COM 405: Mobile Networks – Fall 2024
Homework 3 Solutions
EPFL

1 Cellular Systems (30 points)

We consider the task of designing a cellular network with frequency reuse. The number of available frequency channels in the system is $C = 100$ and the cell radius is $R_o = 1$ Km.

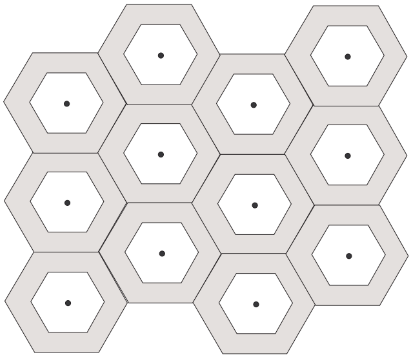
Throughout the whole exercise:

- The propagation loss is modeled as $P_{RX} = P_{TX} d^{-4}$ where P_{RX} is the received power, P_{TX} is the transmitted power and d is the distance between the mobile device and the base station.
- The interferences loss due to frequency reuse, is modeled by considering only the 6 closest interferers and assume that their distance from anywhere in the cell is always equal to the reuse distance D (the distance between two base-stations using the same frequency band).
- Refer to equations in lecture 9 slides 8 - 17 to solve this problem.

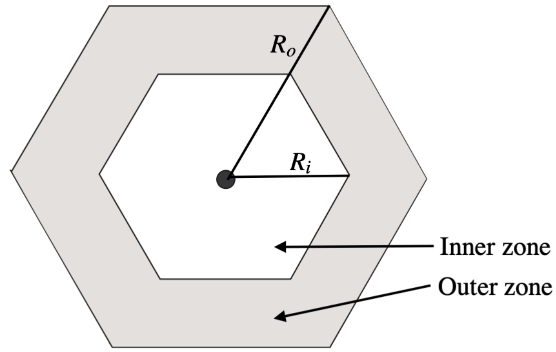
1. Given that the service requires an SIR of at least 17 dB anywhere in the cell, compute the maximum number of channels η each cell can be attributed.

> We first compute the cluster size K_o. Under the considered assumptions, we can use the following relation for the SIR at the edge of a cell (Lec. 9 Slide 17):
>
> $$\Gamma \approx \frac{(\sqrt{3K_o})^\alpha}{i_0} = \frac{(\sqrt{3K_o})^4}{6}$$
>
> 17 dB SIR is equivalent to 50.12 since $10\log_{10}(50.12) = 17$. Hence, we want $\Gamma \geq 50.12$ and we have $K \geq 5.78$. However, K_o must satisfy $K_o = (i+j)^2 - ij$. The closest minimum K_o that satisfies the inequality is 7. This provides each cell with a radio capacity of:
>
> $$\eta = \left\lfloor \frac{C}{K_o} \right\rfloor = \left\lfloor \frac{100}{7} \right\rfloor = 14 \; channels/cell$$

We now consider an optimization called reuse partitioning. This technique aims at increasing the radio capacity by splitting each cell into two concentric zones (inner and outer) of different radiuses R_i and R_o, as shown in the figure below. In practice, this means that there are two antennas on the same pole; one that provides connectivity for stations in the inner zone, and the other one for stations in the outer zone. The available radio channels are separated into two groups for which



(a) Partial network overview



(b) Cell overview

the channel allocation is treated independently. This is, with two different cluster sizes K_i (for the inner zone) and K_o (for the outer zone).

2. Given that $R_o = 1$ Km and $R_i = 0.7$ Km, compute the minimum cluster sizes K_i and K_o to achieve an SIR of at least 17 dB.

> We already computed the value of K_o from the previous question to be 7. For K_i, we use the follow equation (Lec. 9 Slide 16):
>
> $$\Gamma \approx \frac{R_i^{-\alpha}}{6D^{-\alpha}} = \frac{R_i^{-\alpha}}{6(R_o\sqrt{3K_i})^{-\alpha}} = \frac{1}{6}\left(\frac{R_o\sqrt{3K_i}}{R_i}\right)^{\alpha} = \frac{1}{6}\left(\frac{\sqrt{3K_i}}{0.7}\right)^{4} \geq 50.12(17dB)$$
>
> where we used $D = R_o\sqrt{3K_i}$ equation (Lec. 9 Slide 9). This gives us $K_i \geq 2.83$, so $K_i = 3$.

We now consider the allocation of the available radio channels to the inner and outer zones. We denote the number of radio channels allocated to the inner and outer zones of each cells η_i and η_o, respectively. Hence, the following inequality should be satisfied:

$$\eta_i K_i + \eta_o K_o \leq C$$

3. Assuming that the load for the outer zone is expected to be roughly $0.25\times$ the load for the inner one, propose maximal concrete values for η_i and η_o.

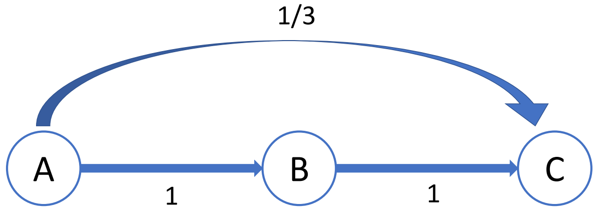
> We have:
>
> $$\eta_i K_i + \eta_o K_o = \eta_i K_i + 0.25\eta_i K_o = 4.75\eta_i \leq C = 100$$
>
> Hence, $\eta_i \leq 21.05$ and $\eta_o \leq 5.2$. Thuse, $\eta_i = 21$ and $\eta_o = 5$.

4. What is the improvement brought by resue partitioning in channels/cell?

> The total radio capacity per cell is $\eta_i + \eta_o = 21 + 5 = 26$ channels/cell, so the optimization improves the radio capacity by $26 - 14 = 12$ channels/cell.

2 Multihop Wireless Network (40 points)

Consider the simple wireless network in the figure below where the number on each link refers to the packet delivery probability on the forward path. (Assume the delivery probability on all reverse links is 1). All nodes are half duplex (Cannot transmit and receive at the same time). Also for this problem assume **no acks**.



1. What is the ETX of each of the above 3 links?

> A → B : 1
>
> B → C : 1
>
> A → C : 3

2. Consider a sender at A and a receiver at C. If the shortest-path routing protocol uses ETX as the link metric, which of the two paths it will pick: $A \to C$, or $A \to B \to C$?
   And, what is the shortest-path ETX? __________

> It will pick $A \to B \to C$ since its ETX is $2 < 3$ for $A \to C$

3. Alyssa Hacker notes that shortest-path routing does not leverage the broadcast nature of the wireless medium. Specifically, when node A broadcasts a packet on the wireless medium, both B and C receive this packet but with different probabilities (B receives the packet with probability 1, and C receives the packet with probability 1/3). Assume no other packets in the network. If the routing protocol is able to leverage the broadcast feature of the wireless medium, what is the expected number of transmissions to deliver a packet from A to C?

> With probability 1/3, it will take the packet a single transmission to reach C and with probability 2/3, it will take the packet 2 transmissions to reach C.
>
> Hence, expected number of transmissions is: $1/3 + 2/3 \times 2 = 5/3$.

4. Ben Bitdiddle tells Alyssa that it is difficult to design a routing protocol that leverages the broadcast nature of the medium. This is because node B does not know which of A's transmissions/packets have been received at C, and hence does not know which packets it should forward to C. He gives Alyssa the following challenge:

Say A has 3 packets to deliver to C. Design a scheme that can deliver all 3 packets to C, while leveraging broadcast to ensure that the expected number of transmissions per packet matches the value in the previous question. **Recall there are no acks in this problem.**

Tell us exactly what nodes A and B transmit in this scheme and why it works.

Hint: Consider using random linear combinations.

Say the packets are P1, P2, and P3.

A transmits P1, P2, P3.

B receives all 3 packets and C receives one of the packets.

B transmits two packets P4 and P5 that are random linear combinations of all 3 packets e.g.
P4 = 7 P1 + 3 P2 + P3
P5 = 4 P1 + P2 + 11 P3

C will now have P4, P5 and one of the 3 packets so it can solve the equations.

In total, we transmitted 5 packets to deliver 3 packets so the expected number of transmission per packet is 5/3 similar to above.

5. Alyssa points out to Ben that using random linear combinations can be computationally intensive as the number of packets increases. She gives back Ben the following challenge:

   Repeat the part (4) using only XOR on the bits of the three packets.

   Tell us exactly what nodes A and B transmit in this scheme and why it works.

Say the packets are P1, P2, and P3.

A transmits P1, P2, P3.

B receives all 3 packets and C receives one of the packets.

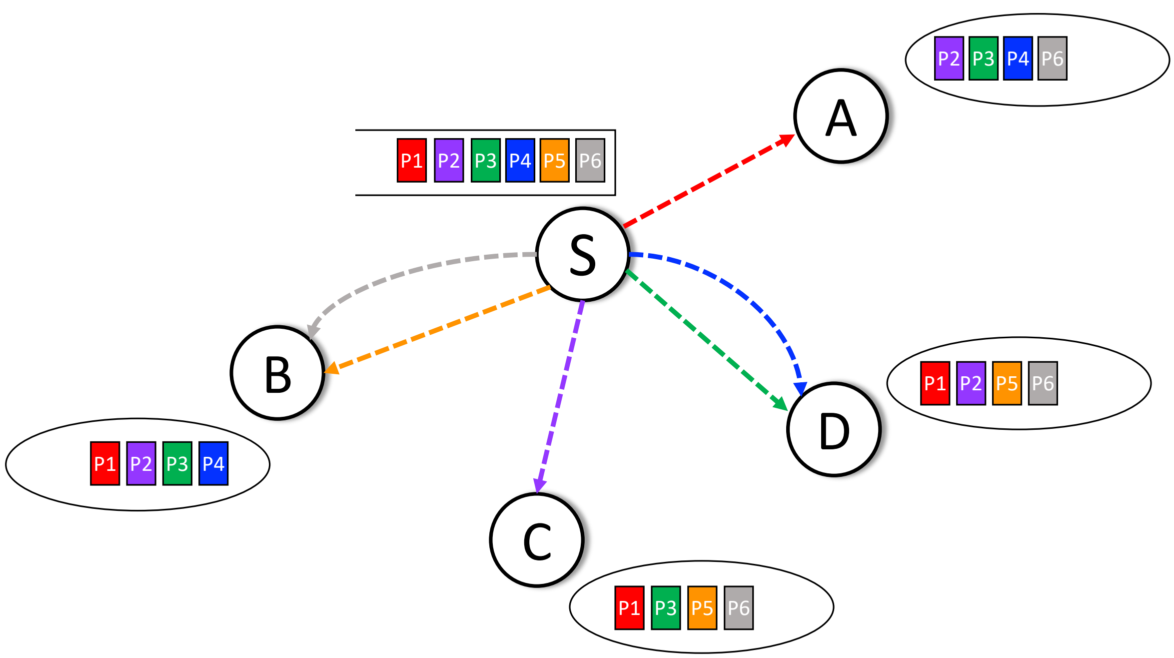
B transmits two packets P4 and P5 that are:
P4 = P1 $\oplus$ P3
P5 = P1 $\oplus$ P2

C will now have P4, P5 and one of the 3 packets.
If C has P1, it can get P2 = P5 $\oplus$ P1 and P3 = P4 $\oplus$ P1.
If C has P2, it can get P1 = P5 $\oplus$ P2 and P3 = P4 $\oplus$ P1.
If C has P3, it can get P1 = P4 $\oplus$ P3 and P2 = P5 $\oplus$ P1.

In total, we transmitted 5 packets to deliver 3 packets so the expected number of transmission per packet is 5/3 similar to above.

Note: B could also transmit P2 XOR P3 in place of one of the two packets.

6. Consider the following network. The buffers of each node is shown with the packets. The different colors represent different packets and where they are destined to go. Node S has 6 packets in its buffer. S has 4 neighbors: A, B, C, & D. Packet P1 destination is A, packet P2 destination is C, packets P3 and P4's destination is D, and packets P5 and P6's destination is B. What is the best coding strategy that S can employ? Explain your answer: how many packets get delivered, which neighbors benefit from such coding strategy and how do they benefit?



The best decoding strategy is to XOR packets: P1 $\oplus$ P2 $\oplus$ P3 $\oplus$ P6.

In this case 4 packets will be delivered and the 4 neighbors benefit from such strategy. This is because:

- A can receive P1 since it has P2, P3, and P6.
- B can receive P6 since it has P1, P3, and P4.
- C can receive P2 since it has P1, P3, and P6.
- D can receiver P3 since it has P1, P2, and P6.

3 Device Free Localization (30 points)

For his thesis, Daniel used 60 GHz millimeter wave signals to enable device free localization. He used FMCW signals to detect range and a single antenna with SAR to detect the angle of arrival. The FMCW signal uses a 5 GHz bandwidth which is swept over a period of 2 ms. The sampling bandwidth is 1 MHz. The SAR moves uniformly at 1 cm/s for 10 seconds.

1. Compute the ranging resolution and maximum detectable range in Daniel's system.

> Ranging resolution $= c/2B_s = 3 \times 10^8/10 \times 10^9 = 3cm$.
> Maximum Range $= cBT_s/2B_s = 3 \times 10^8 \times 10^6 \times 2 \times 10^{-3}/10^{10} = 60m$.

2. Name one advantage and one disadvantage of using millimeter wave frequencies for device free localization as opposed to 2.4 GHz frequencies.

> Advantage: Much more bandwidth $\rightarrow$ higher resolution.
> Disadvantage: Much smaller range since the signal decays quickly.

At this thesis defense, Daniel got a lot of questions and suggestions:

3. Lara pointed out that this is a bad design since the SAR antenna is moving during the sweep of the FMCW signal so the antenna changes location between the time it transmitted the signal and the time it received leading to an error in estimating the range.

   (a) Calculate the maximum ranging error that can result from the moving antenna.

   > The error will always be smaller than the distance moved by the antenna. During a sweep, the antenna moves by $2 \times 10^{-3}cm = 20\mu m$. (Saying $10\mu m$ is considered also correct since the antenna is moving uniformly and we will get the average.)

   (b) Daniel argues that this error in insignificant? Explain why.

   > The error is more than 100× smaller than the resolution of ranging and hence it will not affect the results.

   (c) Lara replied pointing out that even if the error in distance is small, the error in the phase which is used for applications like extracting breathing rate can be large. We consider phase error to be large if it is larger than 5°. Is Lara correct?

   > She is not correct. The wavelength $\lambda = c/f = 3 \times 10^8/60 \times 10^9 = 0.5cm$. The error in phase is $2\pi \times 2 \times 20\mu m/0.5cm = 16\pi \times 10^{-3} \Rightarrow 2.88°$.

4. Given the fact that the sampling rate in the FMCW system is small (1 MS/s) and the 60 GHz band has 14 GHz of unlicensed spectrum, David suggested that Daniel should increase his sweep bandwidth to 14 GHz in order to increase his resolution.

(a) What would the ranging resolution of the system be if we use the full unlicensed bandwidth?

> Ranging resolution $= c/2B_s = 3 \times 10^8/28 \times 10^9 = 1.07cm$.

(b) Suppose we maintain the same sweep slope, what is the new sweep time and the new maximum detectable range?

> Slope $= B_s/T_s = 5 \times 10^9/2 \times 10^{-3} = 2.5 \times 10^{12} Hz/s$.
> New sweep time $= 14 \times 10^9/2.5 \times 10^{12} = 5.6ms$.
> New maximum range $= 3 \times 10^8 \times 10^6 \times 5.6 \times 10^{-3}/28 \times 10^9 = 60m$.

(c) Name one advantage and one disadvantage of maintaining the same slope versus maintaining the same sweep time?

> Advantage: It is typically harder to build hardware that sweeps faster i.e. has the same sweep time for larger bandwidth.
> Disadvantage: It takes longer to sweep → delays detection and harder to use with fast moving objects.

(d) Daniel pointed out that it is very hard to increase the bandwidth to 14 GHz even if the ADC is still sampling at 1 MS/s. Explain why that is the case.

> It is typically harder to build hardware (mixers, filters, antennas) that operate over a large bandwidth like 14 GHz.

5. Sabrina pointed out that there are two problems with Daniel's SAR system:

(a) He needs a second antenna to compensate for the carrier frequency offset.

> This is not true. Since we are looking at reflections of the same signals, there is no CFO.

(b) It is hard to pinpoint the exact position at which the measurement was taken since the antenna is constantly moving. Hence, we do not know when the antenna is exactly at $\lambda/4$ positions so we cannot compute the angle of arrival using the measurements taken at those positions.

> While this is partially true, Daniel's system is not moving by $\lambda/4$ and then sampling the channel (i.e. transmitting the FMCW chirp). It is instead continuously transmitting while continuously moving. In other words it is oversampling. It takes a measurement every sweep $= 2ms$ which corresponds to $20\mu m = (\lambda/4)/62.5$. Hence, by using all the measurements, Daniel would be able to compensate for any error. Note, that had Daniel employed a stop and go strategy where he moves by $\lambda/4$ stops takes a measurement and moves again, Sabrina would have been correct.

Help Daniel defend his system against each of the above two claims?

6. Ben pointed out that 10 seconds is too long of a time to localize since the person, object, car being localized might have significantly moved with in 10 seconds. Ben suggests using millimeter phased arrays instead.

   (a) What is the size N of the equivalent phased array that would result in the same beamwidth to Daniel's SAR?

   > The SAR emulates an array of size $10cm/(\lambda/4) = 80$. Hence, Daniel would need an array of size 80.

   (b) Suppose the phased array sweeps $2N$ sectors. What is the minimum time to sweep all sectors? How does it compare to the time taken by SAR. (You can assume that the time is takes to change the beam pattern of the array is negligible.)

   > At the very least the array needs to stay for one full sweep in each sector. (It could stay more in order to average.) So the time would be $160 \times 2ms = 320ms \ll 10sec$. It is much smaller than 10 seconds and far more practical.