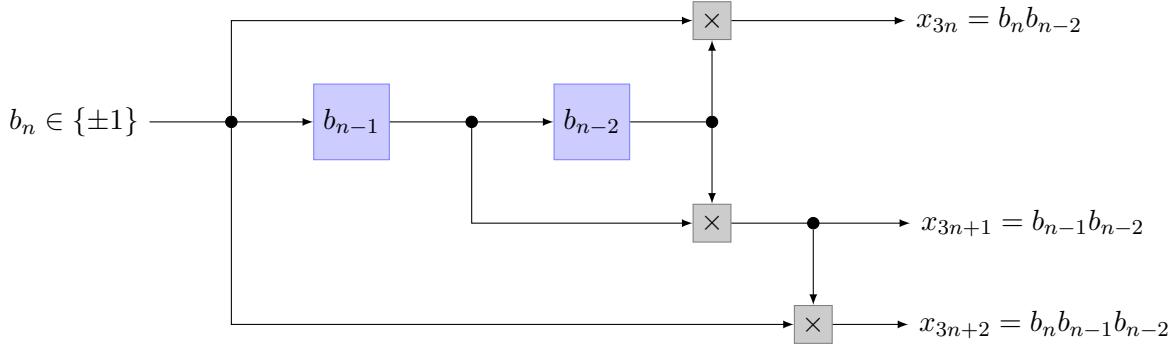


PROBLEM 1. Consider the following convolutional encoder:



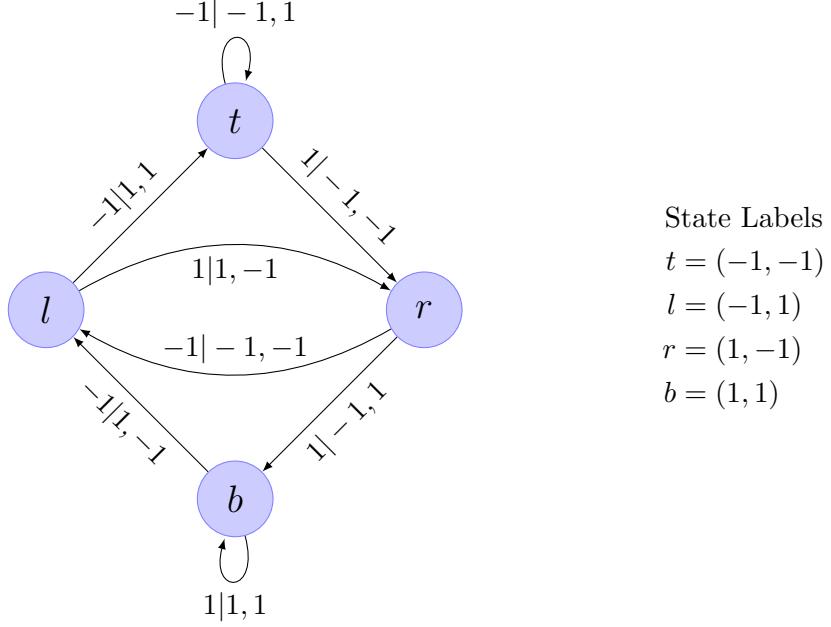
- (a) Draw the state diagram and the detour flow graph.
- (b) Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is \mathcal{E}_b and are sent over the discrete-time AWGN channel of noise variance $\sigma^2 = \frac{N_0}{2}$. Derive an upper bound to the bit-error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 2. The following equations describe the output sequence of a convolutional encoder that in each epoch takes $k_0 = 2$ input symbols from $\{\pm 1\}$ and outputs $n_0 = 3$ symbols from the same alphabet.

$$\begin{aligned} x_{3n} &= b_{2n} b_{2n-1} b_{2n-2} \\ x_{3n+1} &= b_{2n+1} b_{2n-2} \\ x_{3n+2} &= b_{2n+1} b_{2n} b_{2n-2} \end{aligned}$$

- (a) Draw an implementation of the encoder based on delay elements and multipliers.
- (b) Draw the state diagram.
- (c) Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is \mathcal{E}_b and are sent over the discrete-time AWGN channel of noise variance $\sigma^2 = \frac{N_0}{2}$. Derive an upper bound to the bit-error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 3. For the convolutional code described by the state diagram shown below:

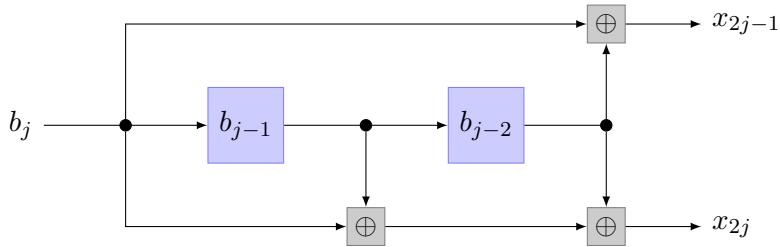


- (a) draw the encoder;
- (b) as a function of the energy per bit \mathcal{E}_b , upper-bound the bit-error probability of the Viterbi algorithm when the scaled encoder output sequence is transmitted over the discrete-time AWGN channel of noise variance $\sigma^2 = \frac{N_0}{2}$.

PROBLEM 4. Consider the convolutional encoder shown below with inputs and outputs over $\{0, 1\}$ and addition modulo 2. Its output is sent over the binary erasure channel described by

$$\begin{aligned} P_{Y|X}(0|0) &= P_{Y|X}(1|1) = 1 - \epsilon \\ P_{Y|X}(?|0) &= P_{Y|X}(?|1) = \epsilon \\ P_{Y|X}(1|0) &= P_{Y|X}(0|1) = 0, \end{aligned}$$

where $0 < \epsilon < \frac{1}{2}$.



- (a) Draw a trellis section that describes the encoder map.
- (b) Derive the branch metric and specify whether a maximum likelihood decoder chooses the path with largest or smallest path metric.
- (c) Suppose that the initial encoder state is $(0, 0)$ and that the channel output is $\{0, ?, ?, 1, 0, 1\}$. What is the most likely information sequence?
- (d) Derive an upper bound to the bit-error probability.