Appendix C. Discrete-Time Fourier Transform (DTFT)

used for:	finite-energy, infinite-length signals ($x[n] \in \ell_2(\mathbb{Z})$)
analysis formula:	$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
synthesis formula:	$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$
symmetries:	$x[-n] \stackrel{\text{DTFT}}{\longleftrightarrow} X(-\omega)$
	$x^*[n] \stackrel{\text{DTFT}}{\longleftrightarrow} X^*(-\omega)$
shifts:	$x[n-n_0] \stackrel{\text{DTFT}}{\longleftrightarrow} e^{-j\omega n_0}X(\omega)$
	$e^{j\omega_0 n}x[n] \stackrel{\text{DTFT}}{\longleftrightarrow} X(\omega-\omega_0)$
convolution:	$(x*y)[n] \stackrel{\text{DTFT}}{\longleftrightarrow} X(\omega)Y(\omega)$
modulation:	$x[n]y[n] \stackrel{\text{DTFT}}{\longleftrightarrow} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\theta)Y(\omega-\theta)d\theta$
differentiation:	$nx[n] \stackrel{\text{DTFT}}{\longleftrightarrow} j\frac{d}{d\omega}X(\omega)$
Parseval's Relation:	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(\omega)\rvert^2 d\omega$

Appendix D. Notable DTFT Pairs:

$x[n] = \delta[n-k]$	$X(\omega) = e^{-j\omega k}$
$x[n] = 1$	$X(\omega) = \tilde{\delta}(\omega)$
	where $\tilde{\delta}(\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$x[n] = u[n]$	$X(\omega) = \dfrac{1}{1-e^{-j\omega}} + \dfrac{1}{2}\tilde{\delta}(\omega)$
$x[n] = a^n u[n] \quad \lvert a\rvert < 1$	$X(\omega) = \dfrac{1}{1-ae^{-j\omega}}$
$x[n] = e^{j\omega_0 n}$	$X(\omega) = \tilde{\delta}(\omega - \omega_0)$
$x[n] = \cos(\omega_0 n + \phi)$	$X(\omega) = (1/2)[e^{j\phi}\tilde{\delta}(\omega-\omega_0) + e^{-j\phi}\tilde{\delta}(\omega+\omega_0)]$
$x[n] = \sin(\omega_0 n + \phi)$	$X(\omega) = (-j/2)[e^{j\phi}\tilde{\delta}(\omega-\omega_0) - e^{-j\phi}\tilde{\delta}(\omega+\omega_0)]$
$x[n] = \begin{cases} 1 & 0 \le n < M \\ 0 & \text{otherwise} \end{cases}$	$X(\omega) = \dfrac{\sin((M/2)\omega)}{\sin(\omega/2)} e^{-j\frac{M-1}{2}\omega}$
$x[n] = \frac{\omega_c}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c}{\pi} n\right)$	$X(\omega) = \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right)$ for $-\pi \le \omega \le \pi$
where $\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$	and $\mathrm{rect}(\omega) = \begin{cases} 1 & \lvert\omega\rvert < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$