
Appendix A. Discrete Fourier Transform (DFT)

used for:	finite-length signals, $x[n] \in \mathbb{C}^N$
analysis formula:	$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} nk}, \quad k = 0, \dots, N-1$
synthesis formula:	$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi}{N} nk}, \quad n = 0, \dots, N-1$
time and frequency reversal:	$x[-n \bmod N] \xleftrightarrow{\text{DFT}} X[-k \bmod N]$ $x^*[n] \xleftrightarrow{\text{DFT}} X^*[-k \bmod N]$
circular shifts:	$x[(n - n_0) \bmod N] \xleftrightarrow{\text{DFT}} e^{-j \frac{2\pi}{N} n_0 k} X[k]$ $e^{j \frac{2\pi}{N} n k_0} x[n] \xleftrightarrow{\text{DFT}} X[(k - k_0) \bmod N]$
energy conservation:	$\sum_{n=0}^{N-1} x[n] ^2 = \frac{1}{N} \sum_{k=0}^{N-1} X[k] ^2$

Appendix B. Notable DFT Pairs

$(n, k = 0, 1, \dots, N-1)$

$x[n] = \delta[n - L]$	$X[k] = e^{-j \frac{2\pi}{N} Lk}$
$x[n] = 1$	$X[k] = N \delta[k]$
$x[n] = e^{j \frac{2\pi}{N} Ln}$	$X[k] = N \delta[k - L]$
$x[n] = \cos\left(\frac{2\pi}{N} Ln + \phi\right)$	$X[k] = (N/2)[e^{j\phi} \delta[k - L] + e^{-j\phi} \delta[k - N + L]]$
$x[n] = \sin\left(\frac{2\pi}{N} Ln + \phi\right)$	$X[k] = (-jN/2)[e^{j\phi} \delta[k - L] - e^{-j\phi} \delta[k - N + L]]$
$x[n] = \begin{cases} 1 & n < M \\ 0 & M \leq n < N \end{cases}$	$X[k] = \frac{\sin((\pi/N)Mk)}{\sin((\pi/N)k)} e^{-j \frac{\pi}{N}(M-1)k}$
