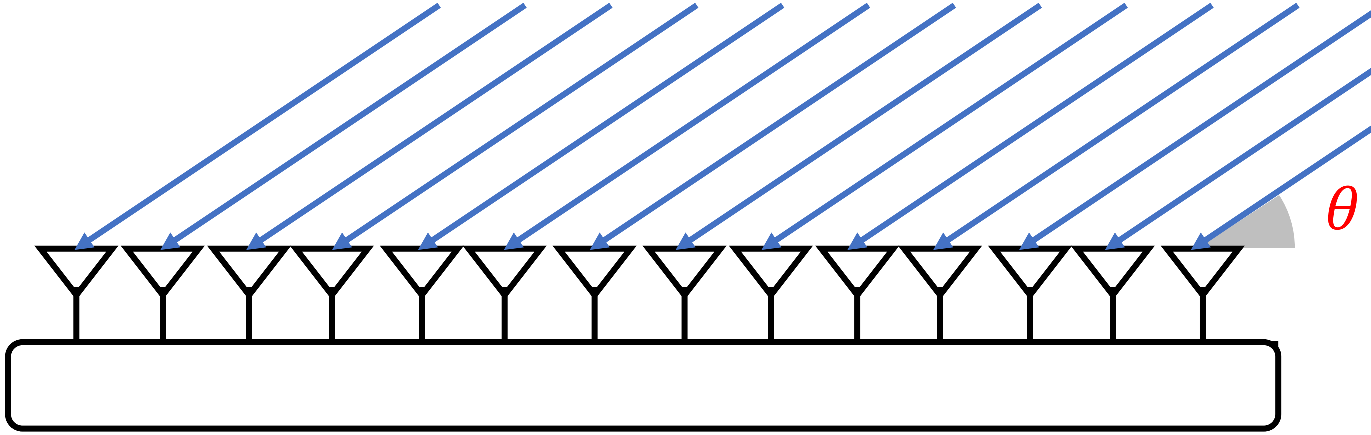
Phased Antenna Array that is always Scanning.



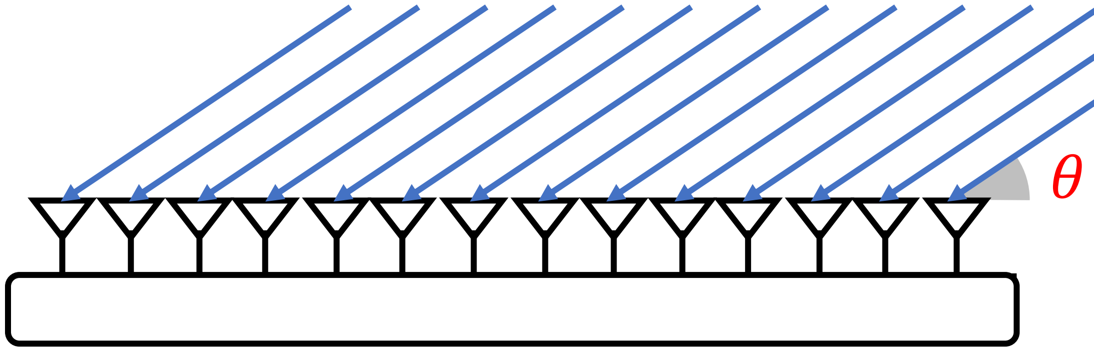
Phased Arrays Primer



$$h_k = \alpha_1 e^{-j2\pi\frac{d_1 - k\ s\ \cos\theta_1}{\lambda}}$$

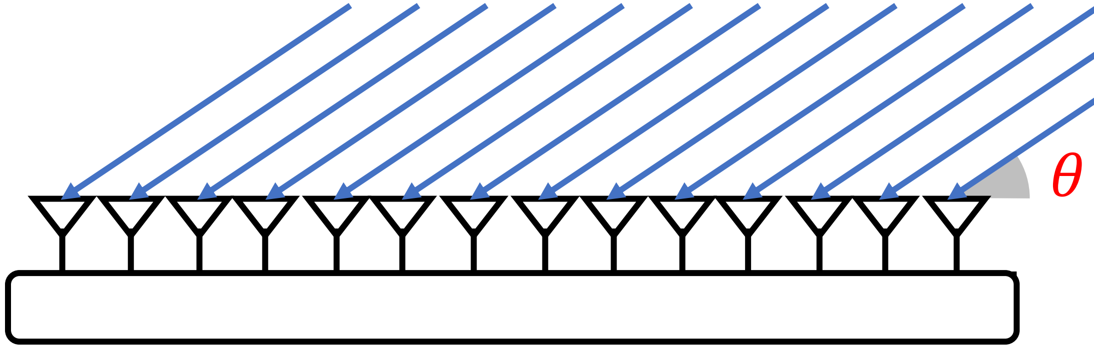
Phased Arrays Primer

Digital Phased Arrays



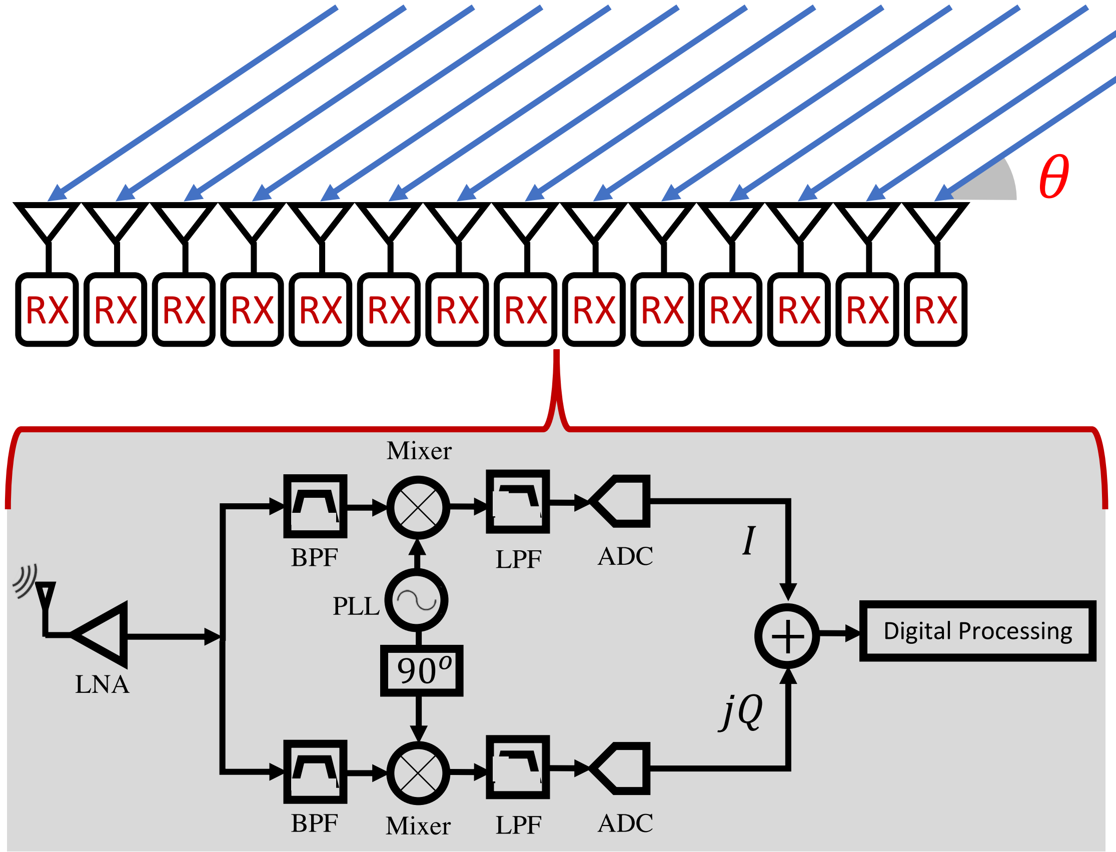
Phased Arrays Primer

Digital Phased Arrays



Phased Arrays Primer

Digital Phased Arrays

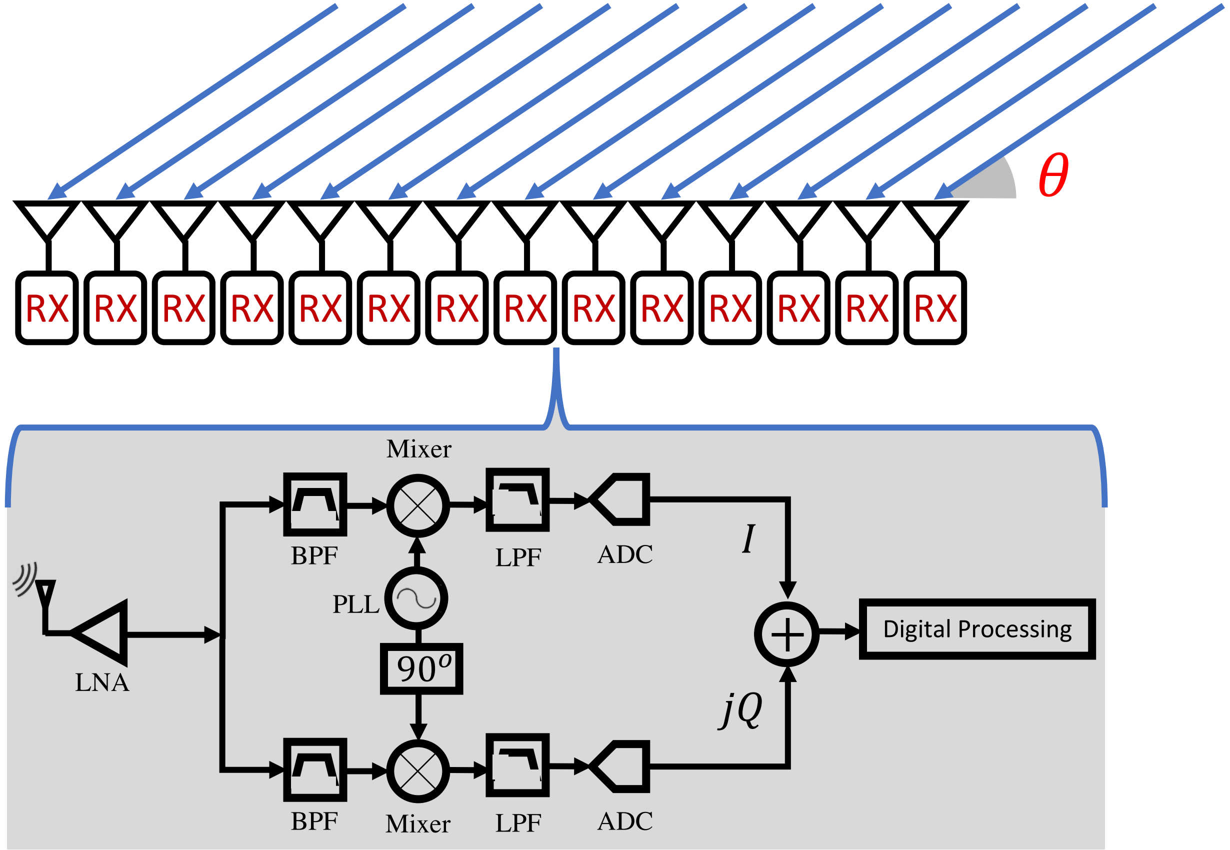


Every antenna connected to full fledged RX

Sample & Process Signals in Digital

Phased Arrays Primer

Digital Phased Arrays

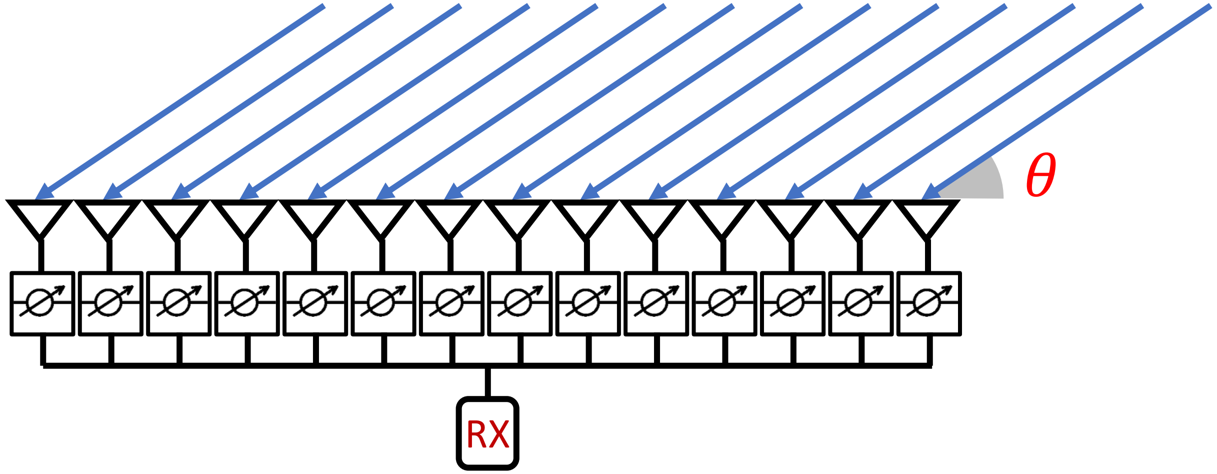


Every antenna connected to full fledged RX

Sample & Process Signals in Digital

Very expensive and high power for large arrays & mmWave

Analog Phased Arrays

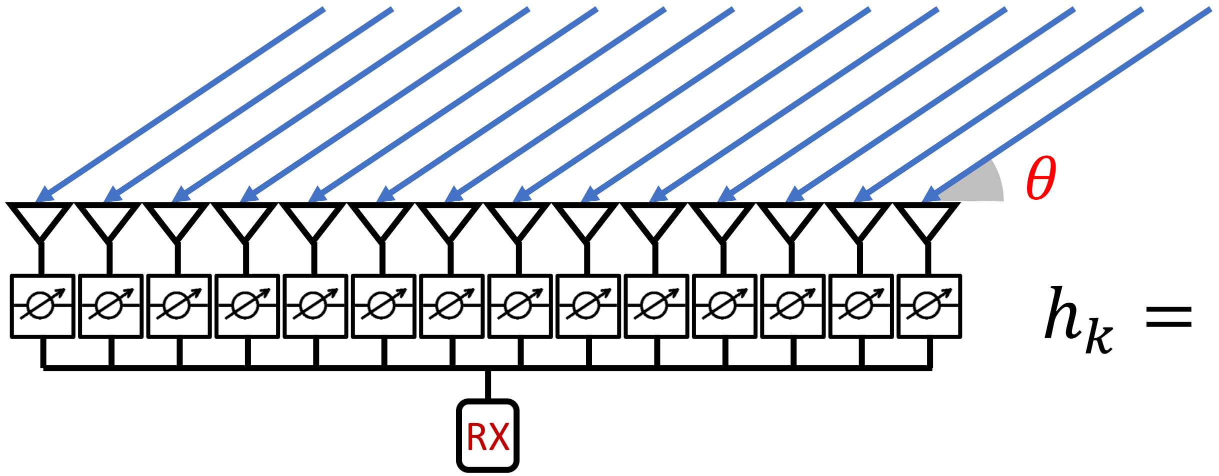


All antennas connected to a single receiver

- Each antenna connected to a phase shifter.
- Phase shifter changes the phase of the signal on each antenna by multiplying with $e^{j\phi}$.
- Steer the beam electronically by changing the phases of the signals.
- Get the sum along a certain direction.

Cheap & low power but requires scanning

Analog Phased Arrays



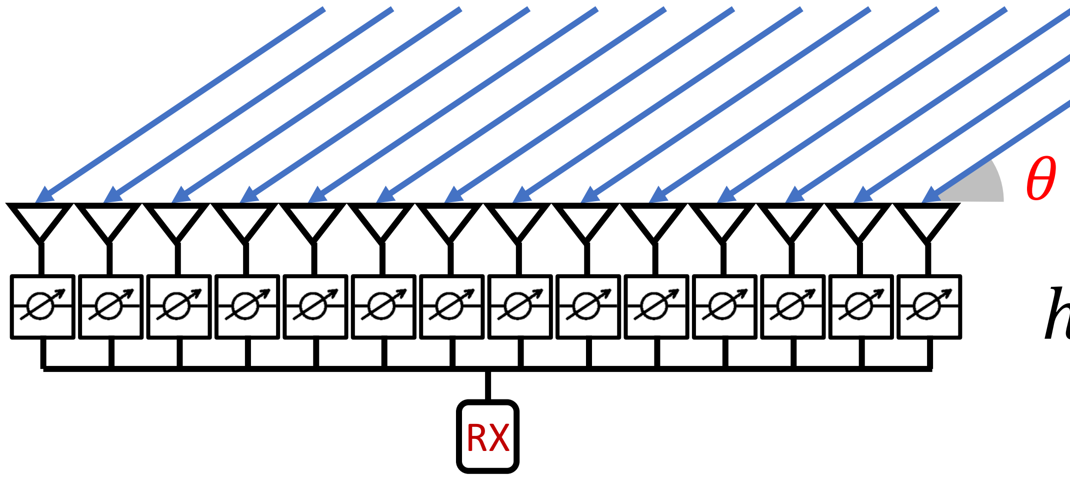
$$h_k = \alpha e^{-j2\pi\frac{d-k\, s \cos\theta}{\lambda}}$$

$$y(t) = \sum_{1}^{N} y_k(t) e^{j\phi_k} = \sum_{1}^{N} h_k x(t) e^{j\phi_k} = \sum_{1}^{N} \alpha e^{-j2\pi\frac{d-k\, s \cos\theta}{\lambda}} x(t) e^{j\phi_k}$$

$$= x(t)\alpha e^{-j2\pi\frac{d}{\lambda}} \sum_{1}^{N} e^{j\pi k\ \cos\theta} e^{j\phi_k}$$

To get signal along direction θ_1, set the phases on the phase shifters to $\phi_k = -\pi k \cos\theta$

Analog Phased Arrays



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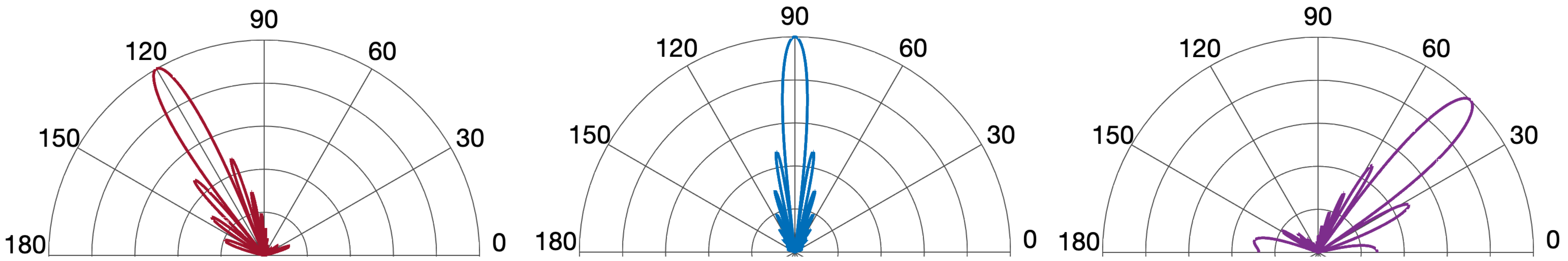
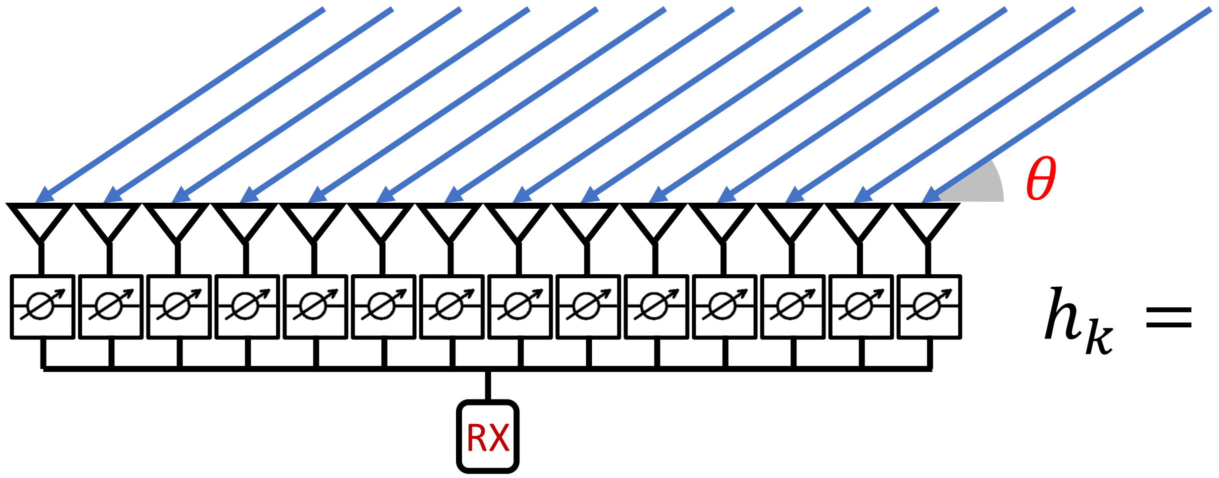


Analog Phased Arrays



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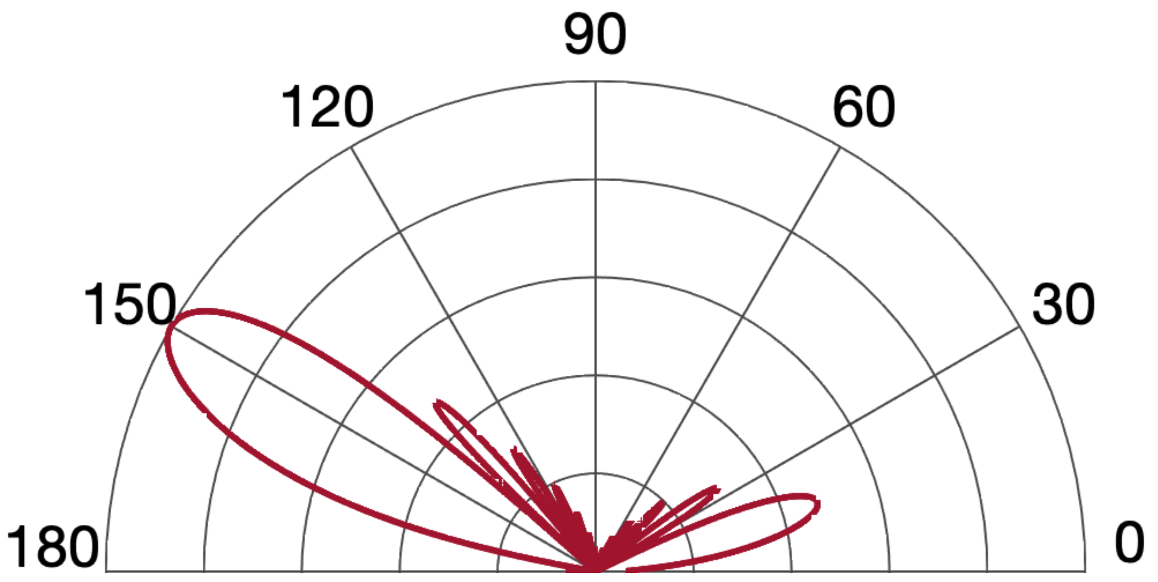


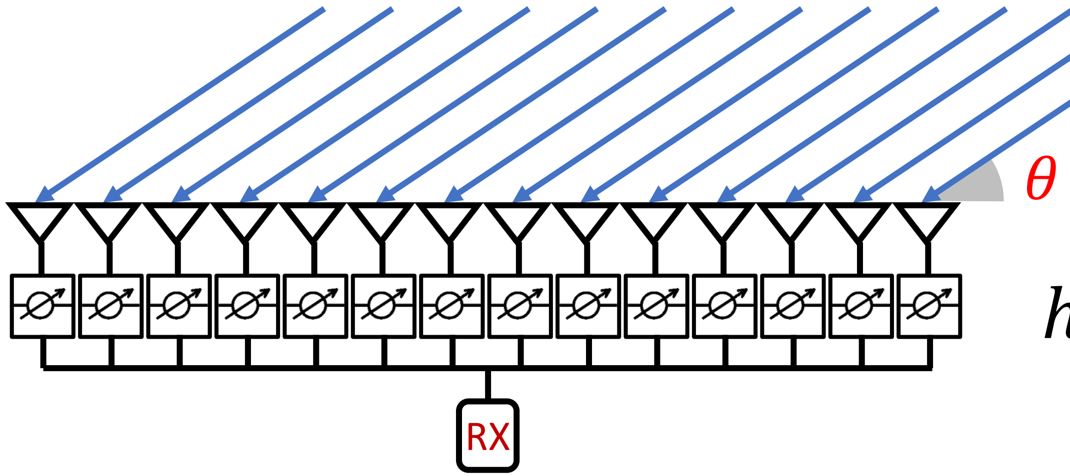
(1) Set the phase shifters to receive signals from a given direction.

(2) Send FMCW signals.

(3) Receiver FMCW reflections, down convert, sample and compute range FFT.

(4) Repeat until you get 2D range image: AoA + Range

Analog Phased Arrays



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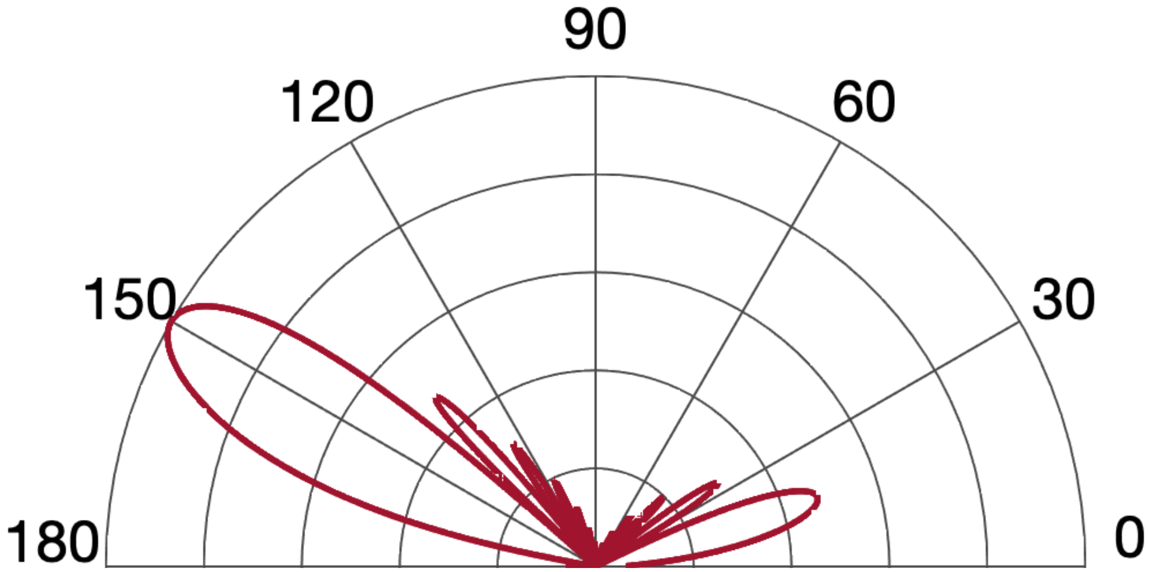
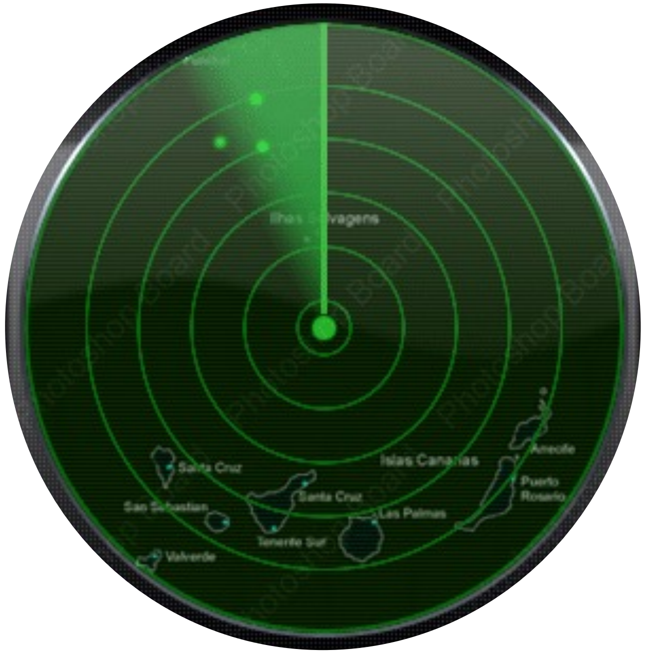
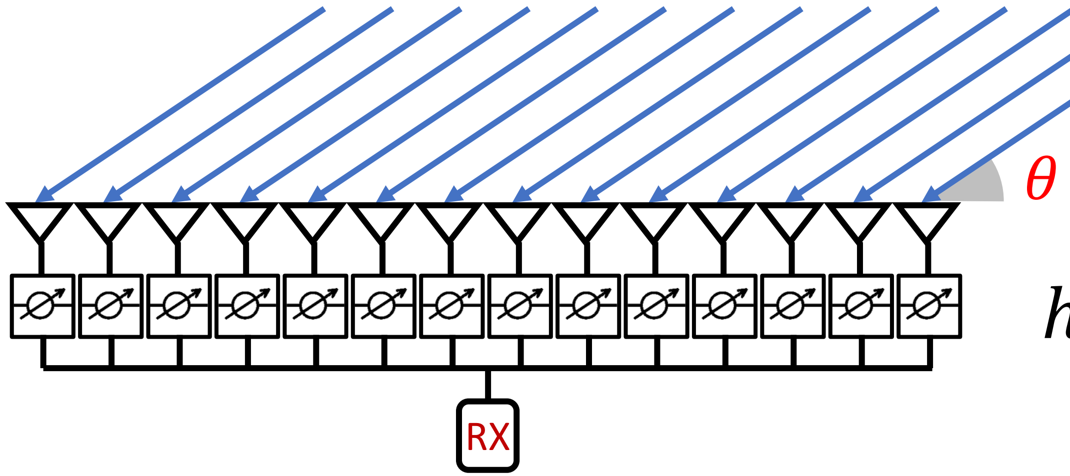




Analog Phased Arrays



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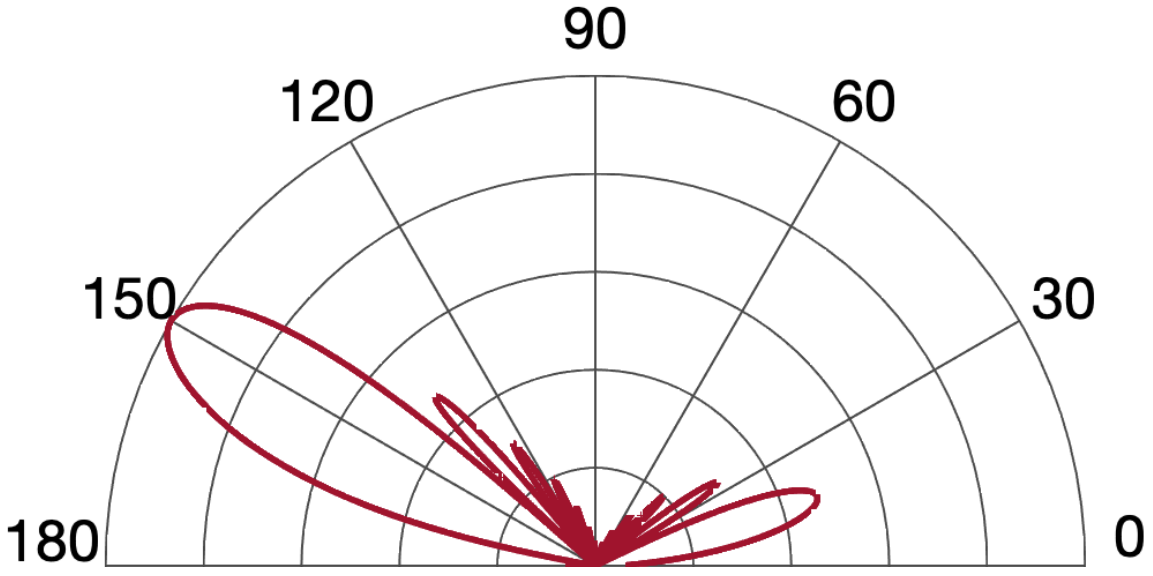

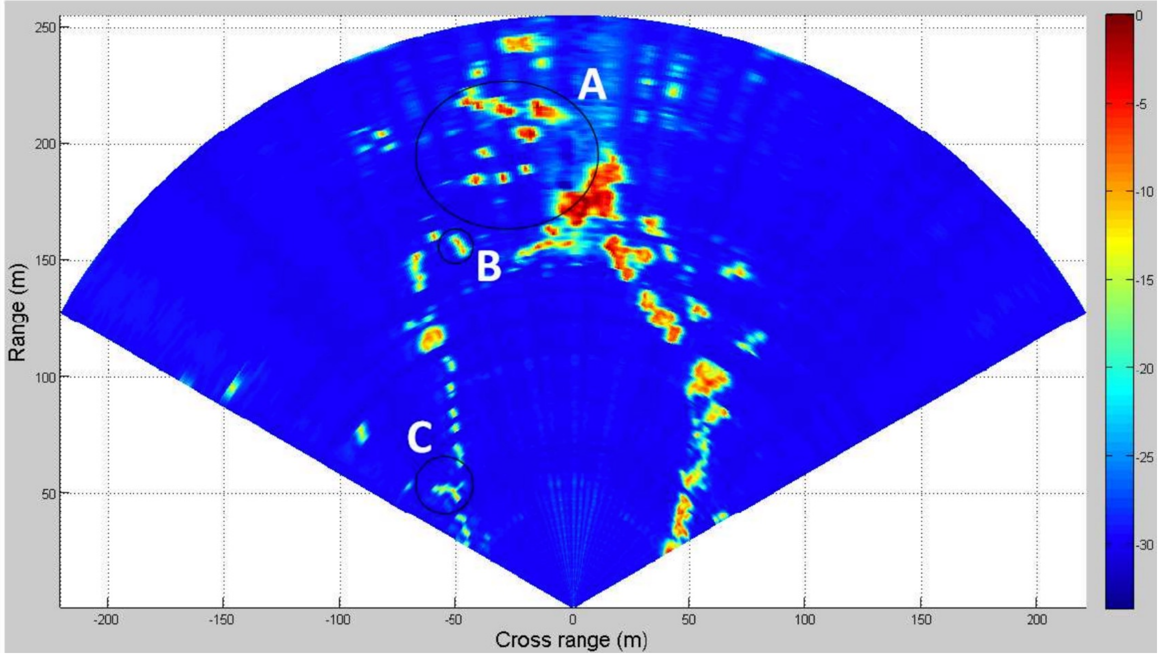

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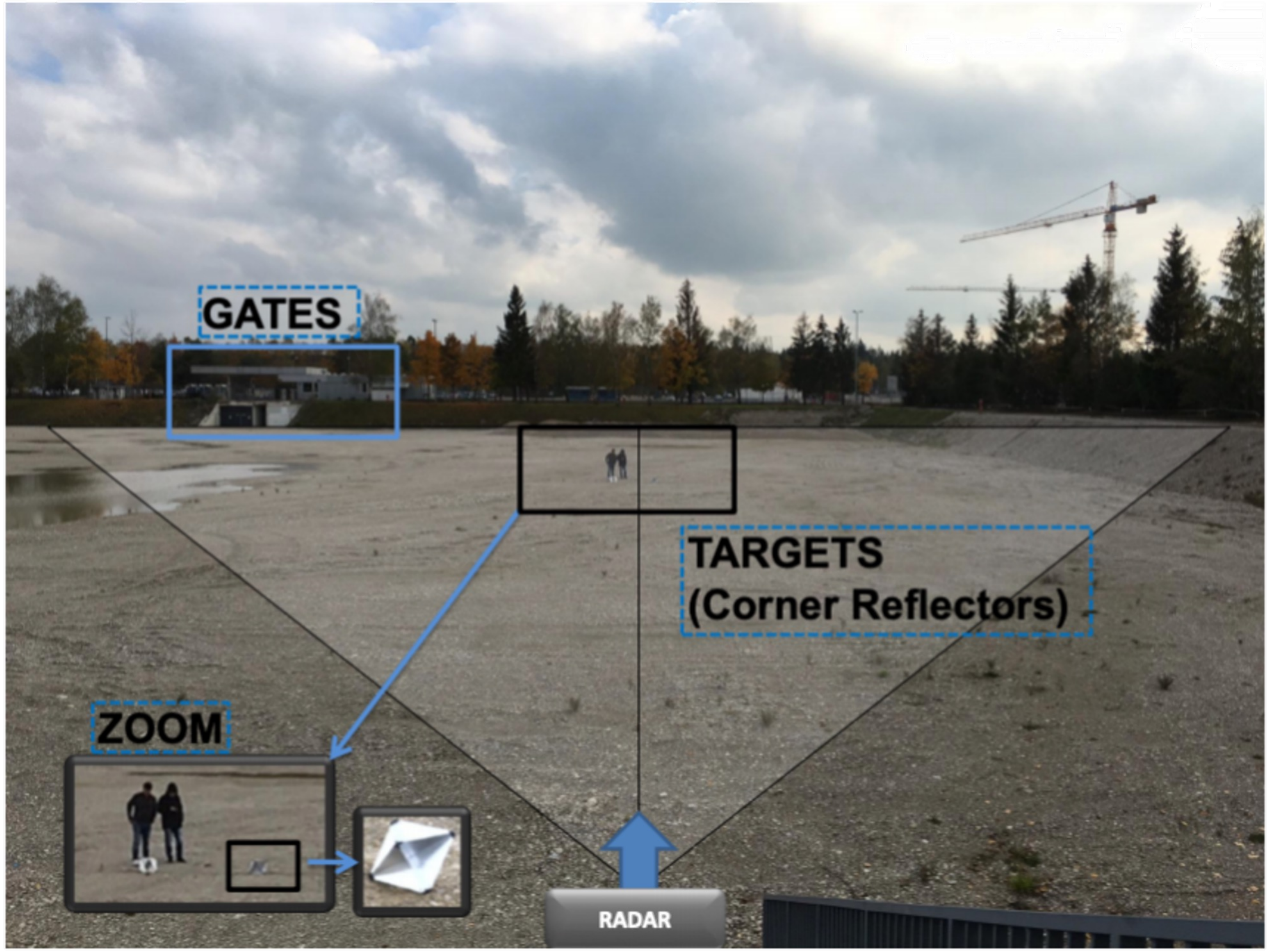
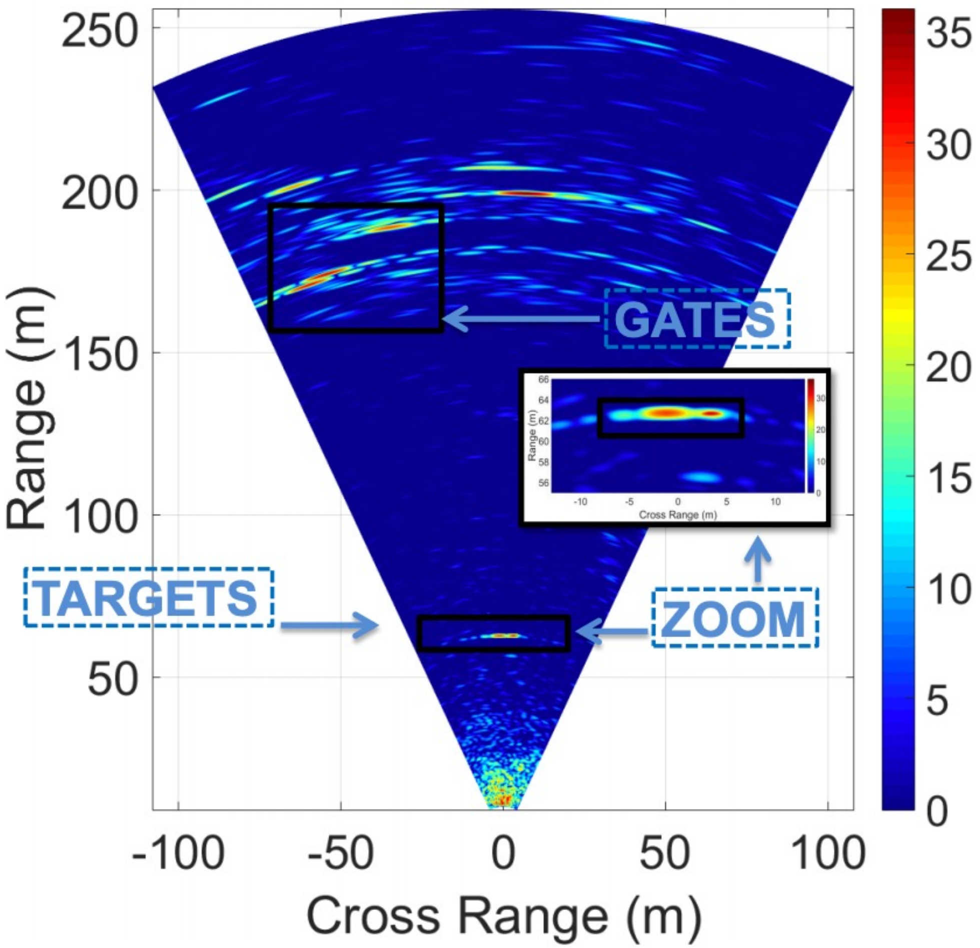
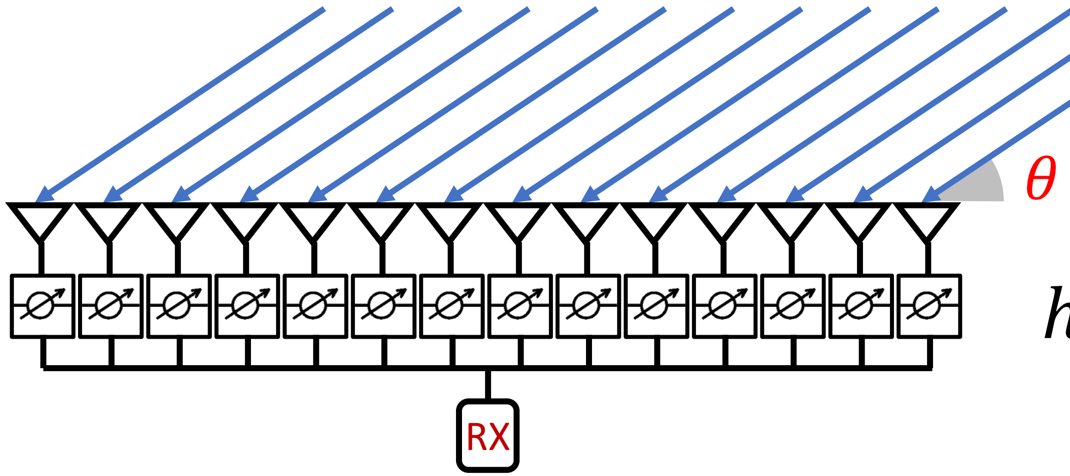


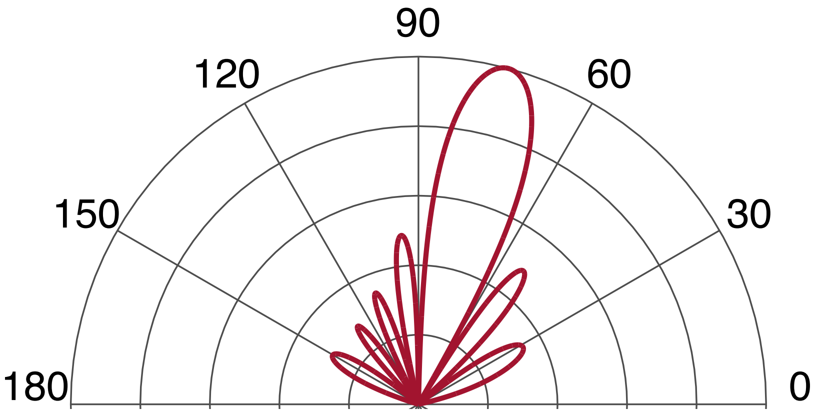


Ganis, A.; Miralles-Navarro, E.; Schoenlinner, B.; Prechtel, U.; Meusling, A.; Heller, C.; Spreng, T.... (2018). A portable 3D Imaging FMCW MIMO Radar Demonstrator with a 24x24 Antenna Array for Medium Range Applications. IEEE Transactions on Geoscience and Remote Sensing. 56(1):298-312. https://doi.org/10.1109/TGRS.2017.2746739

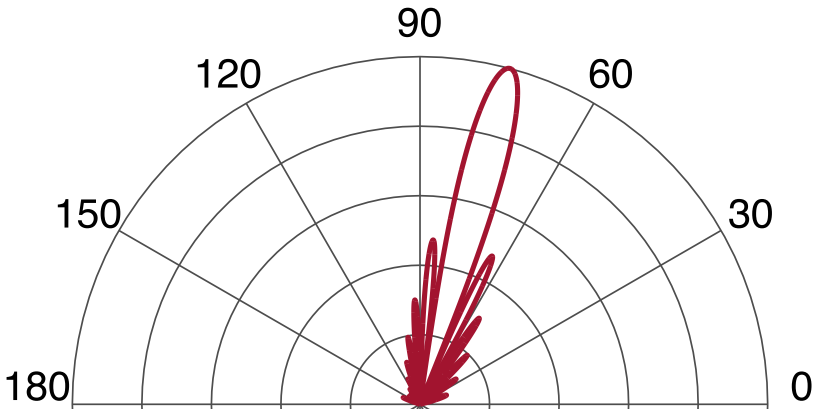
Analog Phased Arrays



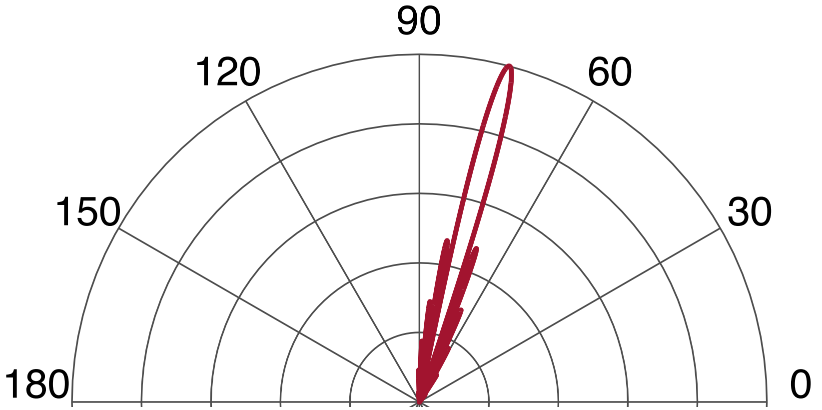
$$h_k = \alpha e^{-j2\pi \frac{d - k\, s \cos\theta}{\lambda}}$$



8 Antenna



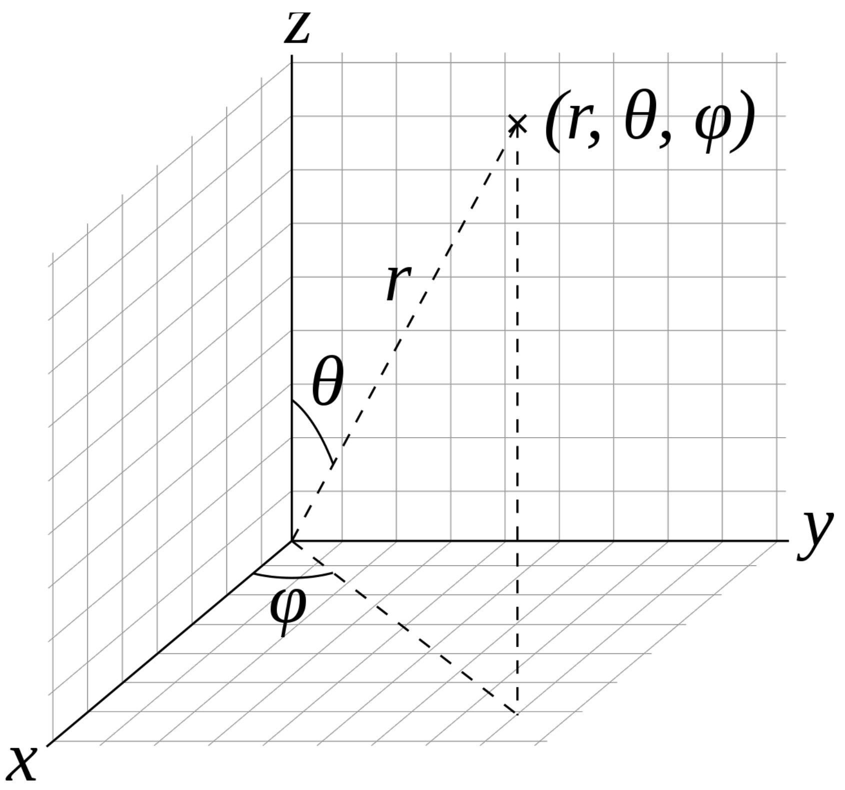
16 Antenna

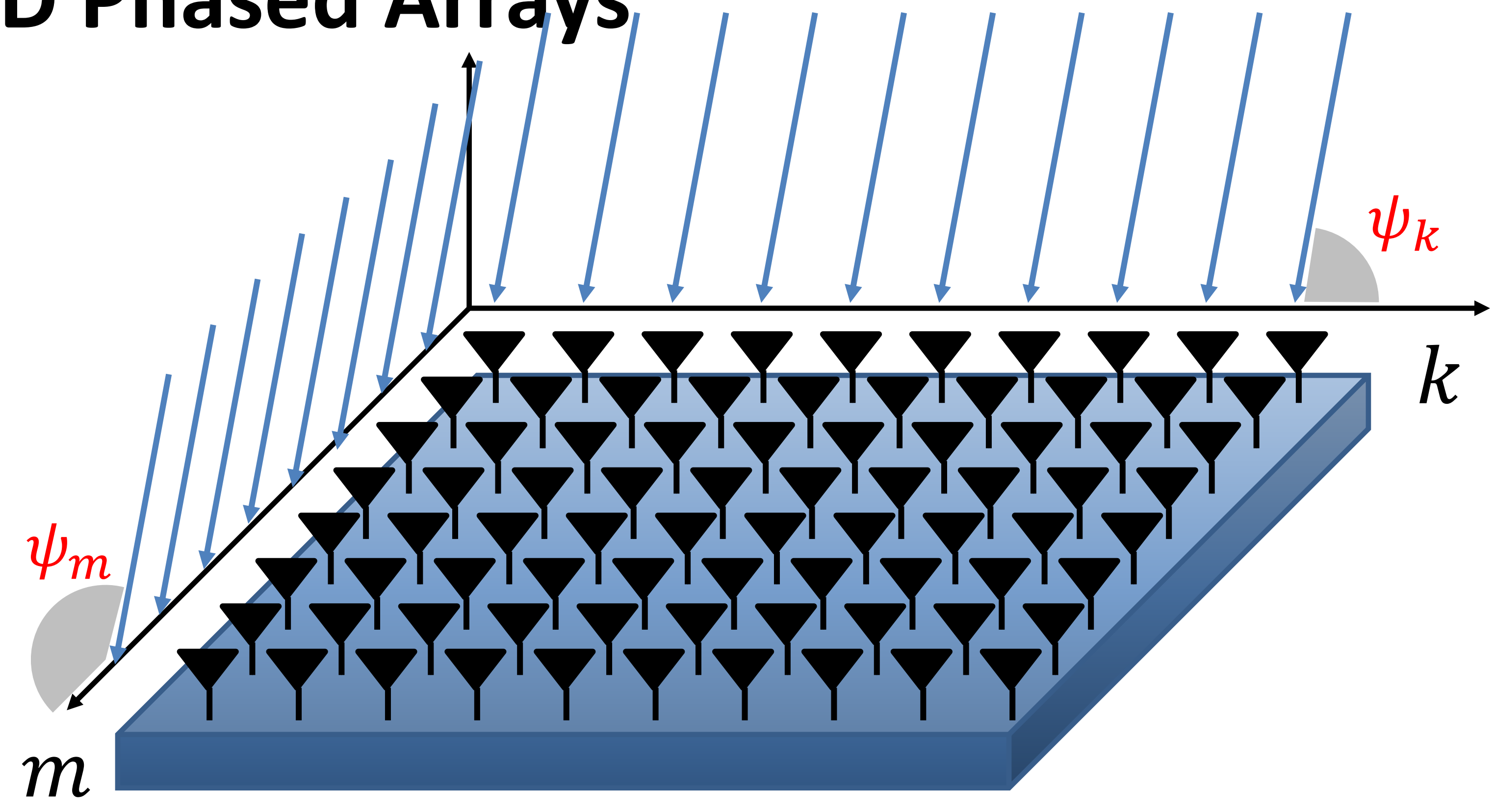


32 Antenna

Larger Array → Narrower Beams → Higher Resolution

2D Phased Arrays





$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+f(m)+f(k))}$$

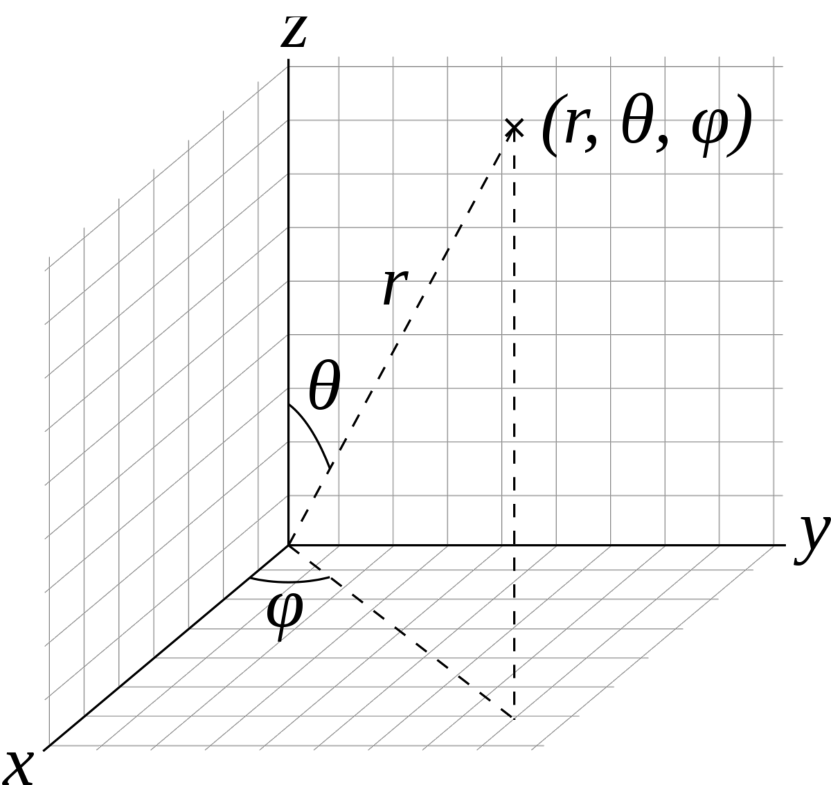
Fix m: $h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+f(m)+ks\cos\psi_k)}$

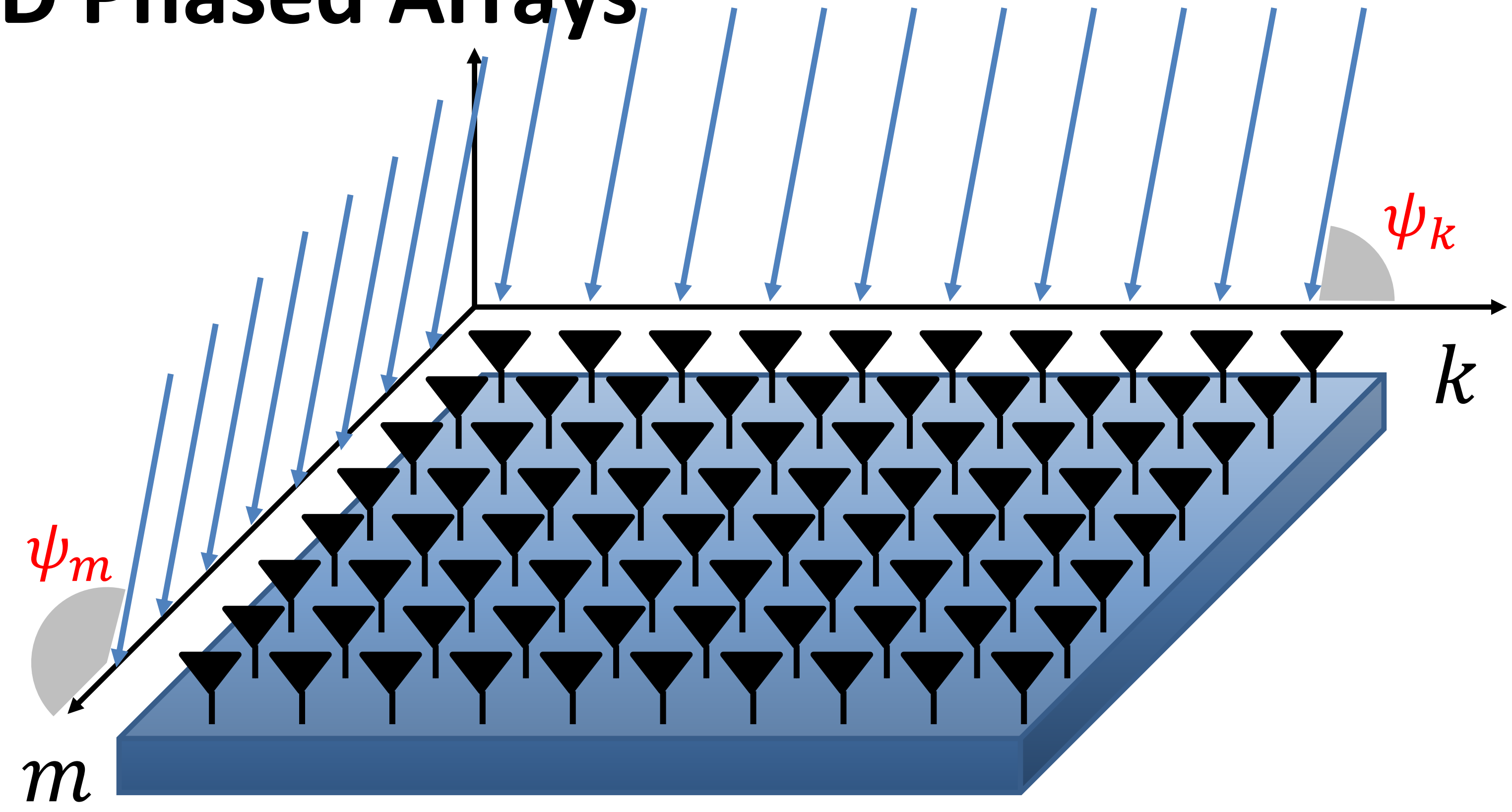
Fix k: $h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+ms\cos\psi_m+ks\cos\psi_k)}$

Can recover:

- Range: r
- Azimuth AoA: φ
- Elevation AoA: θ

2D Phased Arrays





Can recover:

- Range: r
- Azimuth AoA: φ
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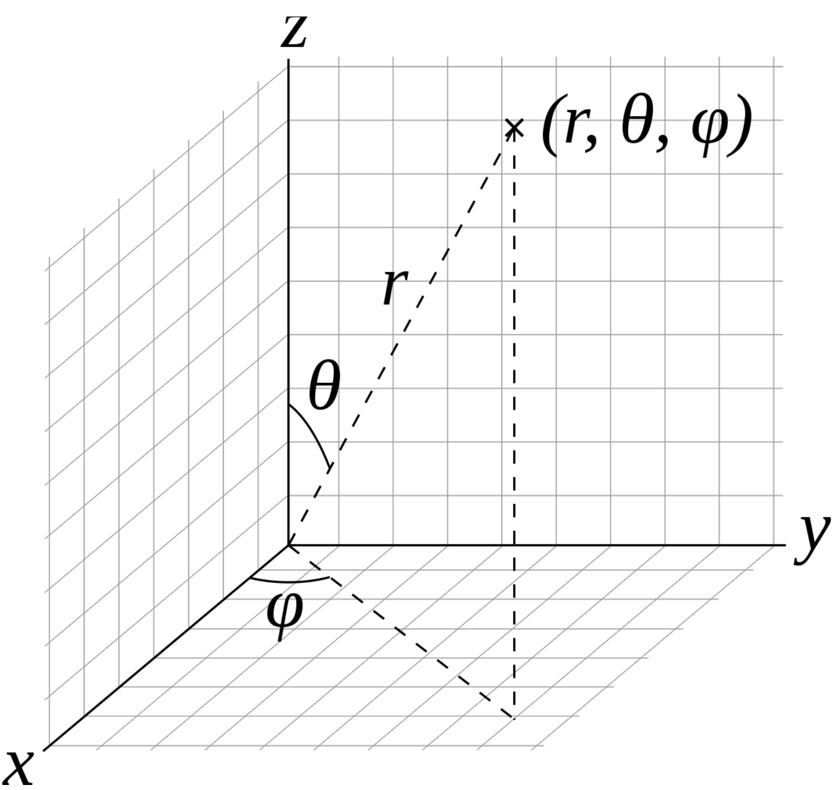
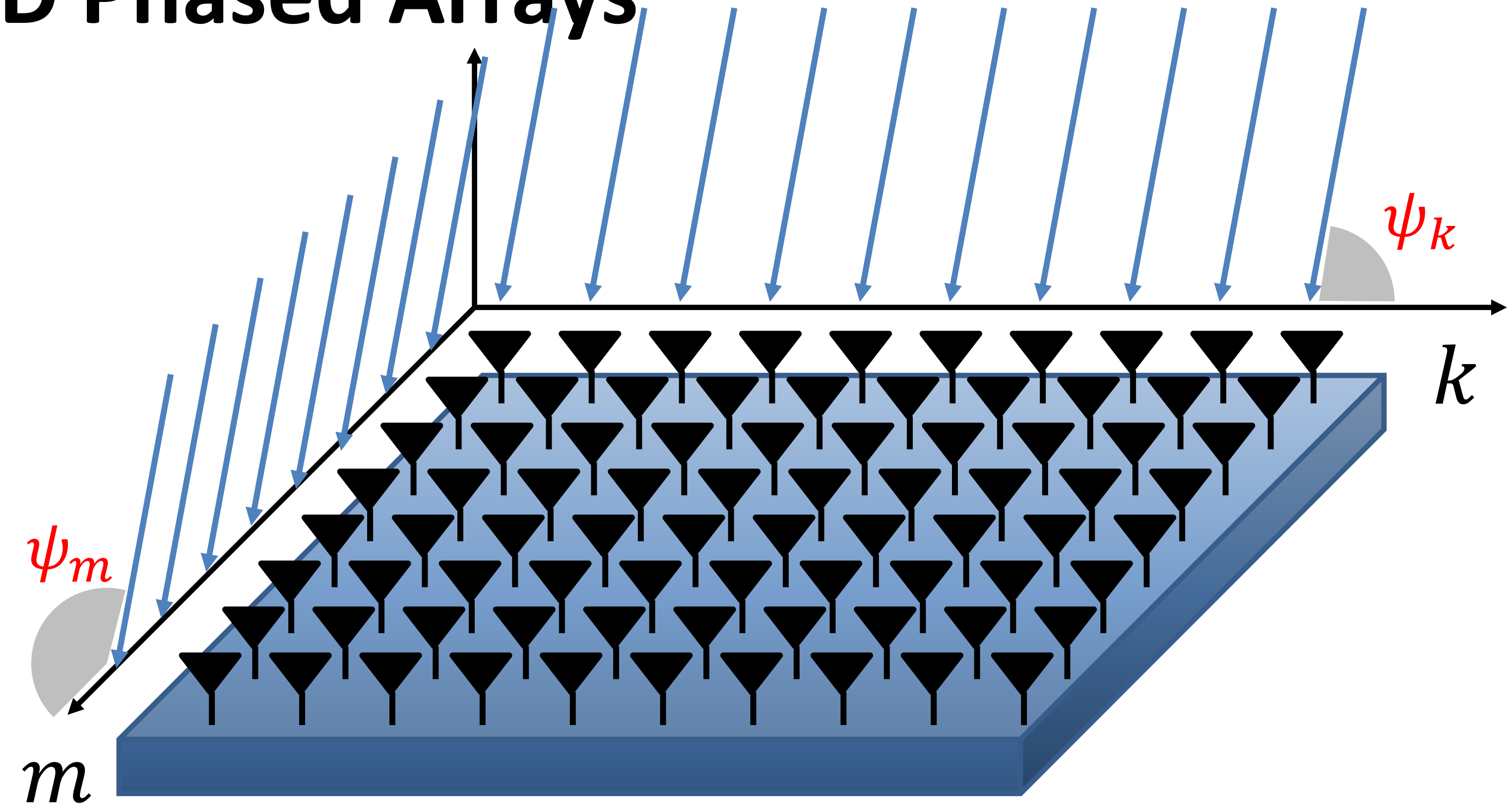
$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r + ms\cos\psi_m + ks\cos\psi_k)}$$

$$\cos\psi_m = \sin\theta\cos\varphi$$

$$\cos\psi_k = \sin\theta\sin\varphi$$

2D Phased Arrays





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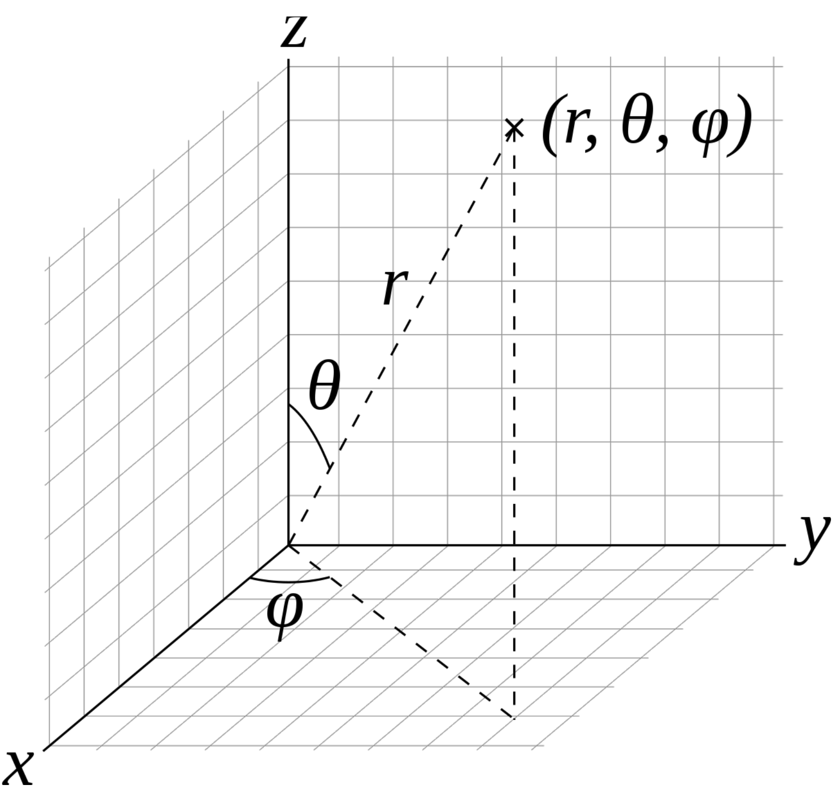
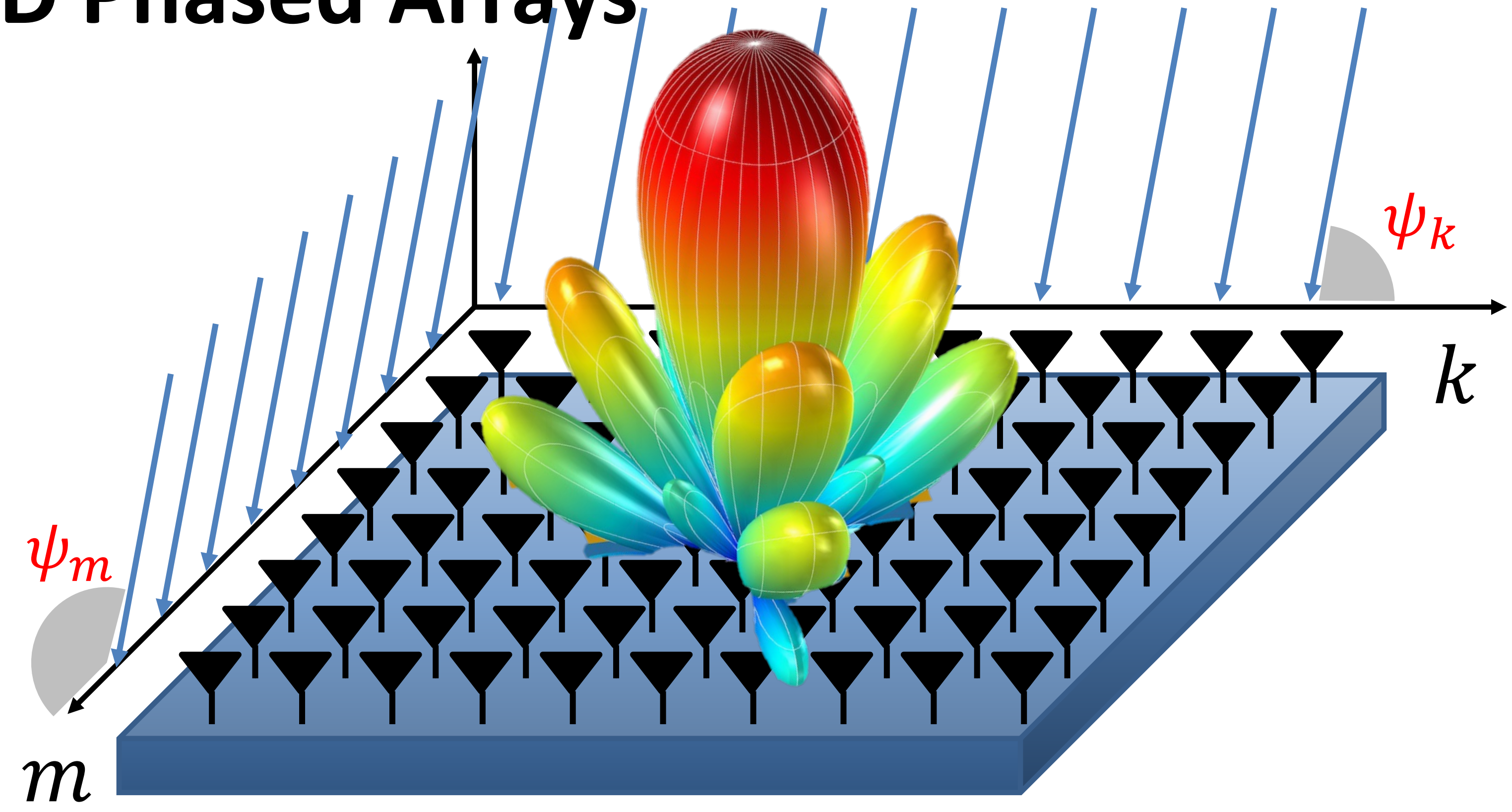
$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+ms\sin\theta\cos\varphi+ks\sin\theta\sin\varphi)}$$

2D Phased Arrays





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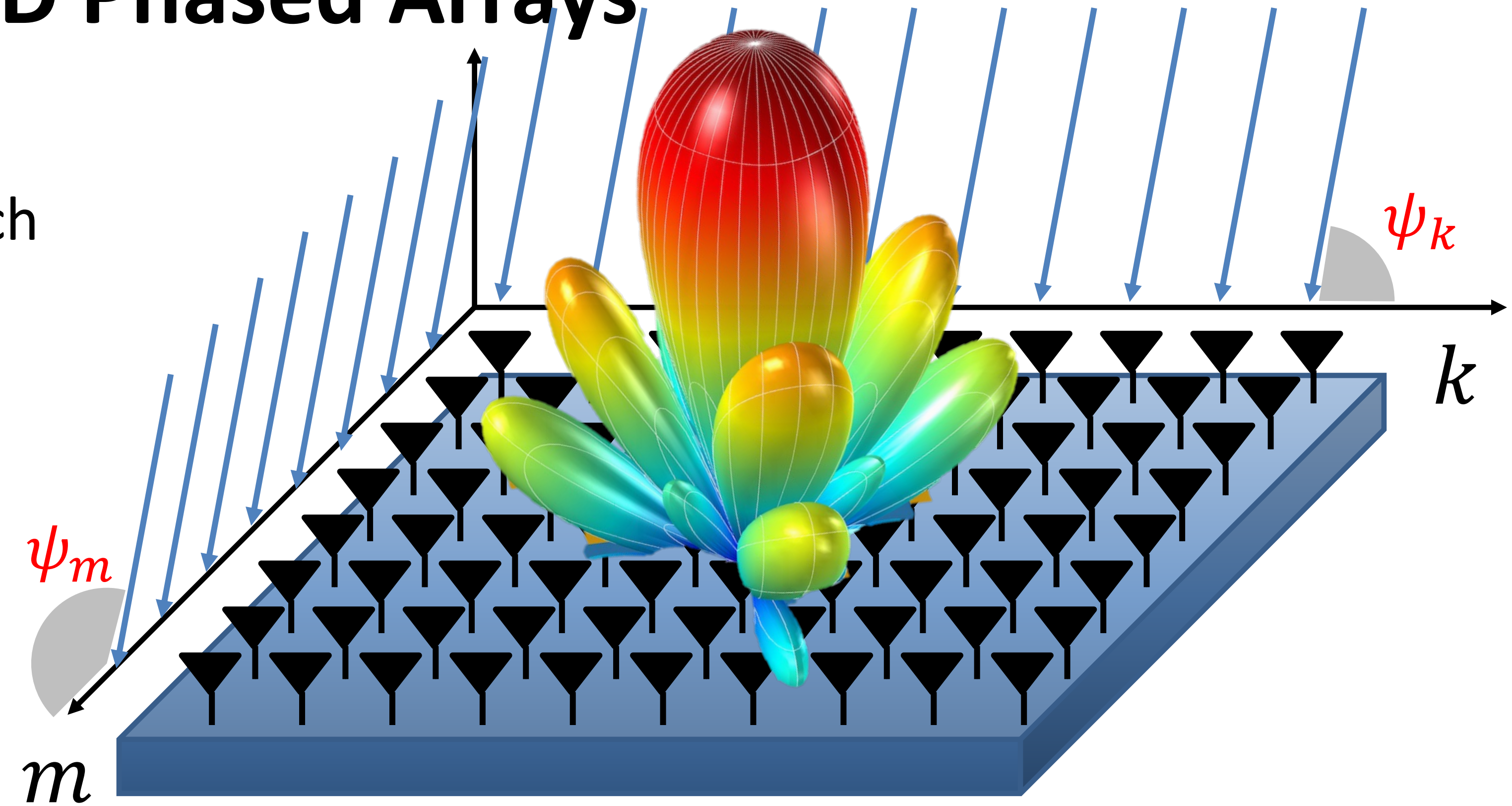
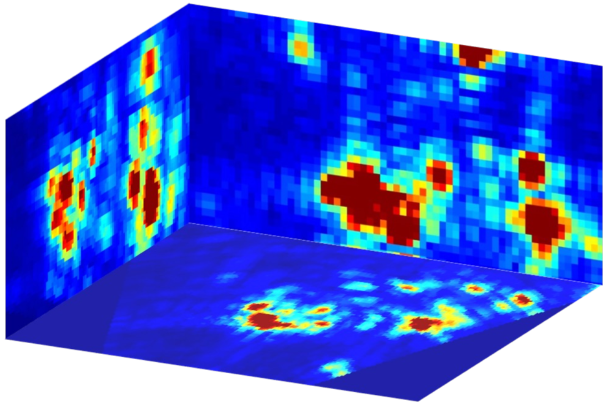
2D Phased Arrays

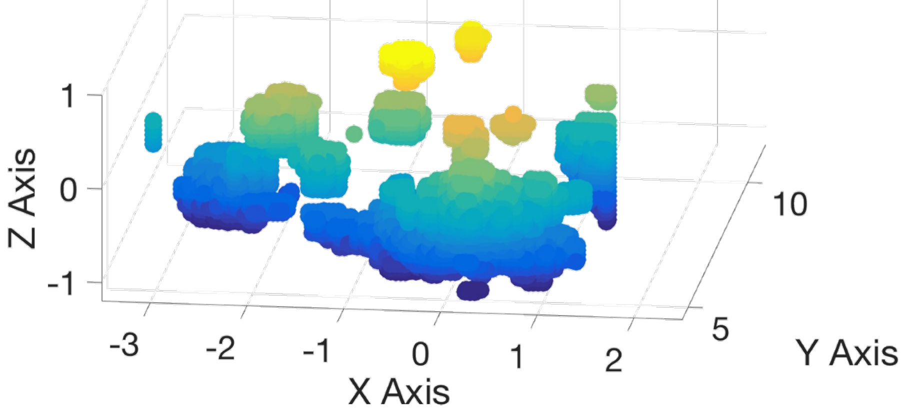


1) Pick the phase shift on each antenna to create a beam in each 3D direction.
2) Transmit FMCW signals and receive reflections.
3) Mix RX signal with TX and take range FFT .
4) Repeat in every direction.

3D Heatmap Image

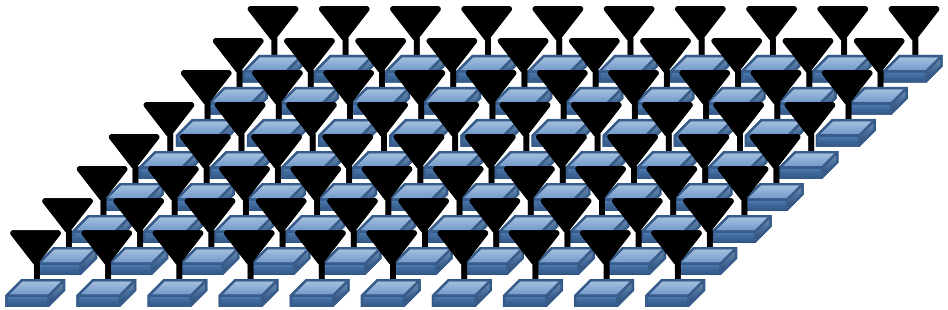


3D Point Cloud



Phased Arrays Primer

Digital Phased Arrays



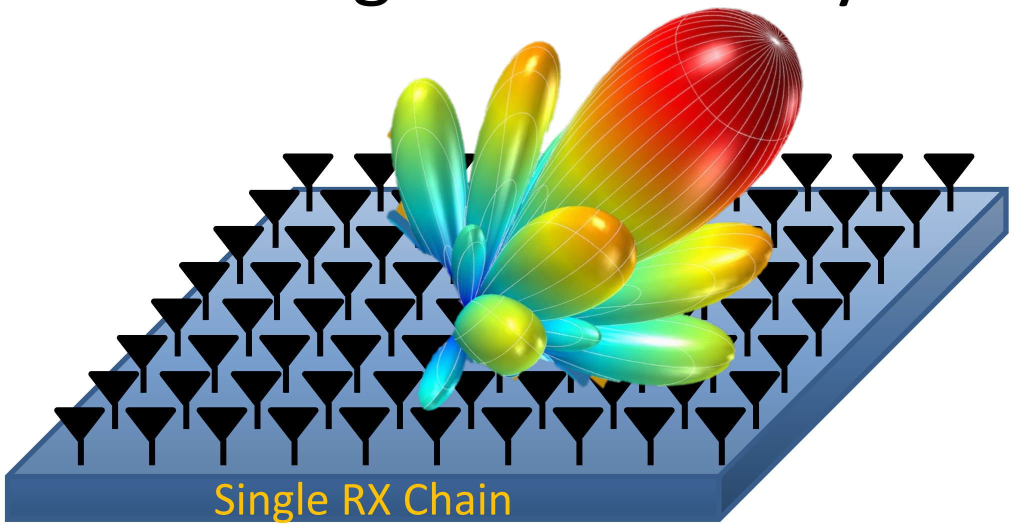
$N \times N$ **RX Chains**

Could Potentially Do the Same thing:

1) Mix the RX signal with TX.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
3) Compute Range FFT.
4) Repeat in every direction.

$$(N \times N \times T + T\log T) \times N \times N = O(N^4 T + N^2 T \log T)$$

Analog Phased Arrays



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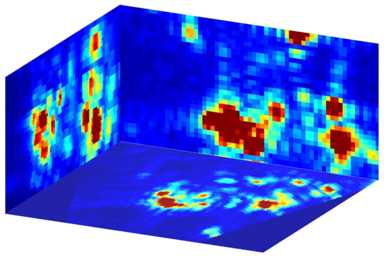
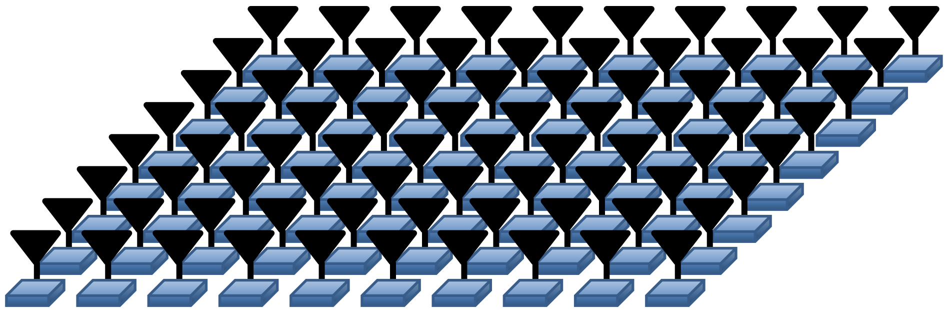
3D Heatmap Image



Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Could Potentially Do the Same thing:

1) Mix the RX signal with TX.

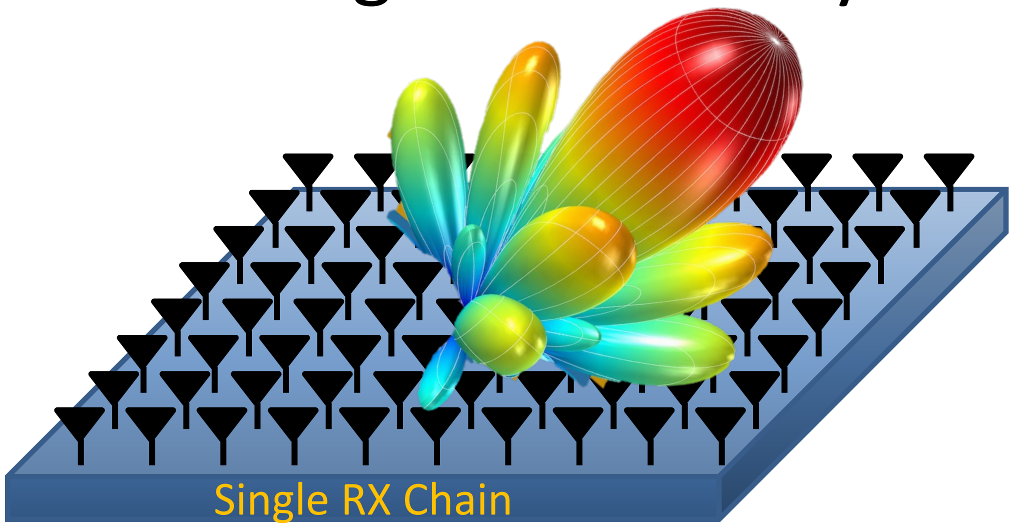
3) Compute Range FFT.

2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.

4) Repeat in every direction.

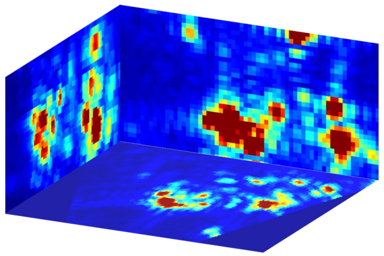
$$N \times N \times T \log T + N \times N \times T \times N \times N = O(N^4 T + N^2 T \log T)$$

Analog Phased Arrays



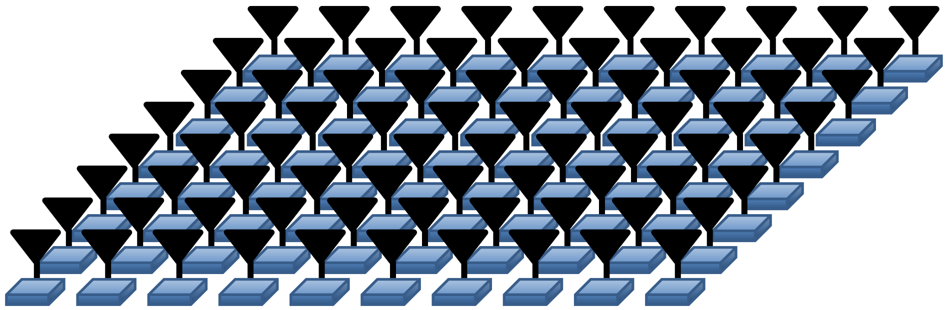
1) Pick the phase shift on each antenna to create a beam in each 3D direction.
2) Transmit FMCW signals and receive reflections.
3) Mix RX signal with TX and take range FFT .
4) Repeat in every direction.

3D Heatmap Image



Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Algorithm 1: $O(N^4 T + N^2 T \log T)$

1) Mix the RX signal with TX.

3) Compute Range FFT.

2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.

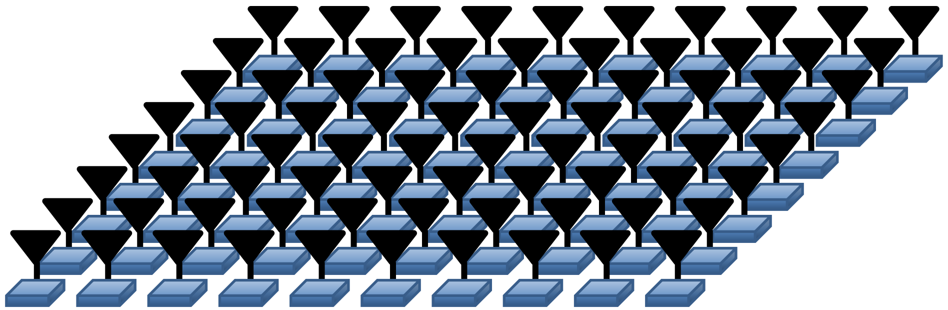
4) Repeat in every direction.

Algorithm 2: (Faster) **IDEA: antenna arrays are Fourier Transforms**

$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r + ms \sin\theta \cos\varphi + ks \sin\theta \sin\varphi)}$$

Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Algorithm 1: $O(N^4 T + N^2 T \log T)$

1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) **IDEA: antenna arrays are Fourier Transforms**

$$h_{m,k} = \sum_{\theta_l} \sum_{\varphi_l} \alpha_l e^{-j\frac{2\pi}{\lambda}(r_l + ms \sin\theta_l \cos\varphi_l + ks \sin\theta_l \sin\varphi_l)}$$

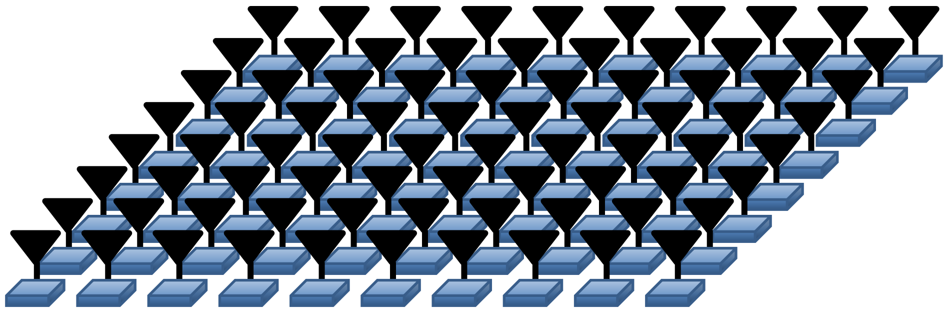
$$h(x,y) = \sum_{f_x} \sum_{f_y} P(f_x, f_y) e^{-j2\pi(xf_x + yf_y)}$$

Use 2D FFT

$$f_x = sin(\theta_l)cos(\phi_l),$$
$$f_y = sin(\theta_l)sin(\phi_l),$$
$$x = ms/\lambda,$$
$$y = ks/\lambda$$
$$P(f_x, f_y) = \alpha_l e^{-j\frac{2\pi r_l}{\lambda}}$$

Phased Arrays Primer

Digital Phased Arrays



$N\times N$ **RX Chains**

Algorithm 1: $O(N^4T + N^2T\log T)$

1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) **IDEA: antenna arrays are Fourier Transforms**

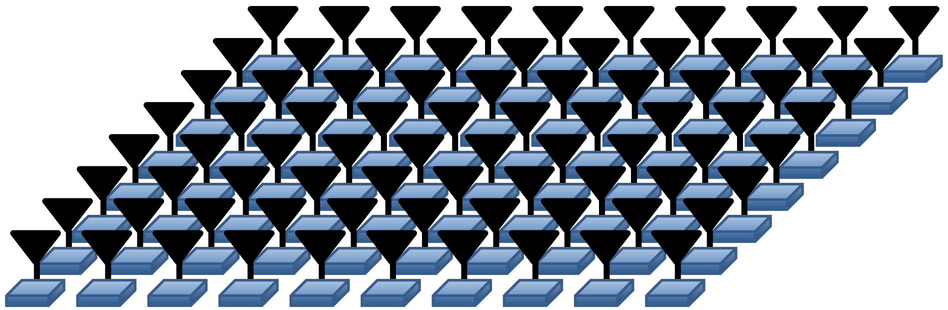
1) Mix the RX signal with TX.

$$\left.\begin{array}{l}\text{2) Compute 2D FFT across antennas}\\[1em] \text{3) Compute Range FFT.}\end{array}\right\}\ \textbf{3D FFT}$$

$$N^2\log N^2\times T + N\times N\times T\log T = O(N^2T\log N^2T)$$

Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Algorithm 3: (More Accurate)

Algorithm 1: $O(N^4 T + N^2 T \log T)$

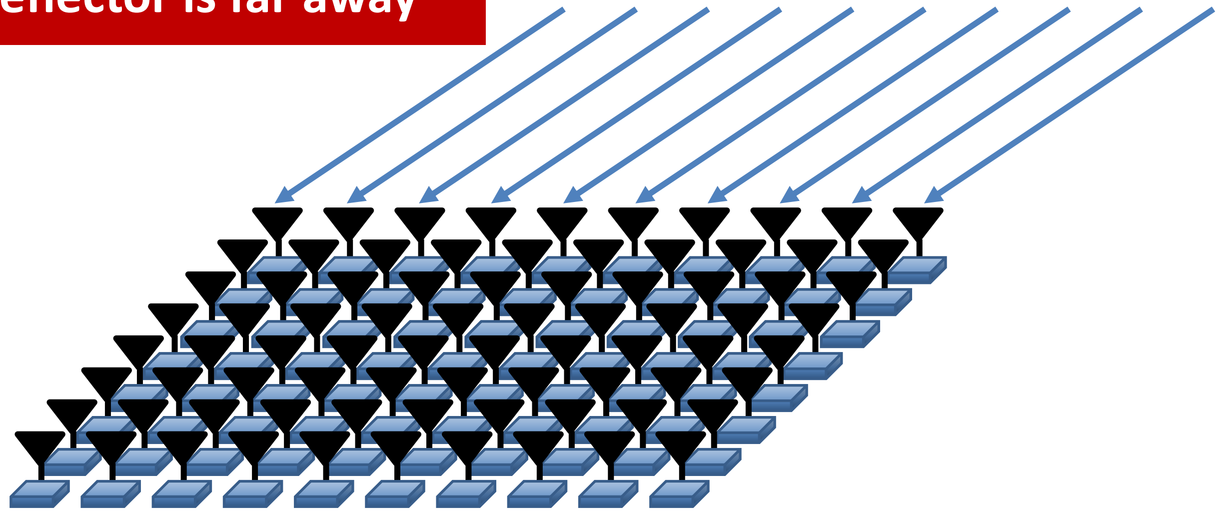
1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) $O(N^2 T \log NT)$ **3D FFT**

1) Mix the RX signal with TX.
2) Compute 2D FFT across antennas
3) Compute Range FFT.

Phased Arrays Primer

Assumes parallel waves
Reflector is far away

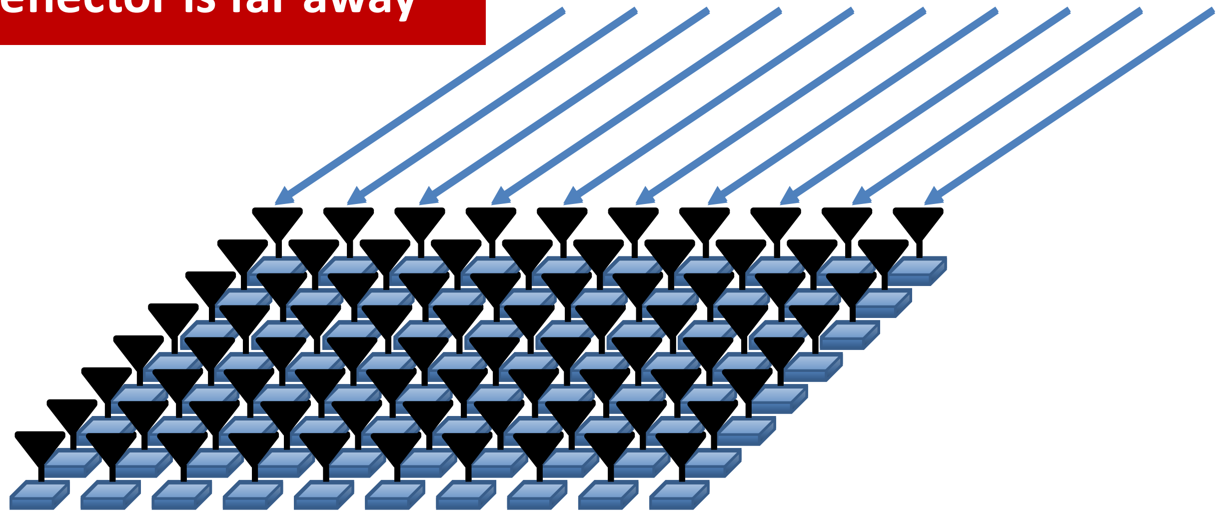


$N \times N$ **RX Chains**

Algorithm 3: (More Accurate)

Phased Arrays Primer

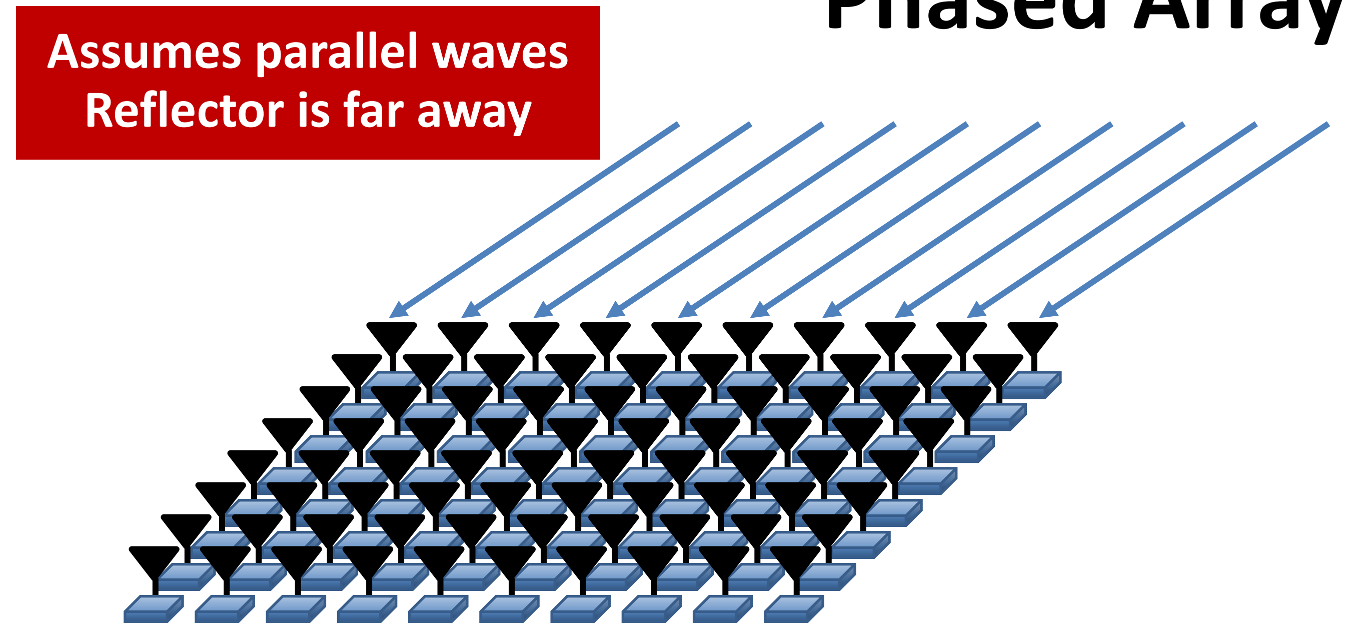
Assumes parallel waves
Reflector is far away



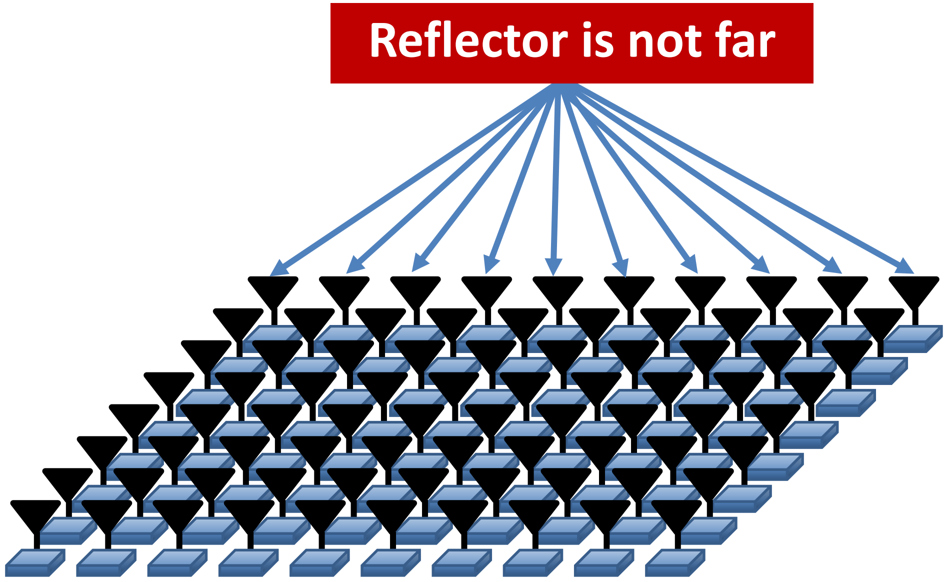
$N \times N$ RX Chains

Algorithm 3: (More Accurate)

Phased Arrays Primer



$N \times N$ **RX Chains**



$N \times N$ **RX Chains**

Algorithm 3: (More Accurate) **IDEA: use the exact equation**

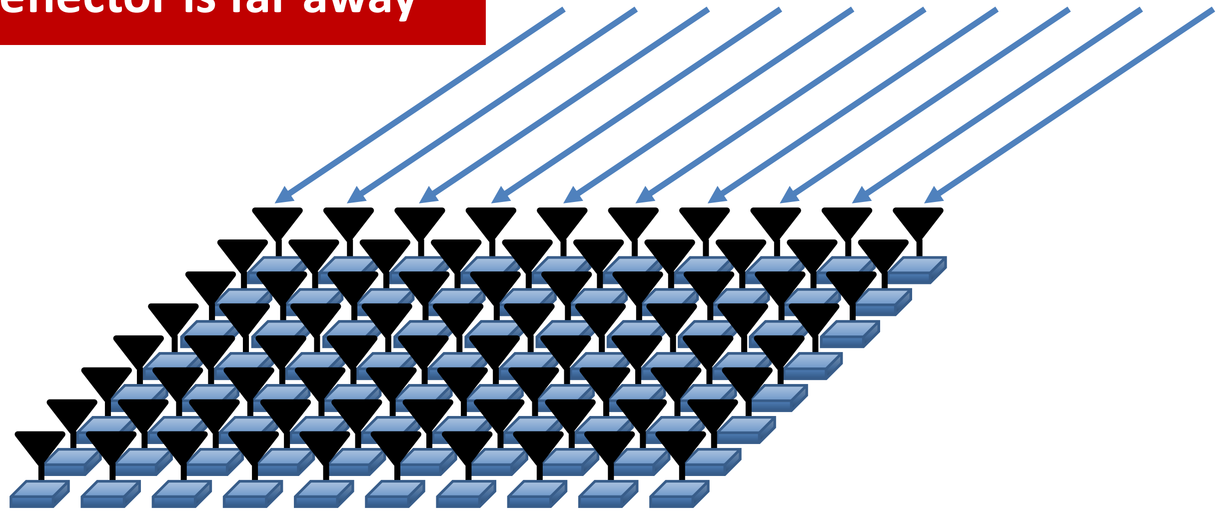
p: **FMCW Slope**

$$s_{m,k}(t) = \alpha_l e^{-j2\pi(p\tau_l t + f_0 \tau_l)} = \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

Received FMCW signal from reflector l after mixing with TX

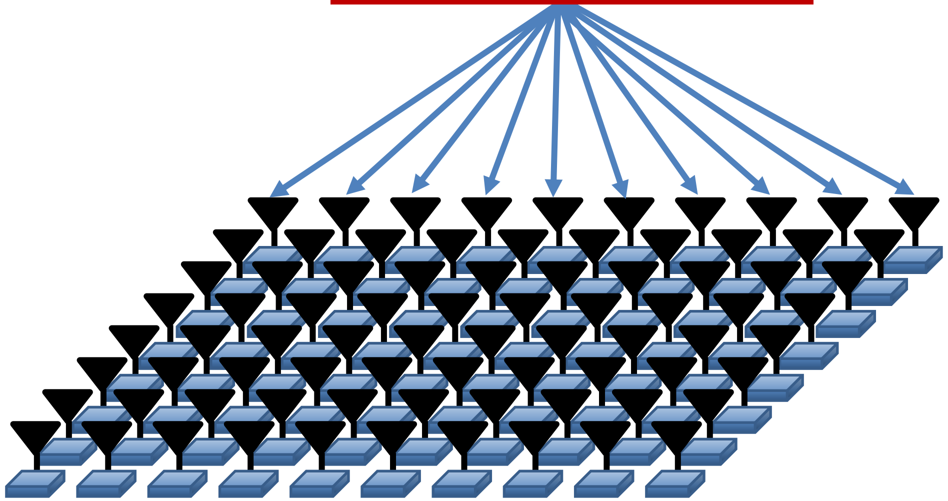
Phased Arrays Primer

Assumes parallel waves
Reflector is far away



$N \times N$ **RX Chains**

Reflector is not far



$N \times N$ **RX Chains**

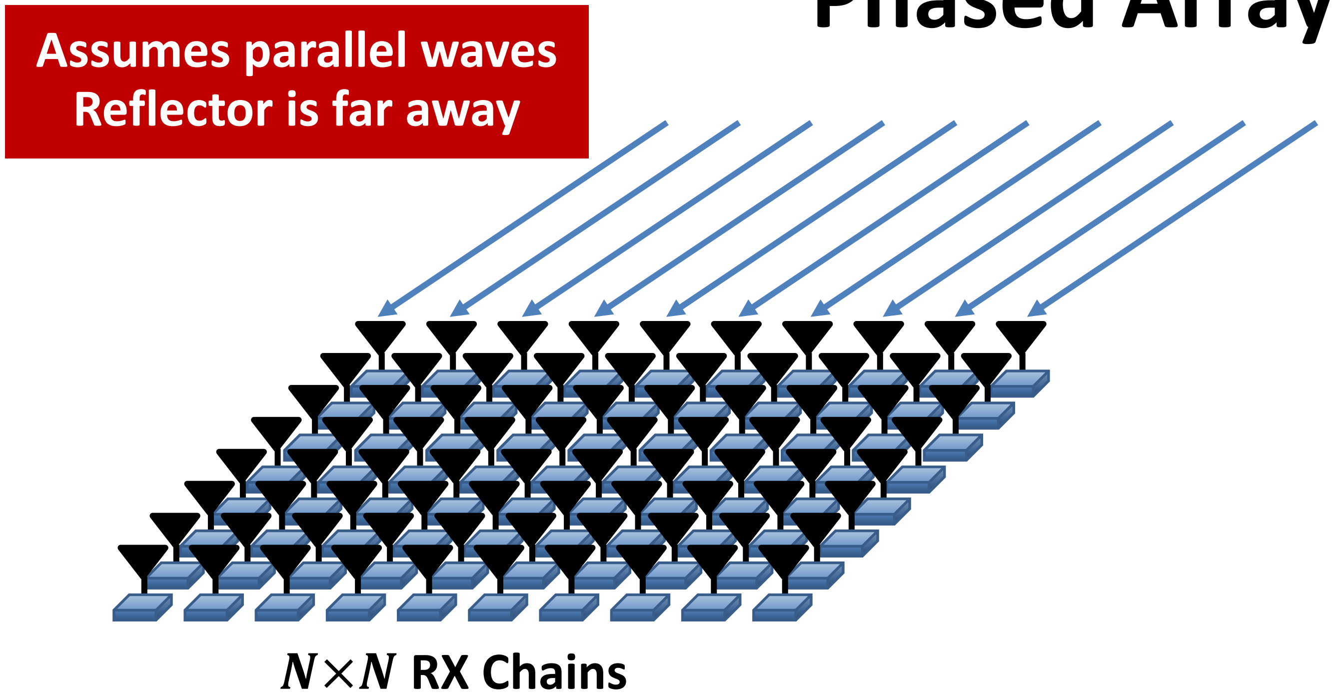
Assume antenna 0,0 at origin
$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

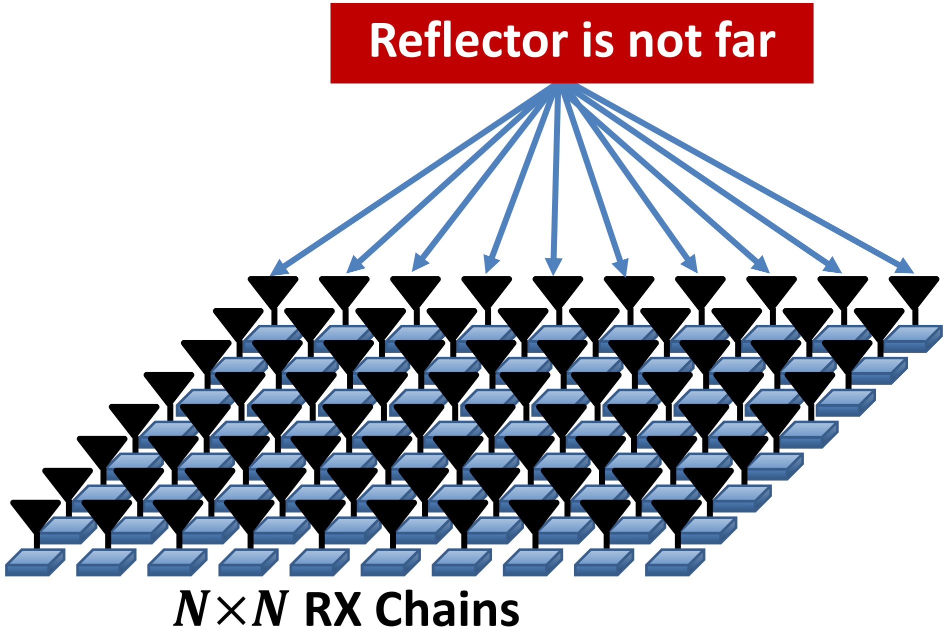
Algorithm 3: (More Accurate) **IDEA: use the exact equation**

$$s_{m,k}(t) = \alpha_l e^{-j2\pi(p\tau_l t + f_0 \tau_l)} = \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

$$d_l = \sqrt{(x_l - x_{m,k})^2 + (y_l - y_{m,k})^2 + (z_l - z_{m,k})^2} = \sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z_c)^2}$$

Phased Arrays Primer





Assume antenna 0,0 at origin
$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

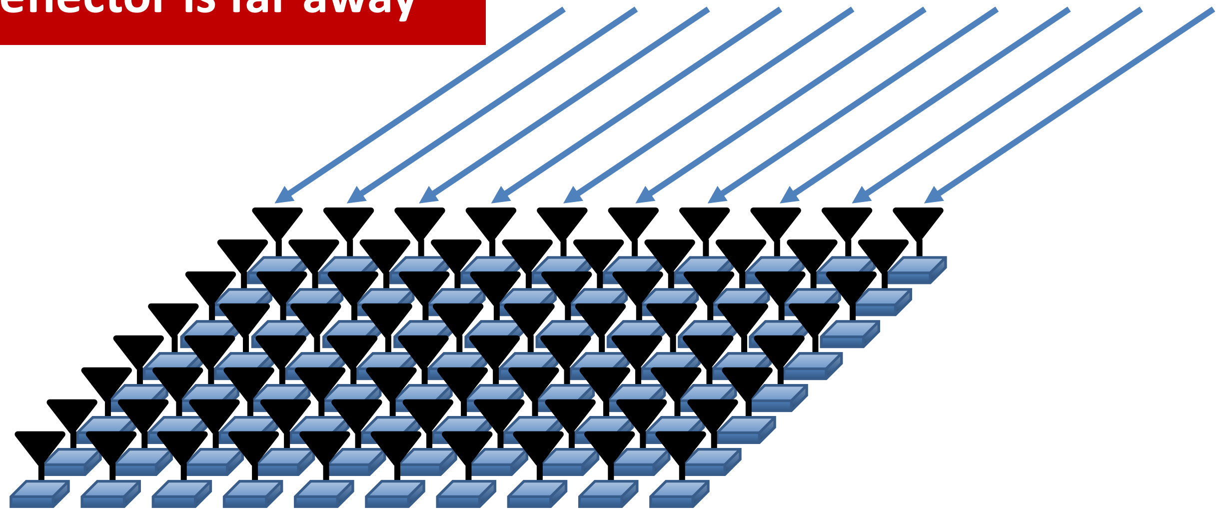
Algorithm 3: (More Accurate) **IDEA: use the exact equation**

$$s_{m,k}(t) = \alpha_l e^{-j2\pi(p\tau_l t + f_0\tau_l)} = \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

$$= \alpha_l e^{-j4\pi(pt/c + 1/\lambda)\sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z_c)^2}}$$

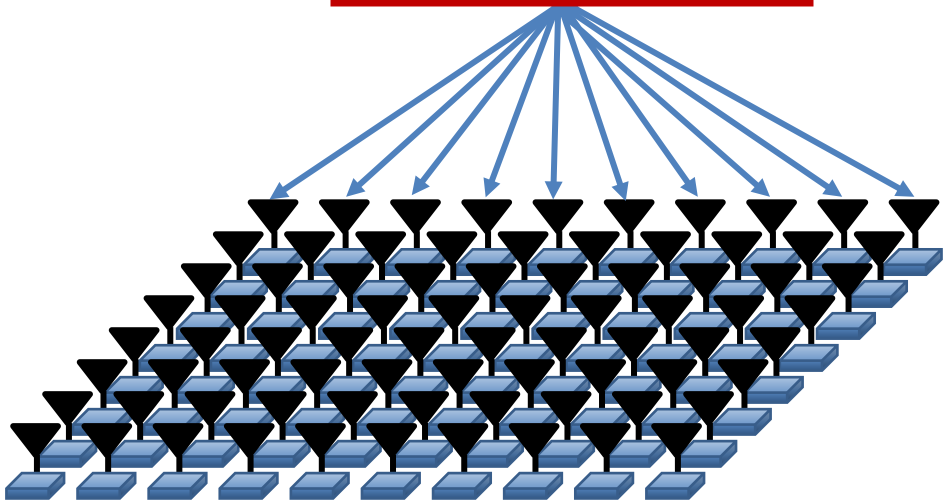
Phased Arrays Primer

Assumes parallel waves
Reflector is far away



$N \times N$ **RX Chains**

Reflector is not far



$N \times N$ **RX Chains**

Assume antenna 0,0 at origin
$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

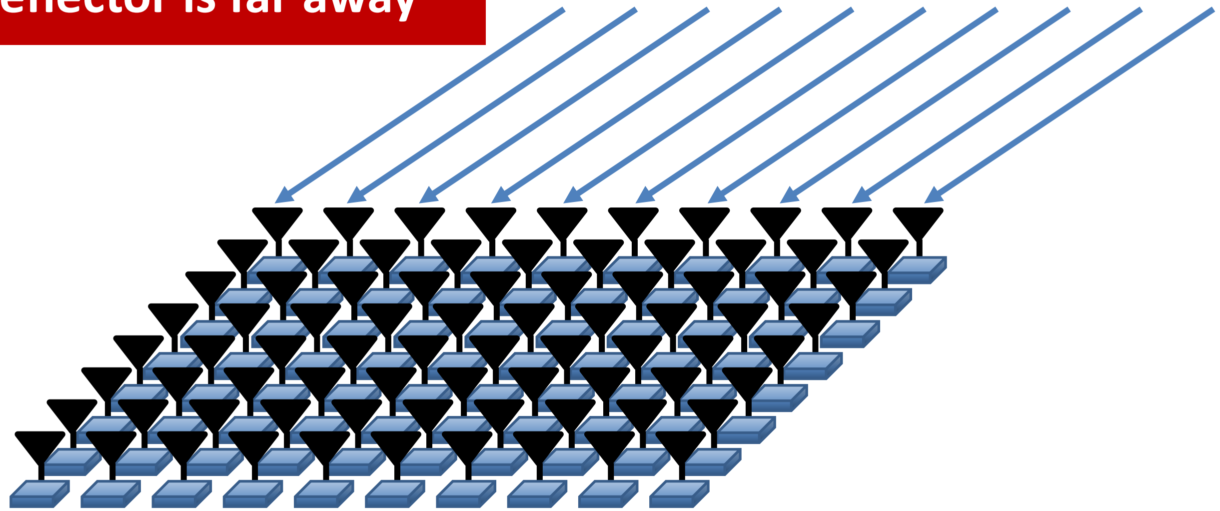
Algorithm 3: (More Accurate) **IDEA: use the exact equation**

$$s_{m,k}(t) = \sum_l \alpha_l e^{-j2\pi(p\tau_l t + f_0 \tau_l)} = \sum_l \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

$$= \sum_l \alpha_l e^{-j4\pi(pt/c + 1/\lambda)\sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z_c)^2}}$$

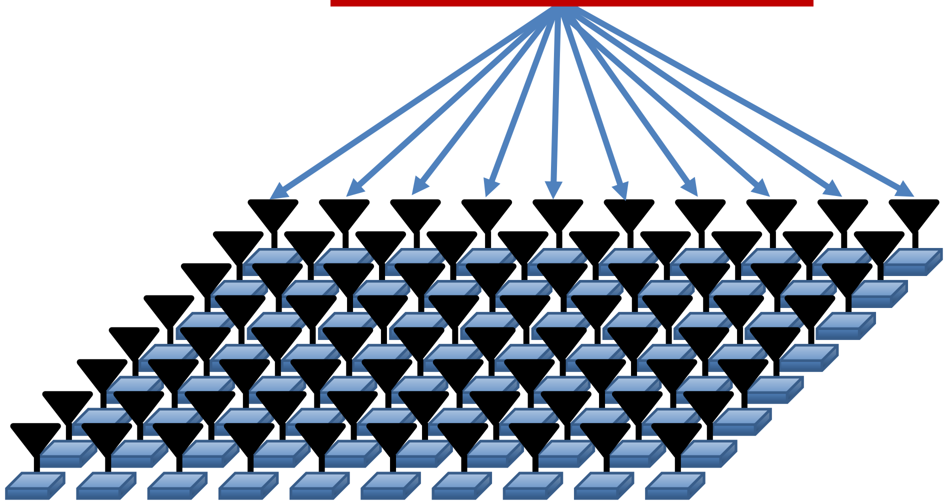
Phased Arrays Primer

Assumes parallel waves
Reflector is far away



$N \times N$ **RX Chains**

Reflector is not far



$N \times N$ **RX Chains**

Assume antenna 0,0 at origin
$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

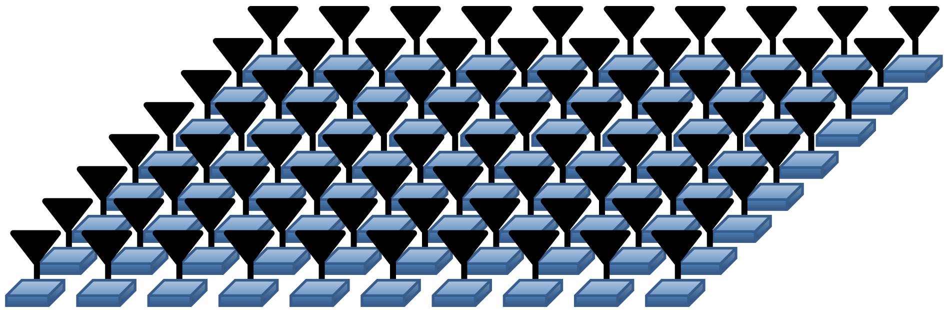
Algorithm 3: (More Accurate) **IDEA: use the exact equation**

$$s_{m,k}(t) = \sum_{l} \alpha_l e^{-j4\pi(pt/c+1/\lambda)\sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z)^2}}$$

$$P(x,y,z) = \sum_{m}^{N}\sum_{k}^{N}\sum_{t}^{T} s_{m,k}(t) \times e^{j4\pi(pt/c+1/\lambda)\sqrt{(x - ms)^2 + (y - ks)^2 + (z - z_c)^2}} = N^2 T \alpha_l$$

Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Algorithm 1: $O(N^4 T + N^2 T \log T)$

1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) $O(N^2 T \log NT)$ **3D FFT**

1) Mix the RX signal with TX.
2) Compute 2D FFT across antennas
3) Compute Range FFT.

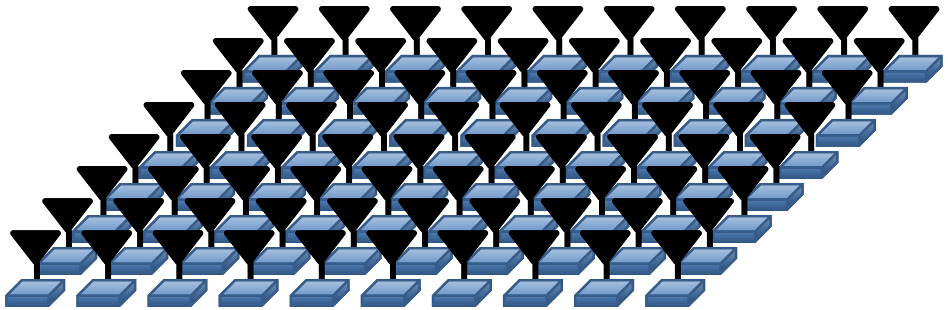
Algorithm 3: (More Accurate) $O(L^3 N^2 T) = o(N^5 T)$

1) Descritize space into $L \times L \times L$ grid.
2) For each point in space compute the received signal using the below equation.

$$P(x,y,z) = \sum_m \sum_k \sum_t s_{m,k}(t) \times e^{j4\pi(kt/c+1/\lambda)\sqrt{(x-ms)^2+(y-ks)^2+(z-z_c)^2}}$$

Phased Arrays Primer

Digital Phased Arrays



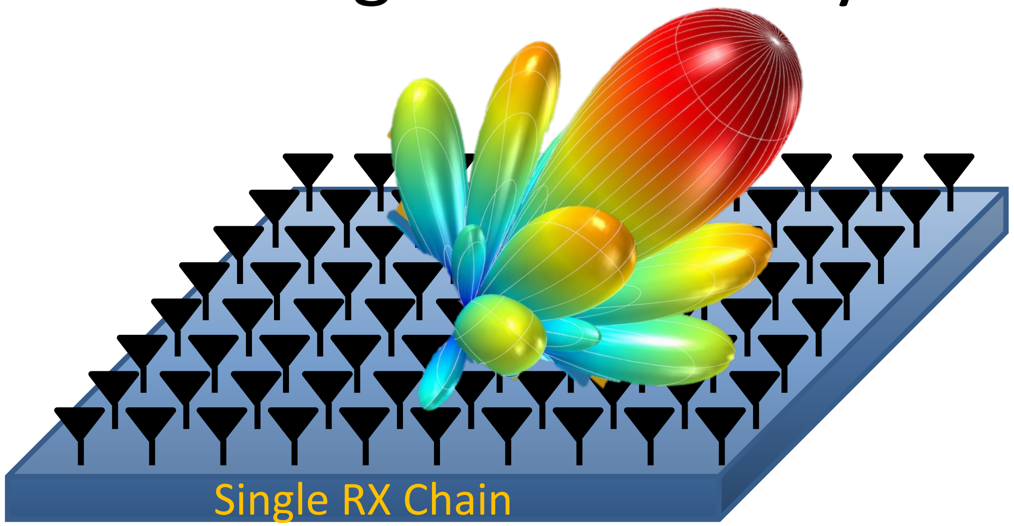
$N \times N$ **RX Chains**

Algorithm 1: $O(N^4T + N^2T\log T)$ **Similar to Analog**

Algorithm 2: (Faster) $O(N^2T\log NT)$ **3D FFT**

Algorithm 3: (More Accurate) $O(L^3N^2T) = o(N^5T)$

Analog Phased Arrays



1) Pick the phase shift on each antenna to create a beam in each 3D direction.
2) Transmit FMCW signals and receive reflections.
3) Mix RX signal with TX and take range FFT .
4) Repeat in every direction.

3D Heatmap Image