Principles of Digital Communication

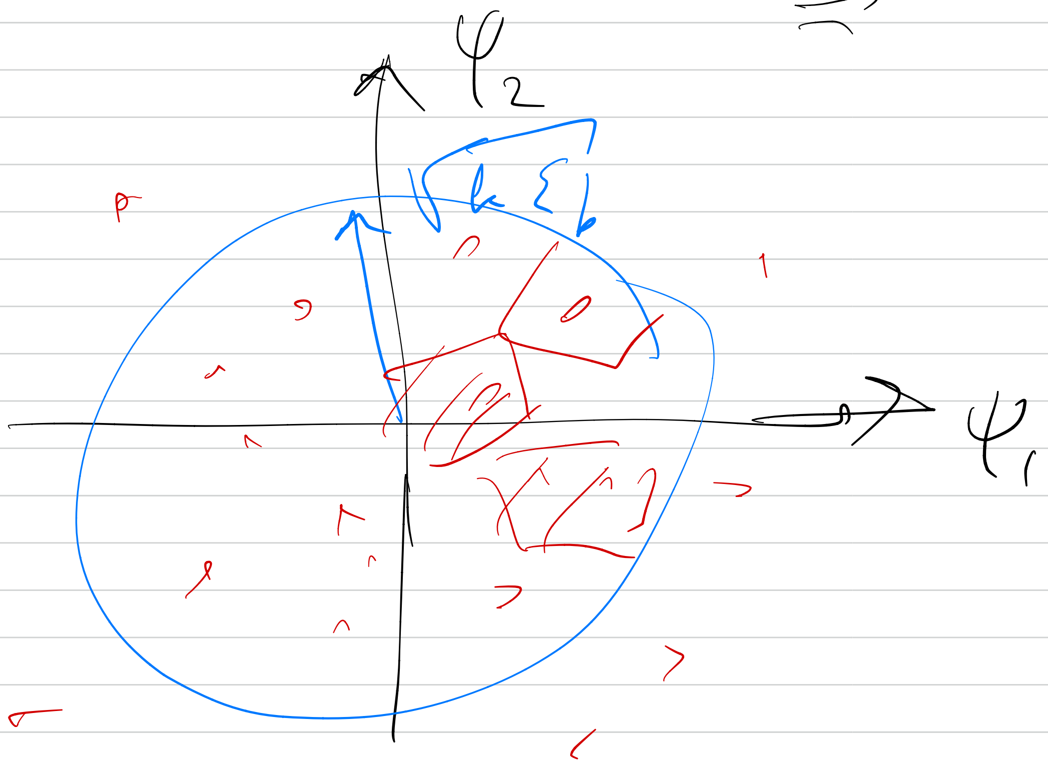
April 16, 2021

Reading Assignments : [4.4.3 – 4.7)

- Understanding the tradeoff between
  - k : # of bits we are sending
  - n : # of dimensions
  - $\mathcal{E}_b$: energy per bits

So far :

4.4.1 : keep n fixed (e.g. 1, 2)
grow k.



need to place 2^k points

average energy is $k\mathcal{E}_b$

$\Longrightarrow$ unreliable system

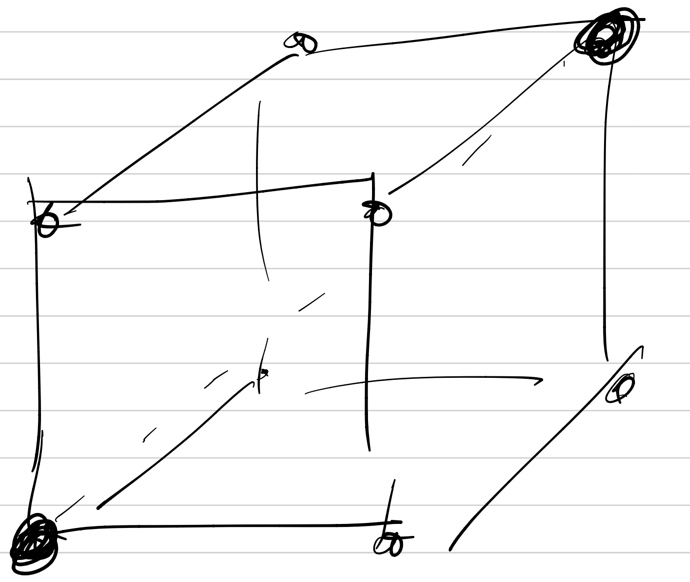
4.4.2 : $k = n$.

$$\boxed{c_i = \underbrace{(\pm 1, \pm 1, \ldots \pm 1)}_{n}\sqrt{\mathcal{E}_b}}$$

$Y = c_i + Z \Longrightarrow$ moderate reliability.

(Aside $\quad k = n/2 \qquad c_i = \underbrace{(\pm 1, \ldots, \pm 1)}_{2^n \text{ possibilities}} \sqrt{E_b}$

we need $2^{(n/2)} = \sqrt{2^n}$



upcoming: $\quad m = 2^k$ messages $\Longrightarrow \; n \le m$

Consider the case when $n = m = 2^k$

Ex. $\quad c_0 = (1, 0, \ldots, 0) \sqrt{k E_b}$

$c_1 = (0, 1, 0, \ldots)$ ''

$\vdots$

$c_{m-1} = \underbrace{(0 \ldots 0, 1)}_{m = 2^k} \sqrt{k E_b}$

as k gets large

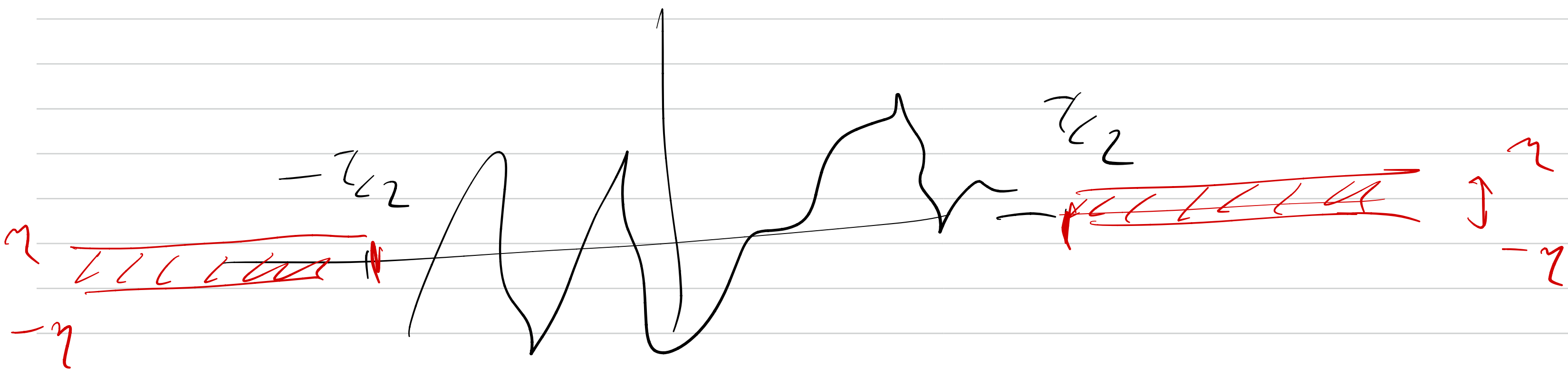
$\Longrightarrow$ will find $P(\text{error}) \to 0$ as long as

$$\boxed{\frac{E_b}{\sigma^2} > (\text{constant})}$$

4.5: the physical meaning of "n". (dimensionality).

- Let us say that two waveforms $x(t)$ and $y(t)$ to be η-indistinguishable (η > 0 number) if $|x(t)-y(t)| < \eta$

- Let us say that $x(t)$ has η-duration $\left(-\frac{T}{2}, \frac{T}{2}\right)$ if $x(t)$ is η-indisting. from $\underline{\underline{x(t)\,\mathbb{1}\{t\in[-T/2, T/2]\}}}$



$x(t)$ has η-bandwidth B if its Fourier Transform $x_F(f)$ is η-indist. from $x_F(f)\,\mathbb{1}\{|f| < B/2\}$.

Notion of (exact) dimension:

- We are given a set S of vectors.
- a collection $\psi_1, \dots \psi_n$ of vectors is a basis for S if every element $x \in S$ is a linear combination of $\psi_1, \dots \psi_n$.
- The dim of S is the smallest n for which a basis with n vectors exists.

Given a collection S of waveforms we say $\{\psi_1(t), \dots, \psi_n(t)\}$ is an ε-basis of

$\varepsilon > 0$

every waveform in S is ε-indisting from some linear comb of $\psi_1, \dots, \psi_n$.

The ε-dim of S is the smallest n for which an ε-basis exists.

Let $S(T, W, \eta)$ be the set of all waveforms that have η-duration $[-\frac{T}{2}, \frac{T}{2}]$,

& " η-bandwidth $(-\frac{W}{2}, \frac{W}{2})$.

Let $n(\varepsilon, \eta, T, W)$ be the ε-dimension of $S(T, W, \eta)$.

Thm (Slepian): as long as $\varepsilon > \eta$,

$$n(\varepsilon, \eta, W, T) \approx WT$$ in the following sense

$$\frac{n(\varepsilon, \eta, W, T)}{WT} \to 1$$

if $WT \to \infty$