Principles of Digital Communication

March 31, 2021

Reading for April 14th : Chap 4. [– 4.4.2]

Last week reading was about alternate formulation of the RCVR structure ~~=~~ matched filter *;

Recall: $w(t) \in \{w_0(t), \ldots, w_{m-1}(t)\}$ is sent.

we express the $w_i(t)$'s in an $\perp$-normal basis $\{\psi_1(t), \ldots, \psi_n(t)\}$ so that

$$w_i(t) = \sum_{j=1}^{n} \underline{c_{ij}}\, \psi_j(t)$$

We had seen that MAP receiver computes from

$R(t)$ (received signal $= w(t) + Z(t)$)

$\underline{Y_j = \langle R, \psi_j \rangle}$ $j = 1 \ldots n$, we then base a decision on $\|Y - c_i\|^2$. We now note

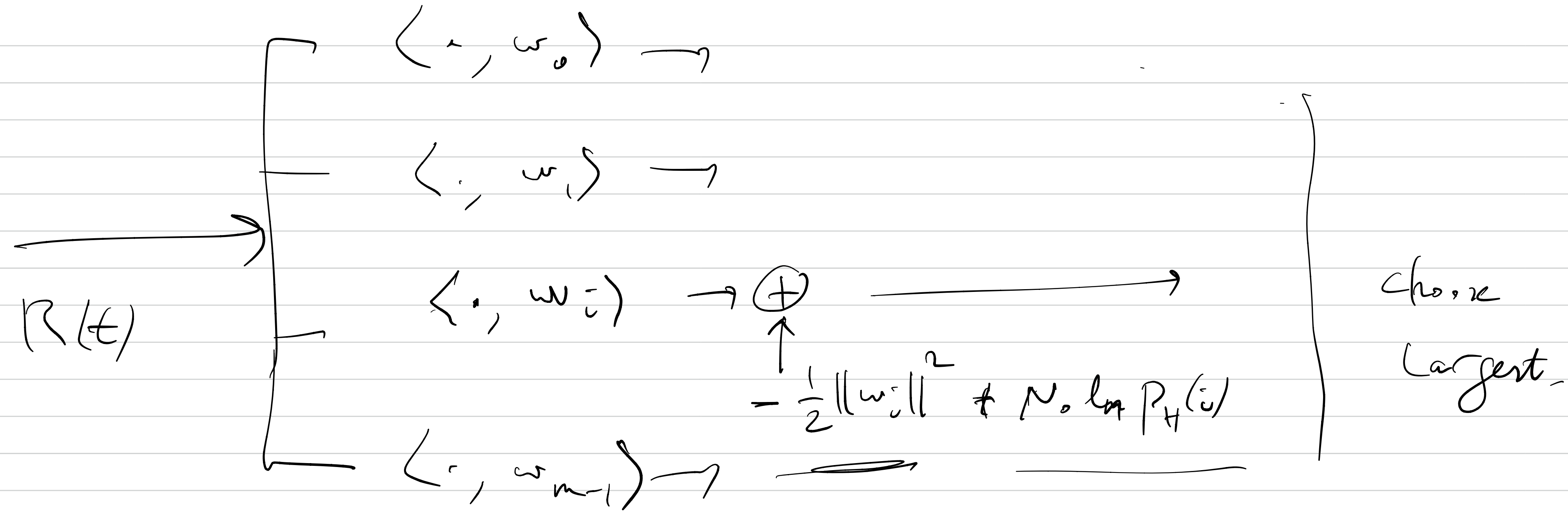
$$\|Y - c_i\|^2 = \cancel{\|Y\|^2} - \underline{2\langle c_i, Y\rangle + \|c_i\|^2}.$$

doesn't effect the MAP decision

& $\langle c_i, Y\rangle = \langle \underline{w_i}, R\rangle = \int w_i(t) R(t)\, dt$

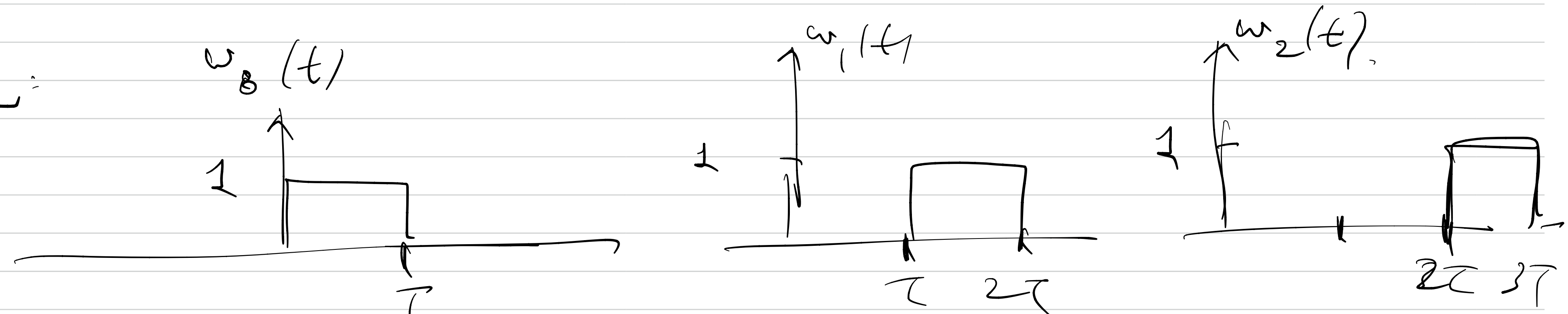
$$\|c_i\|^2 = \langle w_i, w_i\rangle = \int w_i(t)^2\, dt.$$

and we derive an equivalent RCVR structure:



[Essence of last week's reading.]

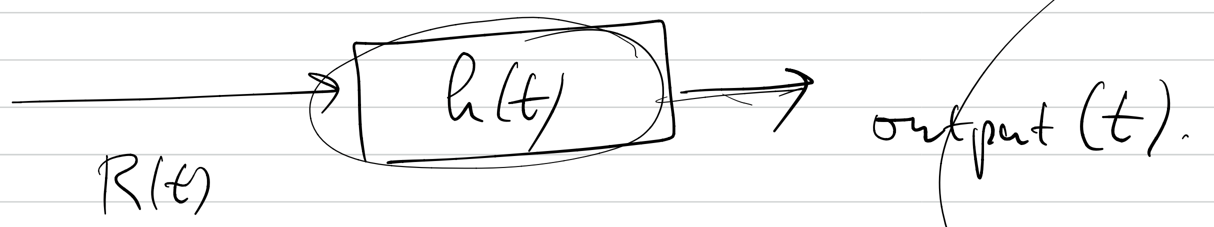
Ex:

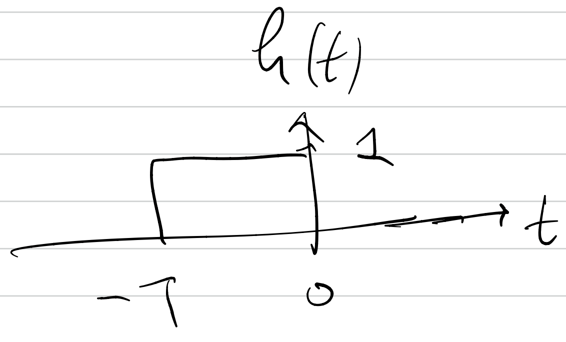


& $p_H(0) = p_H(1) = p_H(2) = \frac{1}{3}$.

Q: How to make a decision based on $R(t)$?

A: we would compute $\displaystyle\int_{iT}^{(i+1)T} R(t)\,dt$, $i=0,1,2$



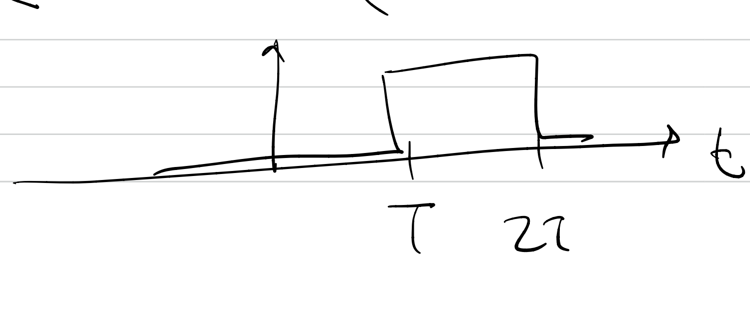


$i=0$

$$\text{output}(0) = \int_0^T R(t)\,dt$$

$i=1$

$$\text{output}(T) = \int h(T-t)\,R(t)\,dt$$



$i=2$

$$\text{output}(2T) = \int_{2T}^{3T} R(t)\,dt$$

We see that "life is easy" if w's are shifts of each other, or if ψ_j's are shifts of each other.

In reading chapter 3, we notice the following:

System 1: m waveforms $w_0(t), \ldots, w_{m-1}(t)$, n $\perp$-normal basis functions $\psi_1(t), \ldots, \psi_n(t)$ with

$$w_i(t) = \sum c_{ij} \psi_j(t)$$

System 2: m waveforms $\tilde{w}_0(t), \ldots, \tilde{w}_{m-1}(t)$, n $\perp$-normal basis functions $\tilde{\psi}_1(t), \ldots, \tilde{\psi}_n(t)$

$$\tilde{w}_i(t) = \sum c_{ij} \tilde{\psi}_j(t)$$

Question: what can we say about

1. $\int |w_i(t)|^2 dt$ vs $\int \tilde{w}_i(t)^2 dt$.

   $= \|c_i\|^2 \quad = \quad = \|c_i\|^2$

2. $Pr(\hat{H} = i' \mid H = i)$ vs $Pr(\hat{\tilde{H}} = i' \mid \tilde{H} = i)$

$Y = c_i + Z$ when $H = i$ $\qquad \tilde{Y} = c_i + \tilde{Z}$ $\hat{H} = i$

So system 1 $\equiv$ System 2 in the "Ch2 repr."

Isometries between two signal sets..

if $\left(\langle w_i, w_j \rangle = \langle \tilde{w}_i, \tilde{w}_j \rangle \text{ for } i, j \right)$

$\Rightarrow \left((w_0, \ldots w_{m-1}) \equiv (\tilde{w}_0, \ldots \tilde{w}_{m-1}) \right)$

$\equiv \left(\|w_i\| = \|\tilde{w}_i\| \quad i = 0, \ldots, m-1, \ \& \ \|w_i - w_j\| = \|\tilde{w}_i - \tilde{w}_j\| \right)$

all $i \neq j$

if either of the above holds we say that

$(w_0 \ldots w_{m-1})$ and $(\tilde{w}_0, \tilde{w}_1 \ldots \tilde{w}_{m-1})$ are isometric.

Also note: From

$(w_0, \ldots w_{m-1})$

we can construct $(\tilde{w}_0, \ldots \tilde{w}_{m-1})$ by

$$\tilde{w}_0(t) = w_0(t) + s(t)$$
$$\tilde{w}_1(t) = w_1(t) + s(t)$$
$$\vdots$$
$$\tilde{w}_{m-1}(t) = w_{m-1}(t) + s(t).$$

by using

RCVR

$$R(t) \longrightarrow \underset{\uparrow s(t)}{\oplus} \longrightarrow \left[\text{RCVR for } (w_0, \ldots w_{m-1}) \right]$$

for $(\tilde{w}_0, \ldots \tilde{w}_{m-1})$ we see that the error probabilities don't change, but we have possibly changed the energy of the signals.

$$\int \tilde{w}_0(t)^2 \, dt \neq \int w_0^2(t) \, dt$$

Perhaps we can choose $s(t)$ so that the new constellation $(\tilde{w}_0, \dots, \tilde{w}_{m-1})$ has smaller average energy than the original constellation $w_0(t), \dots, w_{m-1}(t)$.

You will read in 4.2 that choosing

$$\underbrace{s(t) = -\sum_{i=0}^{m-1} p_H(i)\, w_i(t)}$$

will lead to the smallest value of the average energy.