

# Ad Auctions

Internet Analytics (COM-308)

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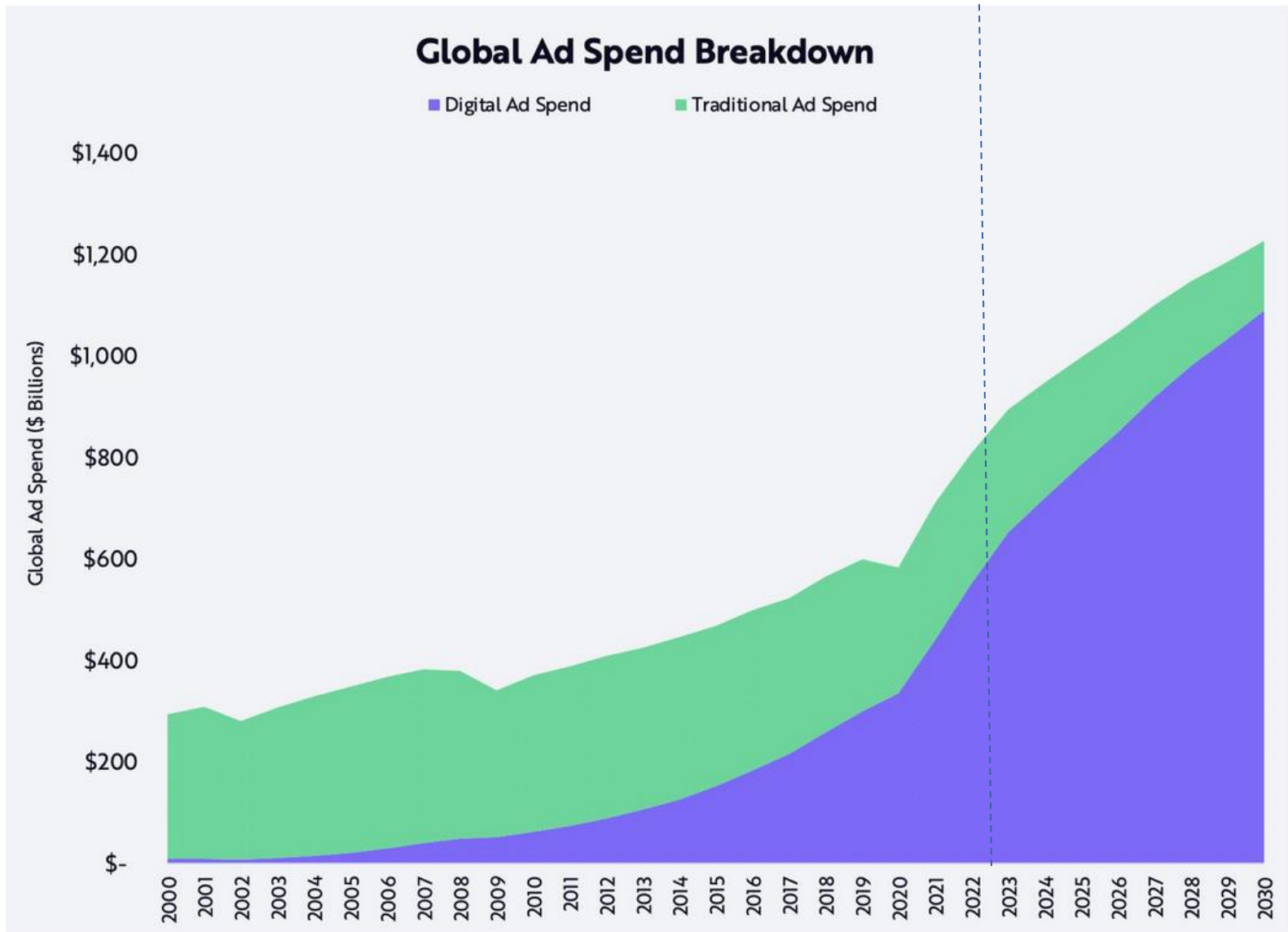
# Overview

- Advertisement is **the** Internet business model
  - Plus a bit of subscription, freemium, etc.
- Most ad impressions are auctioned
  - Google AdSense: banner ads on third-party websites
  - Google AdWords: search ads (“sponsored search result”)
- Classical auctions: single item
  - First vs second price
- Ad auctions: multiple items = several “positions” on web page
  - Generalized Second Price auction
  - VCG auction

# Online advertisement market

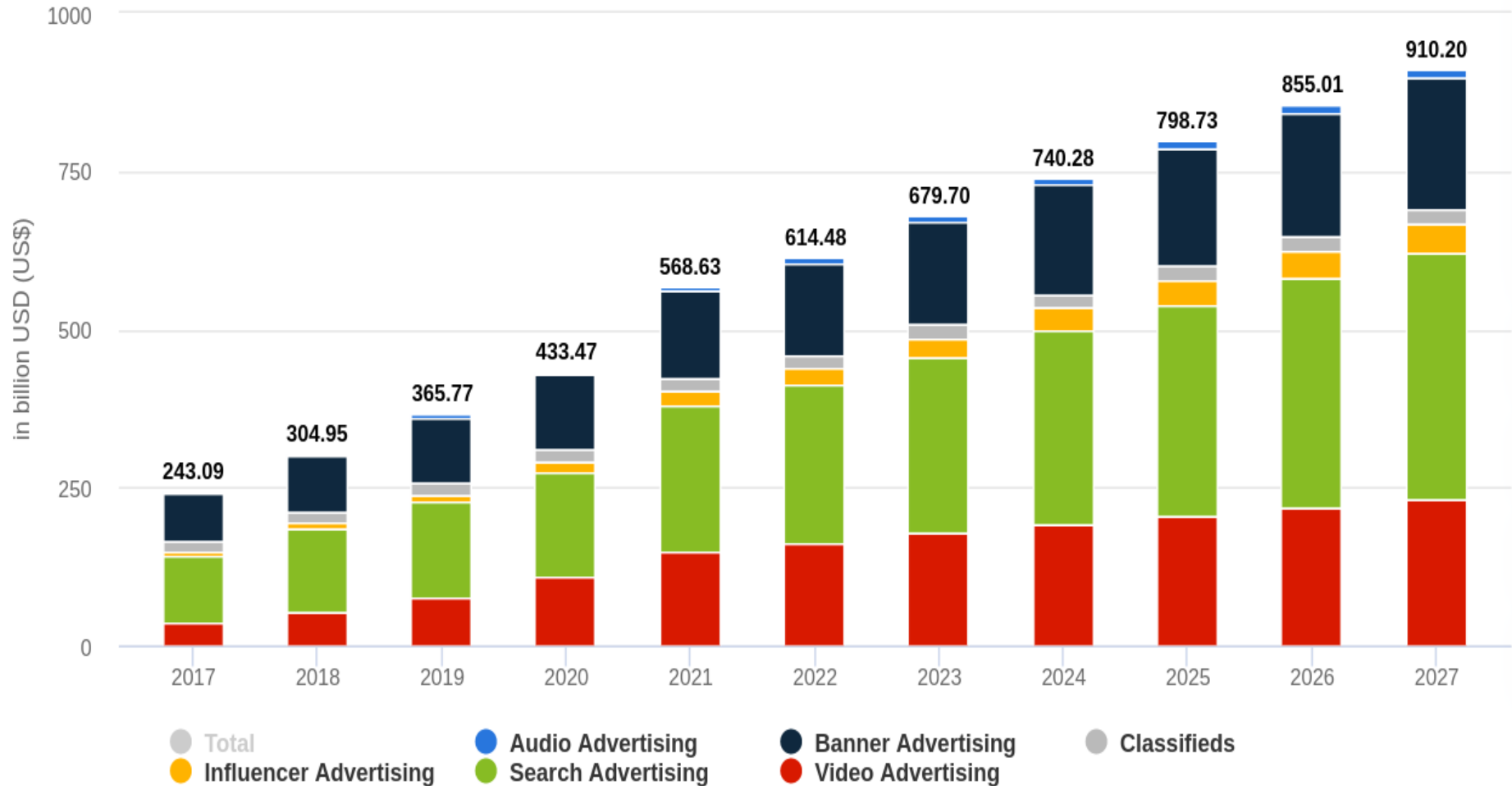
- Business models for the Internet:
  - (a) Paying for it: subscription, per-use, taxes, ...
  - (b) Providing data and consuming advertisement
- “If you’re not paying for it, you’re the product”
  - Willingness to sacrifice privacy and “paying through attention”
  - This model is not just about the money, but ease of use
- Ad market:
  - Globally ~ 800bn\$/year
  - Online: ~ 600bn\$/year (2022)
    - Web: mature; mobile: still growing challenge, different models

# Global advertisement market



[source: finmoov.com, 2023]

# Global digital advertisement market

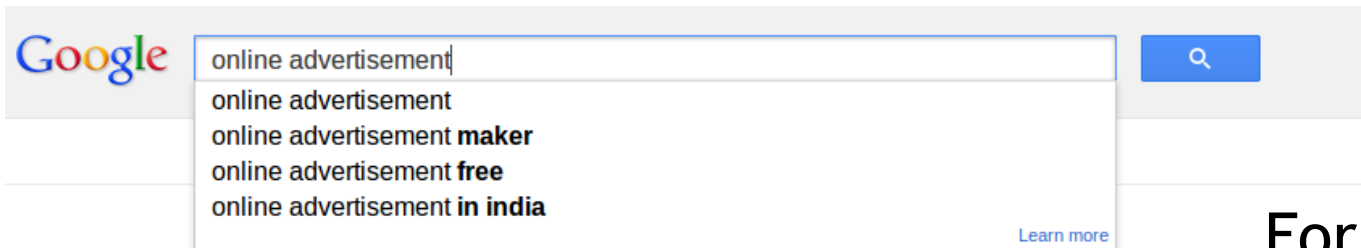


[source: statista.com, 2023]

# Online advertisement market

- Banner ads:
  - Pay-per-impression (user **sees** ad)
    - CPM (cost-per-mille)
  - Pay-per-click (PPC) (user **takes action**)
    - CTR (clickthrough rate = clicks per impression)
    - CPC (cost per click)
- Search ads / sponsored search
  - Google AdWords: bidding for keywords plus other constraints (geographic, max cost,...)
  - Google shows your ads in response to searches, charges for traffic

# Ad auctions



Ads related to **online advertisement** ⓘ

**Online Email Advertising - VerticalResponse.com**  
[www.verticalresponse.com/FreeTrial](http://www.verticalresponse.com/FreeTrial)  
Trusted by 500k+ for Reliable Email Marketing Since 2001. Try it Free!  
238 people +1'd or follow VerticalResponse  
Flexible Pricing Options - Features - Pay As You Go - Start Free Trial Now

**Online Advertising - Create your online marketing strat.**  
[www.agencevirtuelle.com/OnlineMarketing](http://www.agencevirtuelle.com/OnlineMarketing)  
AdWords, SEO, Mobile, Social Media.

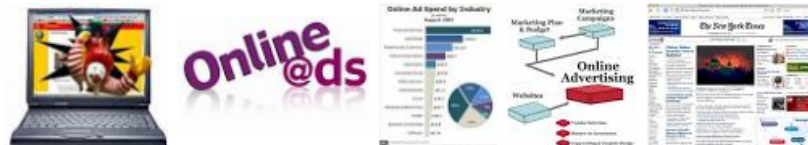
For each search, this table of “sponsored search results” is the result of an online auction

**Online advertising - Wikipedia, the free encyclopedia**  
[en.wikipedia.org/wiki/Online\\_advertising](http://en.wikipedia.org/wiki/Online_advertising)

**Online advertising**, also known as **online advertisement**, internet marketing, online marketing or e-marketing, is the marketing and promotion of products or ...  
[History of online advertising - Competitive advantage over ... - Online advertisement](#)

**Online Advertising: How to Do It Right | Small Business Trends**  
[smallbiztrends.com/2010/11/online-advertising-how-to-do.html](http://smallbiztrends.com/2010/11/online-advertising-how-to-do.html)  
Nov 4, 2010 – Helpful tips on using **online advertising** for small businesses.

**Images for online advertisement** - Report images



**Internet advertising: The ultimate marketing machine | The Economist**  
[www.economist.com/node/7138905](http://www.economist.com/node/7138905)  
Jul 6, 2006 – IN TERMS of efficiency, if not size, the **advertising** industry is only now

# Ad auctions

- The slowly disappearing ad marker... ;)

The screenshot shows a Google search interface. The search bar contains the text 'cruise package'. Below the search bar, the 'All' tab is selected. The search results show 'About 562'000'000 results (0.34 seconds)'. The first result is an advertisement for CruiseDirect.com, titled 'Save Up To 80% Off Cruises | CruiseDirect.com®'. The ad is marked with a red box around the 'Ad' label. The ad text describes the benefits of CruiseDirect.com, including eTickets, flexible payment plans, and low prices. Below the ad text, there are two promotional cards: '7 Night Caribbean Cruises' and 'Norwegian Cruise Deals'. The second result is for Expedia, titled 'Cruises: Find Cheap Cruise Deals & Last Minute Cruises | Expedia', with a link to 'https://www.expedia.com/Cruises'.

Google

cruise package

All Images News Videos Maps More Settings Tools

About 562'000'000 results (0.34 seconds)

**Save Up To 80% Off Cruises | CruiseDirect.com®**

**Ad** [www.cruisedirect.com/](http://www.cruisedirect.com/) ▼

We've Helped Thousands Of People Find The Perfect **Cruise** Deal. Search Online Now. eTickets Available. Flexible Payment Plans. Low Price Guarantee. 15 Years In Business. Highest Rated Agency. Magellan Award Winner. No Booking Fees. Tons Of Freebies. Last Minute Options.

[7 Night Caribbean Cruise](#) - from US\$ 399.00 - Norwegian Cruise Lines · More ▼

**7 Night Caribbean Cruises**  
Free Liquor Package, Up To \$1,000  
Onboard Cash, Sail Norwegian Today!

**Norwegian Cruise Deals**  
Exceptional Dining, Free Open Bar.  
Book Today For Freestyle Cruising!

**Cruises: Find Cheap Cruise Deals & Last Minute Cruises | Expedia**  
<https://www.expedia.com/Cruises> ▼

Expedia has deals on cheap **cruises** from top **cruise** providers so you can explore for ... Search over a million flights, hotels, **packages** and more; No Expedia ...



# Search ads: how to allocate?

- Resource allocation problem:
  - One seller: google, amazon, bing, etc.
  - Many bidders: advertisers buying visibility
  - Several items: multiple locations on results panel
- Auction:
  - Google runs an auction for every search to sell the ad space!
  - Several billion searches per day
  - Very heavy tailed CPC distribution:
    - “mesothelioma” had CPC up to 79 US\$ a few years ago
      - asbestos lawyers

# Auctions

- Open vs sealed envelope
  - Public vs private bids
- Open: ascending (English) vs descending (Dutch)
  - Ascending: increase the price until single bidder left
  - Descending: decrease the price until a bidder calls out
- First-price vs second-price
  - First: winning bidder pays highest bid
  - Second: winning bidder pays second-highest bid
- Equivalences:
  - Descending equivalent to sealed first-price
    - Winning bidder calls out when willing to pay the price
  - Ascending equivalent to sealed second-price
    - Winning bidder stops bidding when second-highest drops out

# Single-item auctions

- One item for sale, many potential buyers
  - $N$  bidders
  - Bid of bidder  $i$ :  $b_i$
  - Internal valuation of bidder  $i$ :  $v_i$ 
    - Advertisement: revenue generated by ad
    - Art: monetary measure for viewing pleasure/pride/envy of friends/...
  - Price paid by bidder  $i$ :  $p_i$ 
    - 0 if lost, price determined by auction mechanism if won
- Payoff (or utility):
  - $$U_i = \begin{cases} 0 & \text{if lost} \\ v_i - p_i & \text{if won} \end{cases}$$

# Single-item auction

- Valuation, price, payoff
  - Valuation  $v_i$ : depends only on bidder (personal preference, business case, etc.)
  - Price  $p_i = p_i(b_1, b_2, \dots, b_N)$ : depends on everybody's bid through the auction mechanism; valuations are hidden and do not directly affect price
- Strategy:
  - Each bidder selects bid  $b_i$  that maximizes  $U_i(b_1, b_2, \dots, b_N)$
  - $b_i$  too low: risk not winning the auction
  - $b_i$  too high: risk paying too much

# Why second-price auction?

- Intuition:

- First price seems natural and reasonable:
  - Bid what you are willing to pay = value
- Second-price seems manipulable:
  - Bid very high to win, pay only 2<sup>nd</sup> price

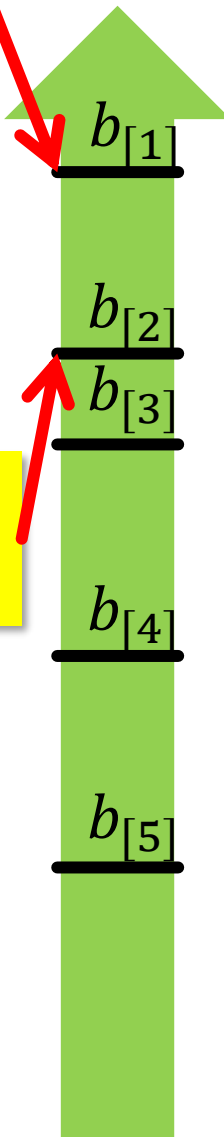
- Theory:

- First price: bidding valuation means zero payoff → must bid less than valuation
- Second-price: bidding too high is a bad strategy, if others follow the same strategy → will pay above value, negative payoff

Price with  
1<sup>st</sup> -price

winner

Price with  
2<sup>nd</sup> -price



# Truthful bidding in second-price

- Theorem: in a second-price auction, truthful bidding ( $b = v$ ) is a dominating strategy
  - Dominating: regardless of what strategy other players use, best strategy for myself
- Proof: assume I bid  $b'$  instead of  $b = v$ 
  - Case  $b' < v$  (“under-bidding”):
    - Affects outcome only if 2<sup>nd</sup>-highest bid  $b_{[2]}$  is  $b' < b_{[2]} < v \rightarrow$  auction lost,  $U = 0$  instead of  $U = v - p = v - b_{[2]} \geq 0$
  - Case  $b' > v$  (“over-bidding”):
    - Affects outcome only if highest bid  $b_{[1]}$  is  $v < b_{[1]} < b' \rightarrow$  auction won,  $U = v - p = v - b_{[1]} < 0$  instead of  $U = 0$

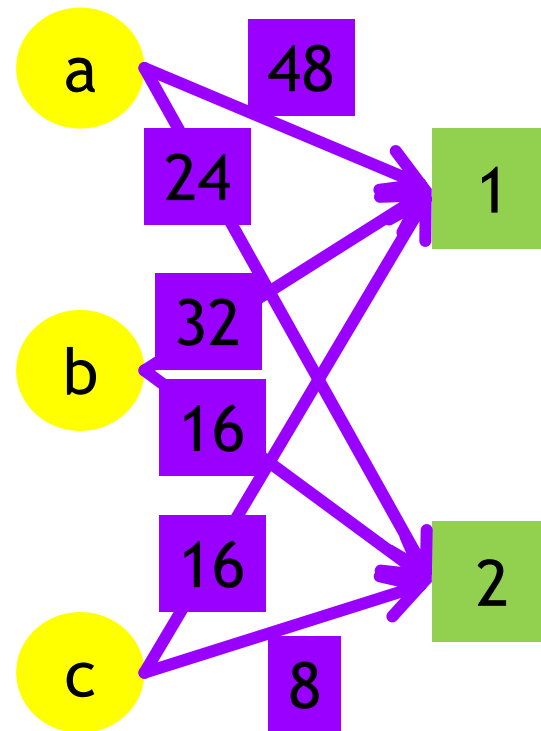
Second price below  $b'$

# Multiple-item auctions

- $K$  spaces for sale (decreasing value)
- Bid vector:  $b_i = (b_{i1}, b_{i2}, \dots, b_{iK})$
- Allocation (who gets what):
  - Maximum matching:  $M$  maximizes  $\sum_{(i,j) \in M} b_{ij}$
- Generalized Second Price (GSP):
  - Winner of  $k$ th item pays  $(k + 1)$ st bid
  - Simple, used in Google AdWords
- Vickrey-Clarke-Groves (VCG):
  - Bidder  $i$  pays its “damage” (externality in economics-speak) on everybody else
  - More on this later...

# Generalized Second-Price (GSP)

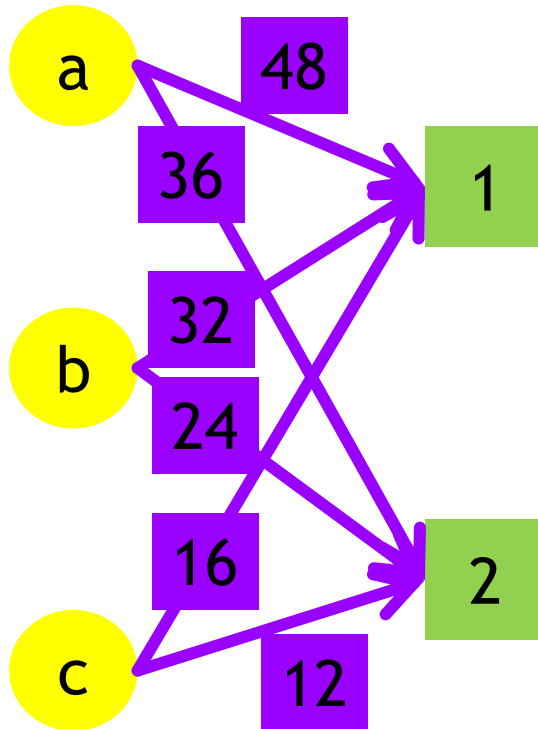
- Ad auctions:
  - Value = CTR x value per click
  - Normally CTR decreases with list, value-per-click assumed independent of position
  - Maximum matching = {(highest bidder, 1<sup>st</sup> position), (2<sup>nd</sup> highest bidder, 2<sup>nd</sup> position), ...}
- Example:
  - Assume CTR of (4%,2%) and valuations per click of (12,8,4)
  - Prices (for truthful bids  $b = v$ ):
    - a pays 32 (8 x 4% CTR)
    - b pays 8 (4 x 2% CTR)





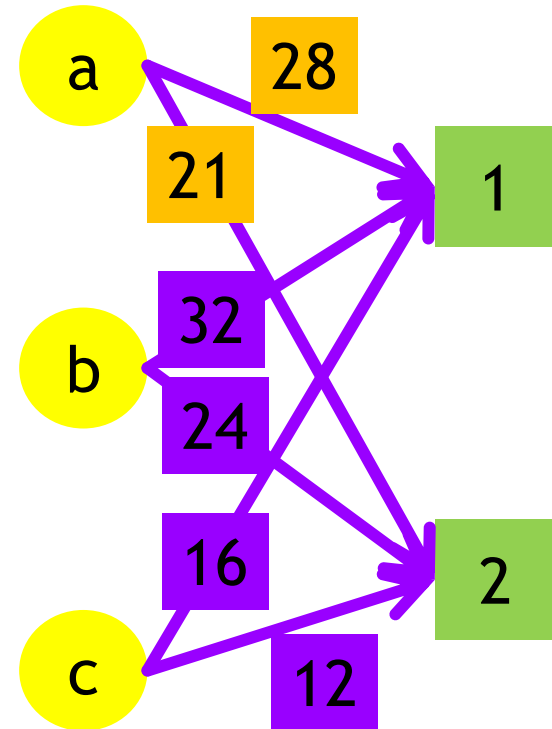
# Non-truthful bidding in GSP

- GSP is not incentive-compatible - example:



Truthful bidding:

$$\begin{aligned} \text{a: } u_a &= v_a - p_a = \\ &= 48 - 32 = 16 \end{aligned}$$



Tactical bidding:

$$\begin{aligned} \text{a: } u_a &= v_a - p_a = \\ &= 28 - 12 = 16 \end{aligned}$$

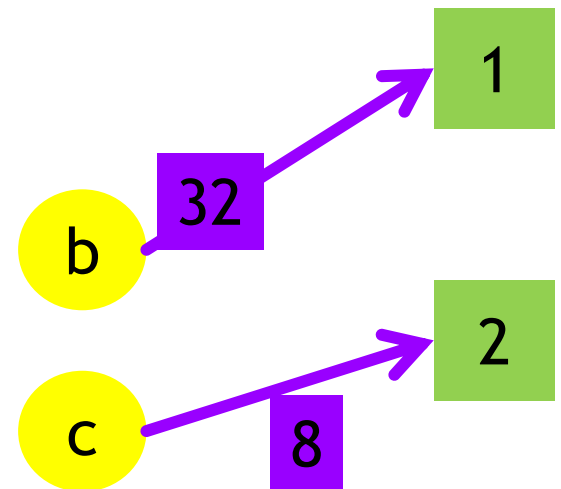
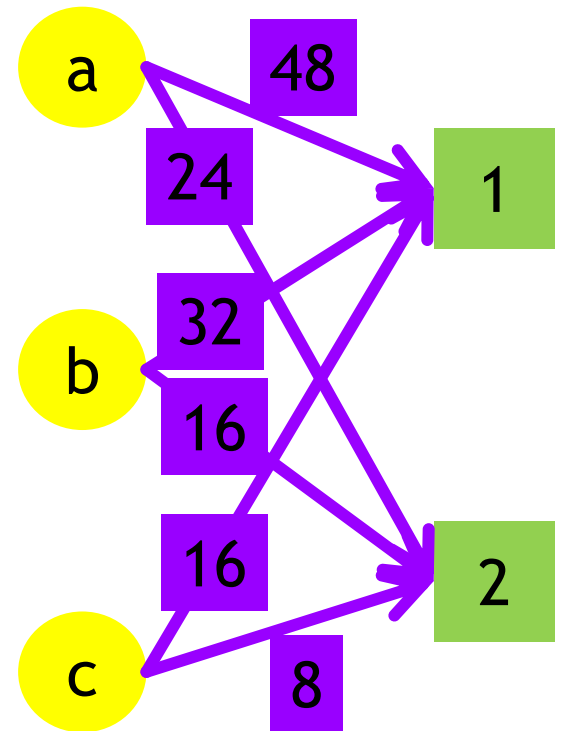
# VCG (Vickrey-Clarke-Groves) auction

- Suppose everybody bids truthfully:  $b_{ij} = v_{ij}$
- Total valuation (optimal matching):  $V = \sum_M v_{ij}$
- Def: if bidder  $i$  gets item  $j$ :  $V_{i \leftarrow j} = \sum_{M-(i,j)} v_{ij}$ 
  - i.e., the value of all bidder-item pairs except for  $(i, j)$
  - Total valuation  $V = v_{ij} + V_{i \leftarrow j}$
- Def: valuation without  $i$ :  $V_{-i}$ 
  - Best total valuation if bidder  $i$  is completely removed (different matching  $M'$ )
- Price:
  - Compute matching  $M$
  - $p_i = V_{-i} - V_{i \leftarrow j}$
  - Interpretation: price for  $i$  is the decrease in total valuation for everybody else due to  $i$ 's participation

# VCG pricing: example


- Allocation identical
  - Prices:
    - $p_a = (32 + 8) - (16 + 0) = 24$ 
      - 16 for reducing b from 32  $\rightarrow$  16
      - 8 for reducing c from 8  $\rightarrow$  0
    - $p_b = 8$ :
      - 8 for reducing c from 8  $\rightarrow$  0
  - Note: VCG with single item is equivalent to second-price
    - $V_{i \leftarrow 1} = 0$
    - $V_{-i} = b_{[2]}$
    - $\rightarrow V_{-i} - V_{i \leftarrow 1} = b_{[2]}$
- $V_{-a} = 40$ :

$V = 64$ :




# Truthful bidding in VCG

- Theorem: in a VCG auction, truthful bidding ( $b = v$ ) is a dominating strategy
- Proof:
  - Suppose  $i$  bids non-truthfully for some item  $j$
  - $V = v_{ij} + V_{i \leftarrow j} \geq v_{ix} + V_{i \leftarrow x}$  for all  $x \neq j$ , because  $V$  is max valuation (best matching)
  - Subtract  $V_{-i}$  from both sides:
$$v_{ij} + V_{i \leftarrow j} - V_{-i} \geq v_{ix} + V_{i \leftarrow x} - V_{-i}$$
  - $v_{ij} - p_{ij} \geq v_{ix} - p_{ix}$ 



payoff when  
truthful



payoff when  
not truthful

# Summary

- Online advertisement: scarce resources + competing interests → auction
- Classification: open/sealed; ascending/descending; 1<sup>st</sup>/2<sup>nd</sup> price
- Truthful bidding: bid = value
- Second-price: truthful bidding is dominant strategy
- Multi-item:
  - GSP: very simple rule, but not incentive-compatible (truthful not dominant); used by Google AdWords, very high frequency
  - VCG: incentive-compatible

# References

- [M. Chiang: Networked Life, Cambridge 2012 (chapter 2)]

# Internet Analytics: conclusion

- Types of data: all about people / user-generated
  - Social and info networks; ratings, likes & preferences; text & language
  - Real & diverse datasets for labs
- Learning outcomes:
  - Key **models** to characterize data in social web, social media, and mobile apps
  - Key **methods**: prediction, filtering, ranking, searching, selling
  - **Tools** of the trade: distributed (non-sql) data processing
    - Spark (make sure to put on resume! ;-))

# Internet Analytics: conclusion

- Fields:
  - Network Science: net structure, evolution, processes
  - Machine Learning: dim reduction; model selection and regularization; Bayesian inference & networks
  - Graph theory & probability: random graph models; MCMC
  - NLP: text classification, topic models, word embeddings
- Breadth & straddling fields:
  - Depth was often limited → probe further!
  - No textbook with full coverage
- Where to go from here?
  - Master Specialization in Data Science
  - Master in Data Science



# Thanks & good luck!

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The EPFL logo is displayed in a bold, red, sans-serif font. The letters are thick and blocky, with the 'E' and 'F' having a distinctive stepped or 'Z' shape.