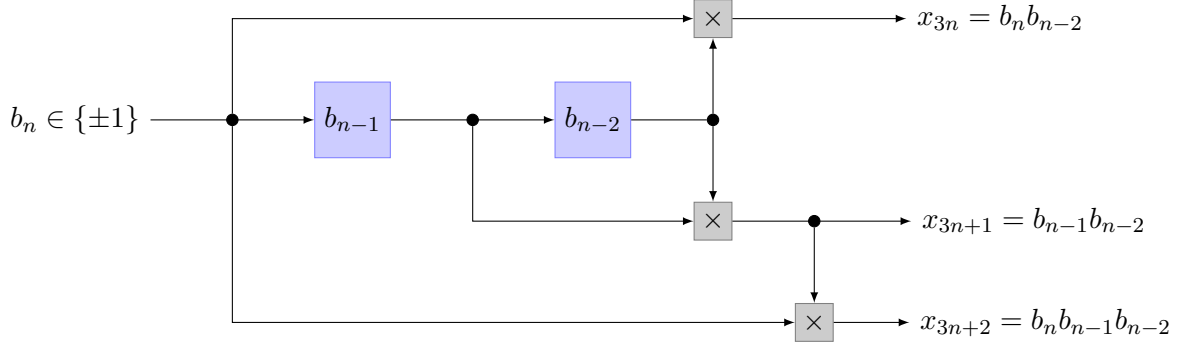


PROBLEM 1. Consider the following convolutional encoder:



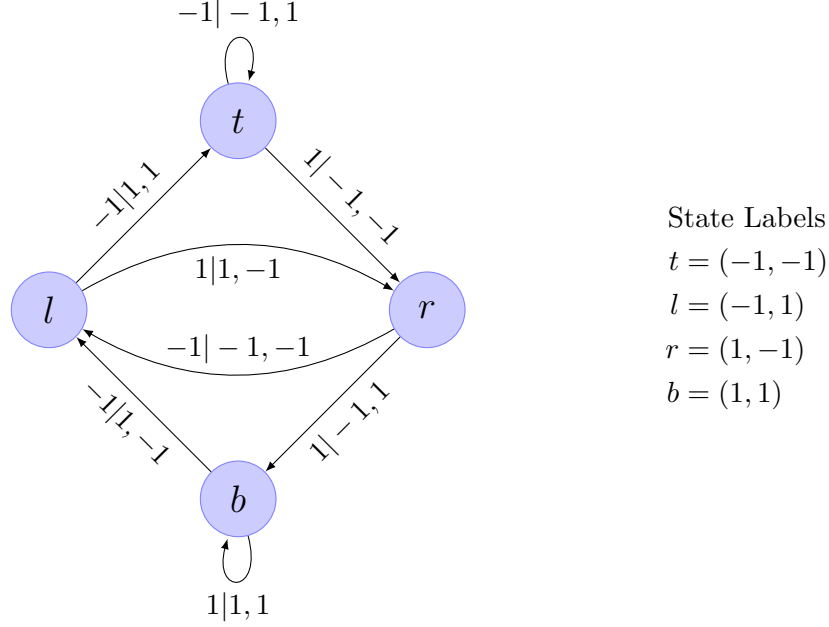
- Draw the state diagram and the detour flow graph.
- Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is  $\mathcal{E}_b$  and are sent over the discrete-time AWGN channel of noise variance  $\sigma^2 = \frac{N_0}{2}$ . Derive an upper bound to the bit-error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 2. The following equations describe the output sequence of a convolutional encoder that in each epoch takes  $k_0 = 2$  input symbols from  $\{\pm 1\}$  and outputs  $n_0 = 3$  symbols from the same alphabet.

$$\begin{aligned} x_{3n} &= b_{2n} b_{2n-1} b_{2n-2} \\ x_{3n+1} &= b_{2n+1} b_{2n-2} \\ x_{3n+2} &= b_{2n+1} b_{2n} b_{2n-2} \end{aligned}$$

- Draw an implementation of the encoder based on delay elements and multipliers.
- Draw the state diagram.
- Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is  $\mathcal{E}_b$  and are sent over the discrete-time AWGN channel of noise variance  $\sigma^2 = \frac{N_0}{2}$ . Derive an upper bound to the bit-error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 3. For the convolutional code described by the state diagram shown below:

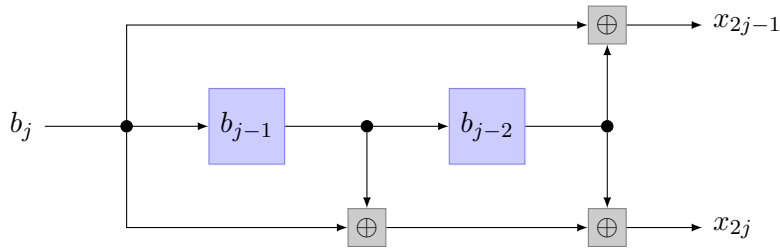


- (a) draw the encoder;
- (b) as a function of the energy per bit  $\mathcal{E}_b$ , upper-bound the bit-error probability of the Viterbi algorithm when the scaled encoder output sequence is transmitted over the discrete-time AWGN channel of noise variance  $\sigma^2 = \frac{N_0}{2}$ .

PROBLEM 4. Consider the convolutional encoder shown below with inputs and outputs over  $\{0, 1\}$  and addition modulo 2. Its output is sent over the binary erasure channel described by

$$\begin{aligned}
 P_{Y|X}(0|0) &= P_{Y|X}(1|1) = 1 - \epsilon \\
 P_{Y|X}(?|0) &= P_{Y|X}(?|1) = \epsilon \\
 P_{Y|X}(1|0) &= P_{Y|X}(0|1) = 0,
 \end{aligned}$$

where  $0 < \epsilon < \frac{1}{2}$ .



- (a) Draw a trellis section that describes the encoder map.
- (b) Derive the branch metric and specify whether a maximum likelihood decoder chooses the path with largest or smallest path metric.
- (c) Suppose that the initial encoder state is  $(0, 0)$  and that the channel output is  $\{0, ?, ?, 1, 0, 1\}$ . What is the most likely information sequence?
- (d) Derive an upper bound to the bit-error probability.