Appendix E. Continuous-Time Fourier Transform (CTFT)

used for: finite-energy, infinite-length signals ($x(t) \in L_2(\mathbb{R})$)

analysis formula: $$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

synthesis formula: $$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

scaling: $$x(at) \overset{\text{CTFT}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

symmetries: $$x(-t) \overset{\text{CTFT}}{\longleftrightarrow} X(-f)$$

$$x^*(t) \overset{\text{CTFT}}{\longleftrightarrow} X^*(-f)$$

shifts: $$x(t-t_0) \overset{\text{CTFT}}{\longleftrightarrow} e^{-j2\pi f t_0} X(f)$$

$$e^{j2\pi f_0 t} x(t) \overset{\text{CTFT}}{\longleftrightarrow} X(f-f_0)$$

convolution: $$(x * y)(t) \overset{\text{CTFT}}{\longleftrightarrow} X(f)Y(f)$$

modulation: $$x(t)y(t) \overset{\text{CTFT}}{\longleftrightarrow} (X * Y)(f)$$

Parseval's Relation: $$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

Appendix F. Notable CTFT Pairs:

$x(t) = \delta(t)$ $\quad$ $X(f) = 1$

$x(t) = \delta(t - t_0)$ $\quad$ $X(f) = e^{-j2\pi f t_0}$

$x(t) = 1$ $\quad$ $X(f) = \delta(f)$

$x(t) = e^{j2\pi f_0 t}$ $\quad$ $X(\omega) = \delta(f - f_0)$

$x(t) = \cos(2\pi f_0 t)$ $\quad$ $X(f) = \frac{1}{2}\left(\delta(f - f_0) + \delta(f + f_0)\right)$

$x(t) = \sin(2\pi f_0 t)$ $\quad$ $X(f) = \frac{1}{2j}\left(\delta(f - f_0) - \delta(f + f_0)\right)$

$x(t) = e^{-at}u(t)$ $\quad$ $X(f) = \frac{1}{a + j2\pi f}$

where $Re(a) > 0$