COM 405: Mobile Networks – Fall 2024
Homework 2
EPFL

Due Friday November 15 at 8:00pm

Instructions

- Homework is due Friday November 15 at 8:00pm on Moodle.
- Homework can be done in groups of two or individually.
- Homework can be submitted handwritten or typed. If handwritten, please make sure to have good handwriting. Anything we do not understand, we do not correct. Scan handwritten homework and Submit as pdf.
- If you find any typos, please do not hesitate to let us know.
- Recall, you do not need to ask us to submit the HW late. You can simply take advantage of the following late submission policy:

  0 – 24 hrs late: -0 points

  24 – 48 hrs late: -20 points

  48 – 72 hrs late: -40 points

  >72 hrs late: -100 points

1 Channel Contention 18 Points

Suppose nodes A and B are ready to send a packet. In the i^{th} round after $(i-1)$ collisions have already occurred, the two nodes can wait $0, 1, \ldots, 2^{i-1}-1$ slots until the next attempt, all 2^{i-1} choices having equal probability.

1. Find the probability q_i of a collision in the i^{th} round, given that there are collisions in the previous $(i-1)$ rounds (i.e., $q_1 = 1$, $q_2 = \frac{1}{2}$), for all $i \geq 1$.
2. Find the probability p_i that exactly i rounds are needed for the first success, and compute $p_1, p_2, \ldots, p_4$.
3. Now assume that after the first collision, node A "wins" the backoff and transmits successfully. After it is finished, both nodes try to transmit again (A has an infinite amount of traffic to send), causing a collision. Now compute the probability that A wins the channel for the next packet.

2 MAC in WiFi 5 vs. WiFi 6 (16 points)

Consider an AP and 4 clients using WiFi 802.11ac with CSMA/CA. The network administrator has disable RTS/CTS. The DIFS = $50\mu s$, the SIFS = $10\mu s$, the ACK takes $30\mu s$. Assume there is only downlink traffic with constant flow of packets to each client. Also assume the MAC is fair and the AP transmits to each client in a round robin schedule. The AP on average has to backoff for 6 slots where each slot is $10\mu s$.

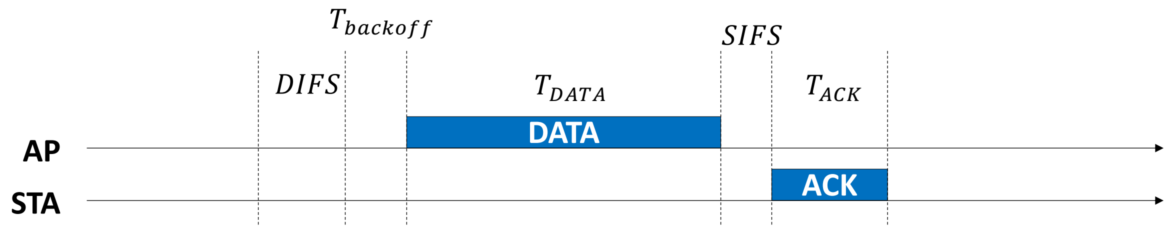

Figure 1: WiFi CSMA/CS without RTS/CTS

1. Compute the actual average throughput of a client in each of the following cases:
   (a) Packet size: 1500 bytes, Data Rate: 6.5 Mbps
   (b) Packet size: 1500 bytes, Data Rate: 390 Mbps

2. The network operator upgrades the AP and clients to 802.11ax with OFDMA. RTS/CTS is still disabled and all the network parameters are the same. Each of the Block ACK Request and Block ACK takes $30\mu s$. Assume that the AP allocates equal number of subcarriers to each client and ignore overhead in terms or guard subcarriers.

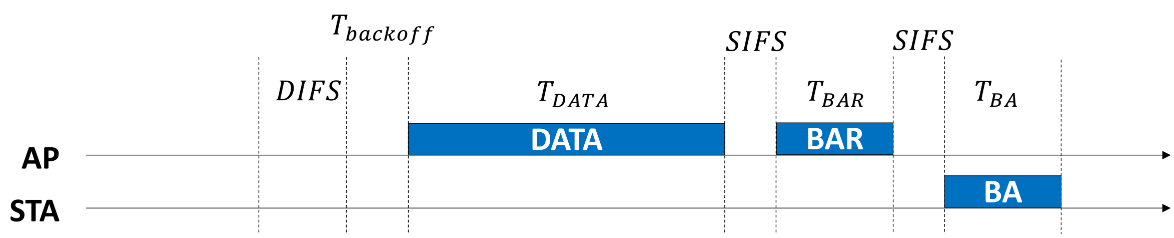

Figure 2: WiFi OFDMA without RTS/CTS

   Compute the actual average throughput of a client in each of the following cases:
   (a) Packet size: 1500 bytes, Single User Data Rate: 6.5 Mbps
   (b) Packet size: 1500 bytes, Single User Data Rate: 390 Mbps

3. In general when does OFDMA yield substantial throughput gain and why?

4. Does it still make sense to use OFDMA in cases where there is no throughput gain? Explain why or why not.

3 MIMO **26 points**

1. Consider a 3×3 MIMO system:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

   (a) (2 points) How many packets can be transmitted in parallel if the channel matrix $\mathbf{H} = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 9 & 3 \\ 6 & 5 & -3 \end{bmatrix}$

   (b) (6 points) Now suppose the channel matrix is complex: $\mathbf{H} = \begin{bmatrix} 1+j & 2 & 3 \\ 8 & 9-j & 2+j \\ 6+2j & 5 & -3 \end{bmatrix}$.
   We want to decode the packets in x_1, x_2, and x_3 using the projection method described in class. What are the vectors $\vec{h}_{23}^{\perp}$, $\vec{h}_{13}^{\perp}$ and $\vec{h}_{12}^{\perp}$ we need to project on to decode the packets.

2. Consider a 1×3 SIMO system where n is the noise:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0.02 - 0.01j \\ 0.05 \\ -0.03 + 0.04j \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

   Suppose $E[|x|^2] = E_s = 100$ and $E[|n_1|^2] = E[|n_2|^2] = E[|n_3|^2] = N_0 = 0.05$.

   (a) (3 points) The receiver decodes using $y = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3$. What values of α_1, α_2, and α_3 maximize the SNR?

   (b) (3 points) Compute the SNR in dB on each antenna if we decode using y_1, y_2, and y_3 independently.

   (c) (2 points) Compute the SNR if we combine the signals using the scheme derived in part (a).

   (d) (2 points) Compute the SNR if we combine the signals by setting $\alpha_1 = \alpha_2 = \alpha_3 = 1$.

3. Consider a 2×1 MISO system:

$$y = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.02 - 0.01j & -0.03 + 0.04j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   (a) (2 points) The transmitter wants to transmit x. It sets $x_1 = \alpha_1 x$ and $x_2 = \alpha_2 x$. What values of α_1 and α_2 maximize the SNR at the receiver?

   (b) (4 points) Suppose the transmitter uses Alamouti Codes to transmit. Suppose it is using 16 QAM and the first symbol is $(1+j)/\sqrt{10}$ and the second symbol is $(-3+j)/\sqrt{10}$. What values will the transmitter transmit on x_1 and x_2 in the first and second time slot?

   (c) (2 points) Name one advantage and one disadvantage of using Alamouti codes compared to scheme derived in the first part.

4 Scheduling (40 points)

1. Consider a wireless TDMA system with 4 mobile client connected to the base station. The data rates of the four mobile clients C_1, C_2, C_3, and C_4 are $R_1 = 20$ Mbps, $R_2 = 50$ Mbps, $R_3 = 70$ Mbps, and $R_4 = 140$ Mbps respectively. The base station divides time into equal time slots of length T and schedules a single mobile client in each time slot. Compute the throughput of each client and the whole network throughput when the following scheduling strategy is used:

   (a) Round-Robin Scheduling

   (b) Max-Throughput Scheduling

   (c) Max-Min Scheduling

2. Recall that the Proportional fair (PF) scheduler is a compromise scheduling policy, trying to balance the completing interests of maximizing the total network throughput and providing all clients with a minimal level of service.

   Let M be the number of clients and S_i the long run throughput of client i. The PF scheduler aims to maximize the following objective function:

   $$\sum_{i=1}^{M} \ln S_i$$

   The throughput in slot $t-1$ is denoted as $S_i[t-1]$. $S_i[t]$ is updated as follows:

   $$S_i[t] = \left(1 - \frac{1}{\tau}\right) \times S_i[t-1] + \frac{1}{\tau} \times R_i[t] I(i,t)$$

   where τ is a constant > 1, $R_i[t]$ is the data rate of user i at time t, and $I(i,t)$ is the indicator function, i.e., $I(i,t) = 1$ if client i is scheduled at time t, and 0 otherwise.

   Prove that in order to maximize the object function, the client with the highest $R_i[t]/S_i[t-1]$ should be scheduled.

3. Consider a wireless TDMA system with 2 mobile clients. The base station divides time into equal time slots of length T and schedules a single mobile client in each time slot using a proportional fair (PF) scheduler. The transmission continues for the next two time slots $t = 2$ and $t = 3$. The data rates of the two mobile clients C_1 and C_2 are for all t, $R_1[t] = 32$ Mbps and $R_2[t] = 64$ Mbps respectively. The average throughput is updated using the following equation:

   $$S_i[t] = \left(1 - \frac{1}{t}\right) \times S_i[t-1] + \frac{1}{t} \times R_i[t] I(i,t)$$

   where $I(i,t) = 1$ if client i is scheduled in time slot t and $I(i,t) = 0$ if client i is not scheduled.

   Assume $S_1[1] = 16$ Mbps and $S_2[1] = 48$ Mbps. Calculate the average throughput of the two clients in the following two time slots: $S_1[2], S_2[2], S_1[3], S_2[3]$.