Signal Processing in the Wild

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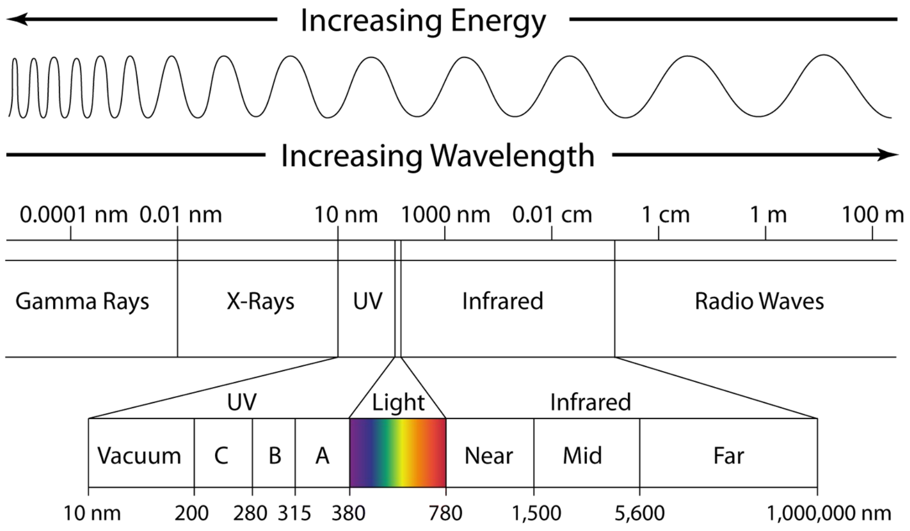
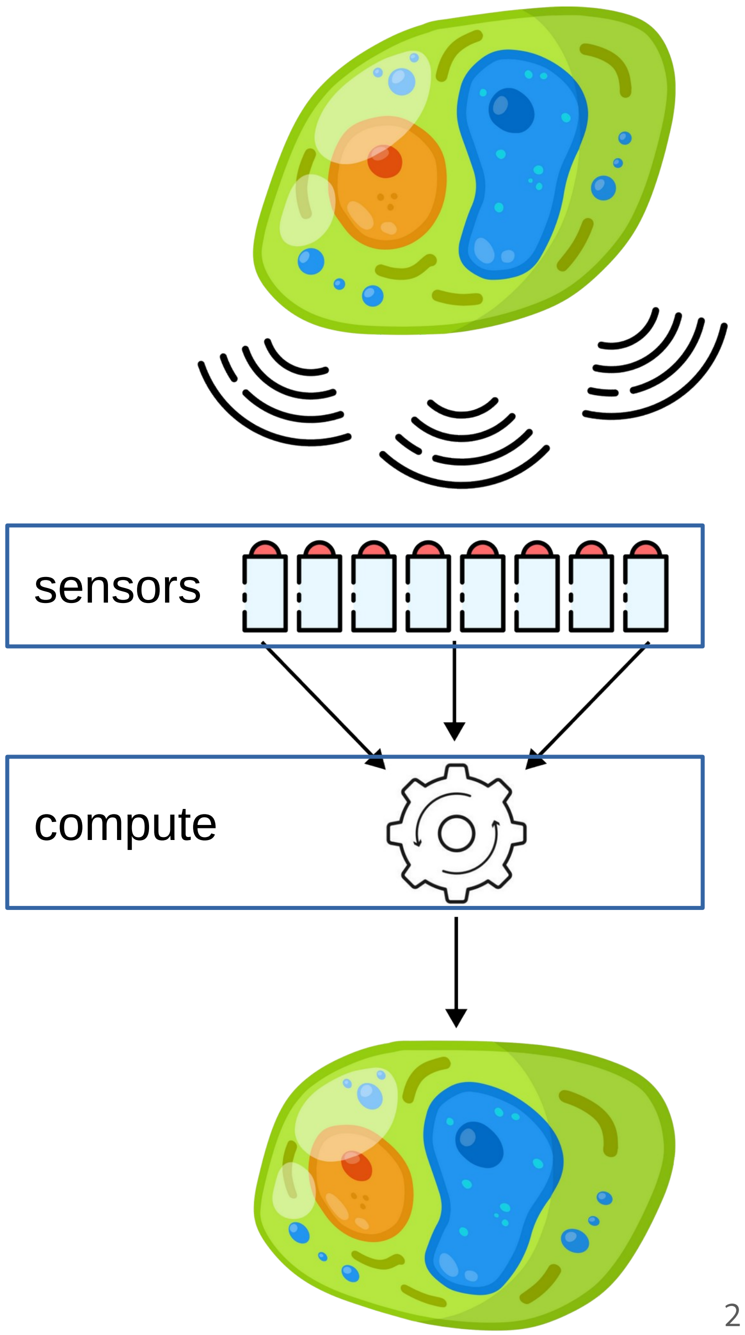
2025.04.14

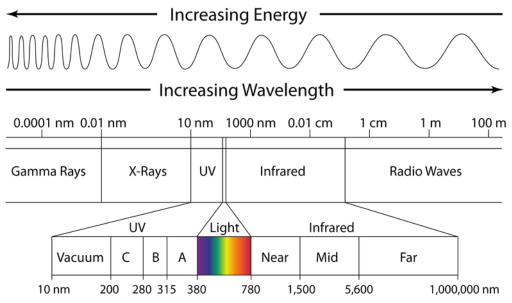
Signal Processing (SP)

- Capture signals from real world with sensors
- Infer something about quantity of interest via **computation**





Acoustic Imaging



- Determine acoustic intensity in different directions.
  - Microphone(s) record time series
  - Process signals to infer spherical intensity

Acoustic emitters

measurement

computation?

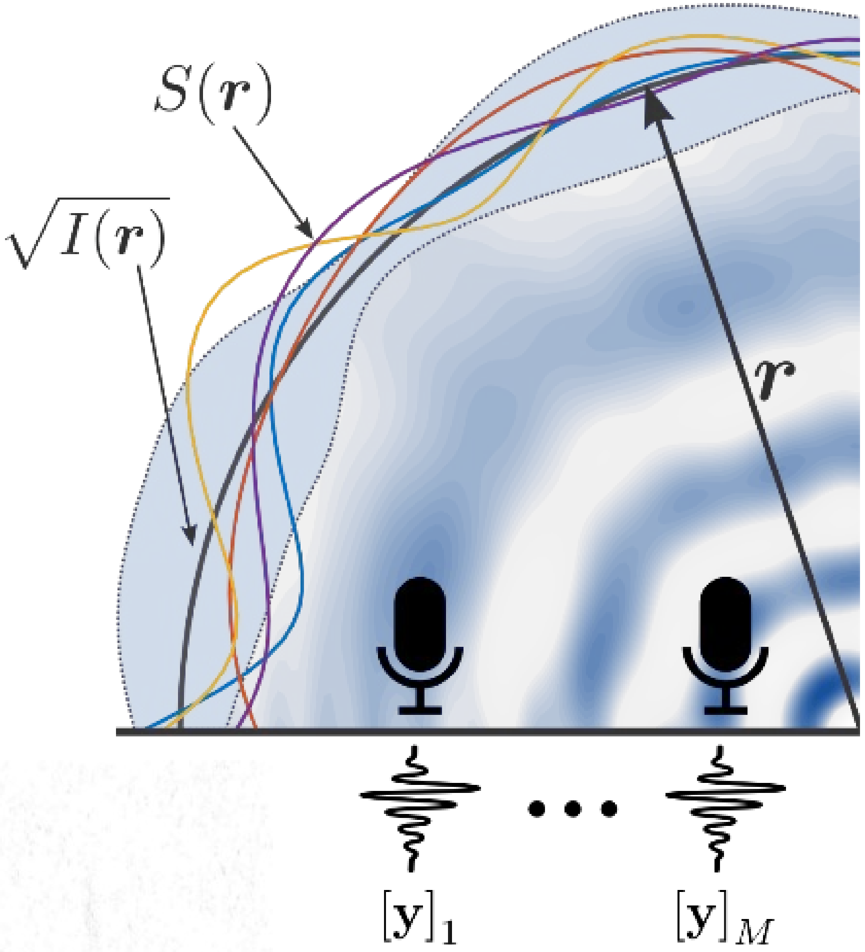
Microphone signals

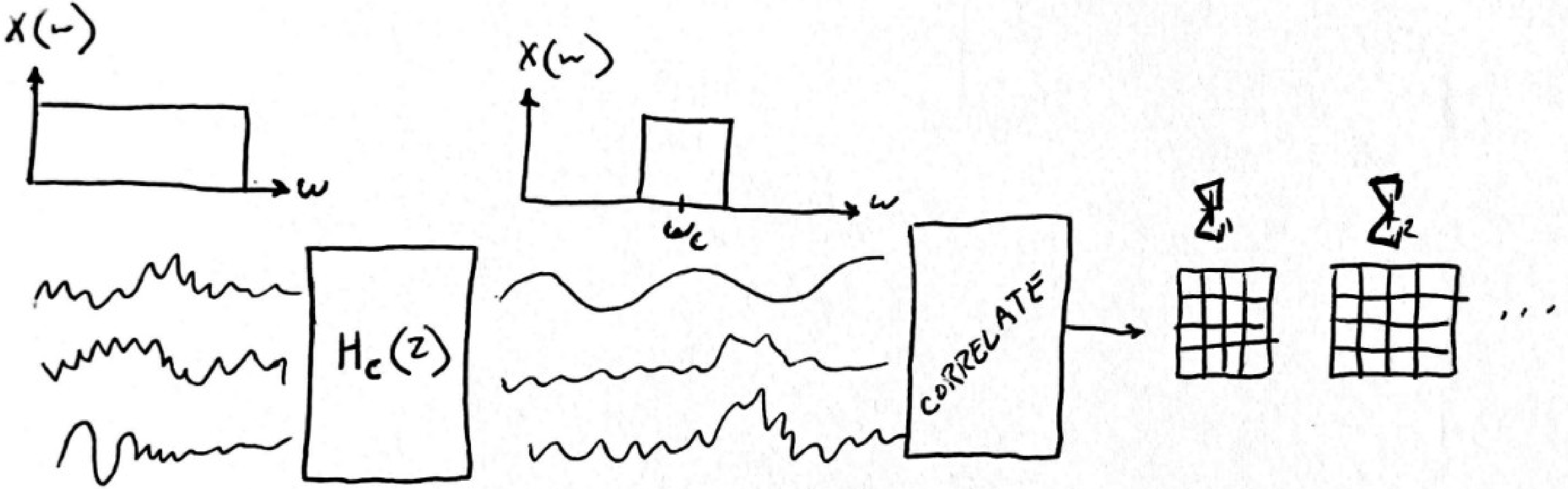
Acoustic Imaging 2

$$\mathbf{y}_m = \int_{\mathbb{S}^2} S(\mathbf{r})\alpha_m^*(\mathbf{r})e^{-j\frac{2\pi}{\lambda}\langle \mathbf{p}_m,\mathbf{r}\rangle}d\mathbf{r}$$

$$= \sum_{\mathbf{r}\in\theta} S(\mathbf{r})\alpha_m^*(\mathbf{r})e^{-j\frac{2\pi}{\lambda}\langle \mathbf{p}_m,\mathbf{r}\rangle}$$

$$\mathbf{\Sigma} = \mathbb{E}\left[\mathbf{y}\mathbf{y}^{\mathrm{H}}\right] = \left(\overline{\mathbf{A}} \circ \mathbf{A}\right)\mathbf{I} + \sigma^2$$





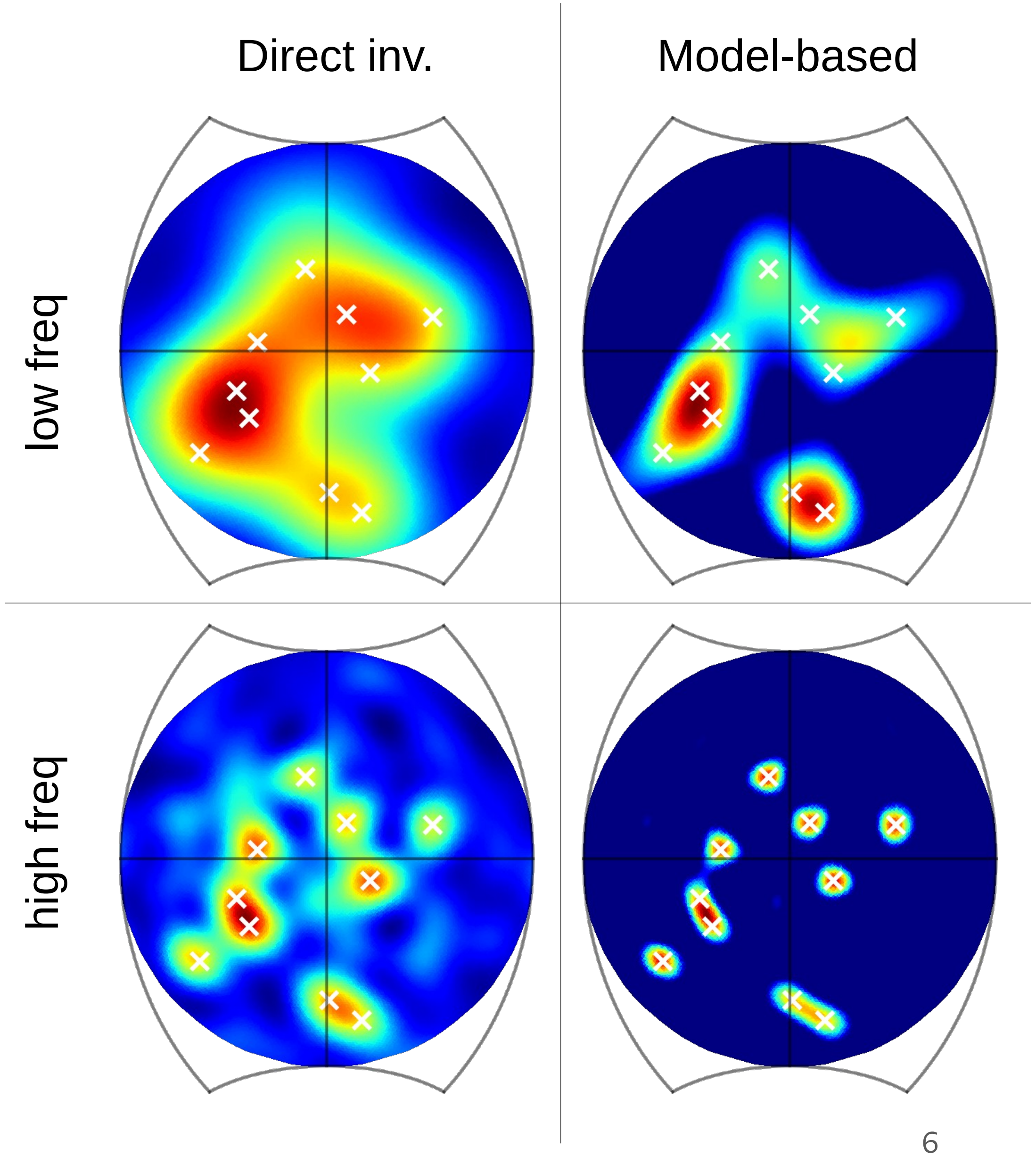
Acoustic Imaging: Real-Time Demo

Acoustic Camera Video

Acoustic Imaging 4

- Least-squares reconstruction to achieve real-time estimation.
  - Poor spatial resolution for compact arrays.
- Solution: inject prior knowledge.

$$\hat{\mathbf{I}} = \underset{\mathbf{x}\in\mathbb{R}^N_+}{\arg\min} \underbrace{\left\| \mathrm{vec}(\hat{\boldsymbol{\Sigma}}) - (\overline{\mathbf{A}} \circ \mathbf{A})\,\mathbf{x} \right\|_2^2}_{\text{Data Fidelity}} + \underbrace{\lambda \left[\gamma \left\| \mathbf{x} \right\|_1 + (1-\gamma) \left\| \mathbf{x} \right\|_2^2 \right]}_{\text{Prior Model}}$$

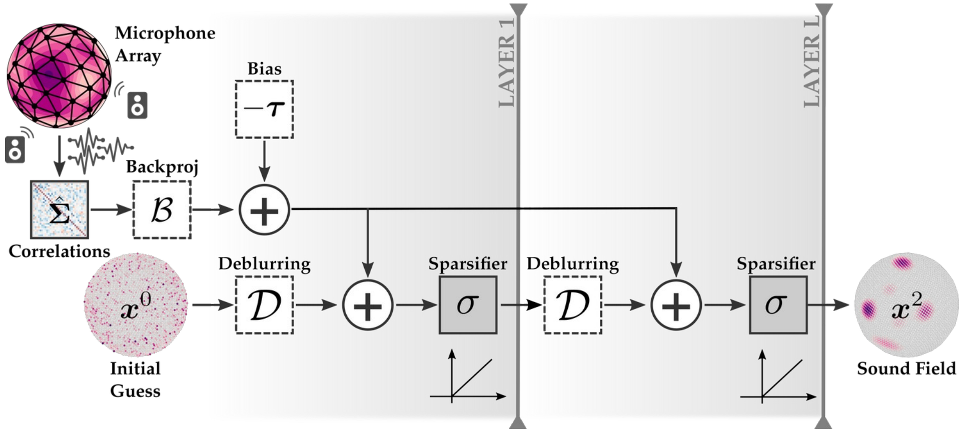


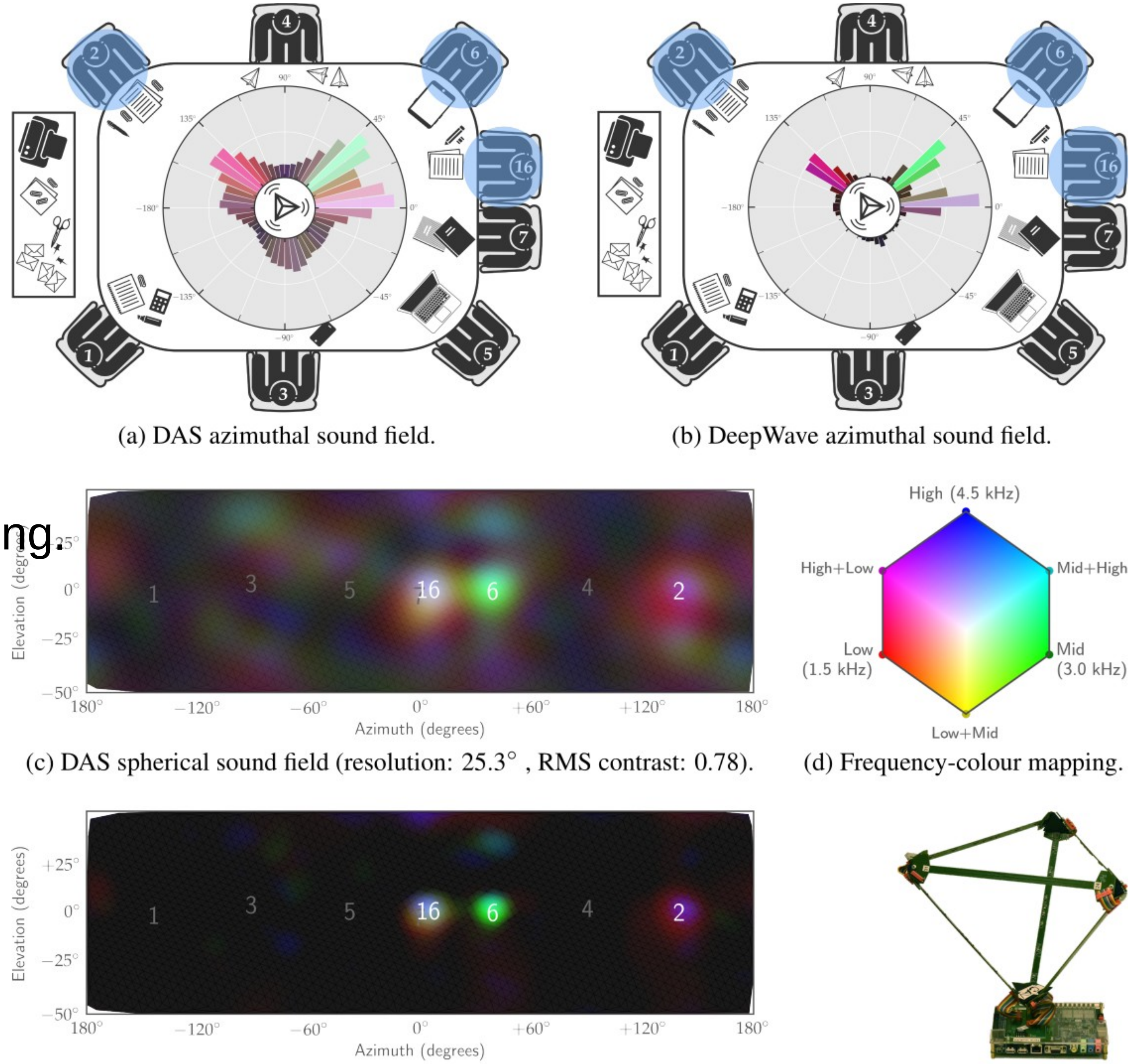
Acoustic Imaging 5

- Solve via iterative methods.

$$\mathbf{x}_{\mathrm{PGD}}^{k} = \mathrm{prox}_g\left(\mathbf{x}^{k-1} - \alpha\nabla f(\mathbf{x}^{k-1})\right)$$
$$= \mathrm{ReLu}\left[\mathcal{D}\mathbf{x}^{k-1} + \mathcal{B}\,\mathrm{vec}(\hat{\boldsymbol{\Sigma}}) - \boldsymbol{\tau}\right]$$

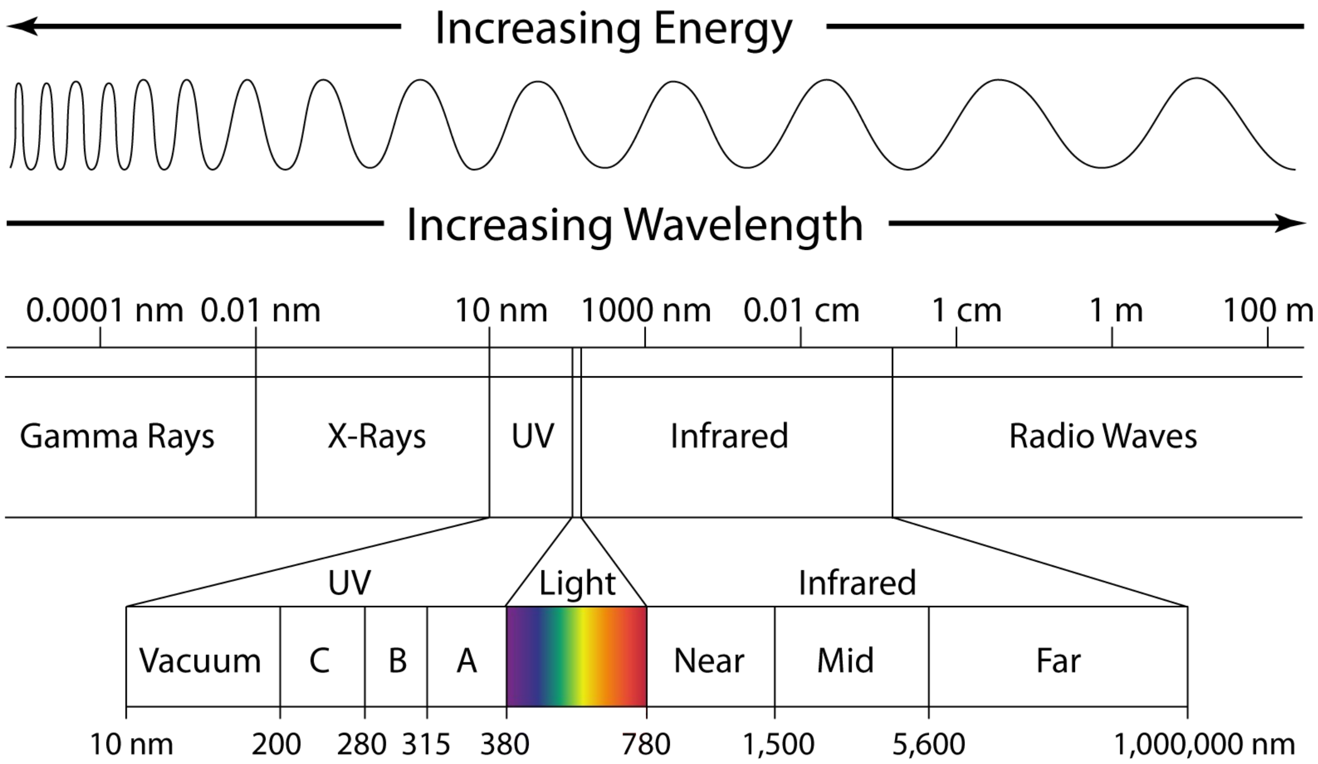
- Can be made real-time via loop-unrolling.



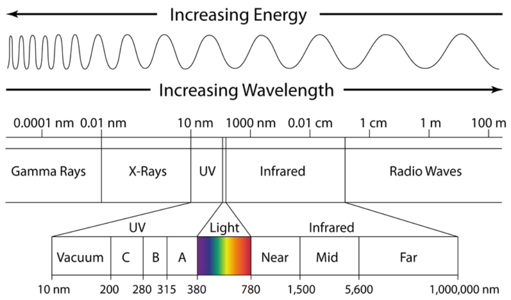


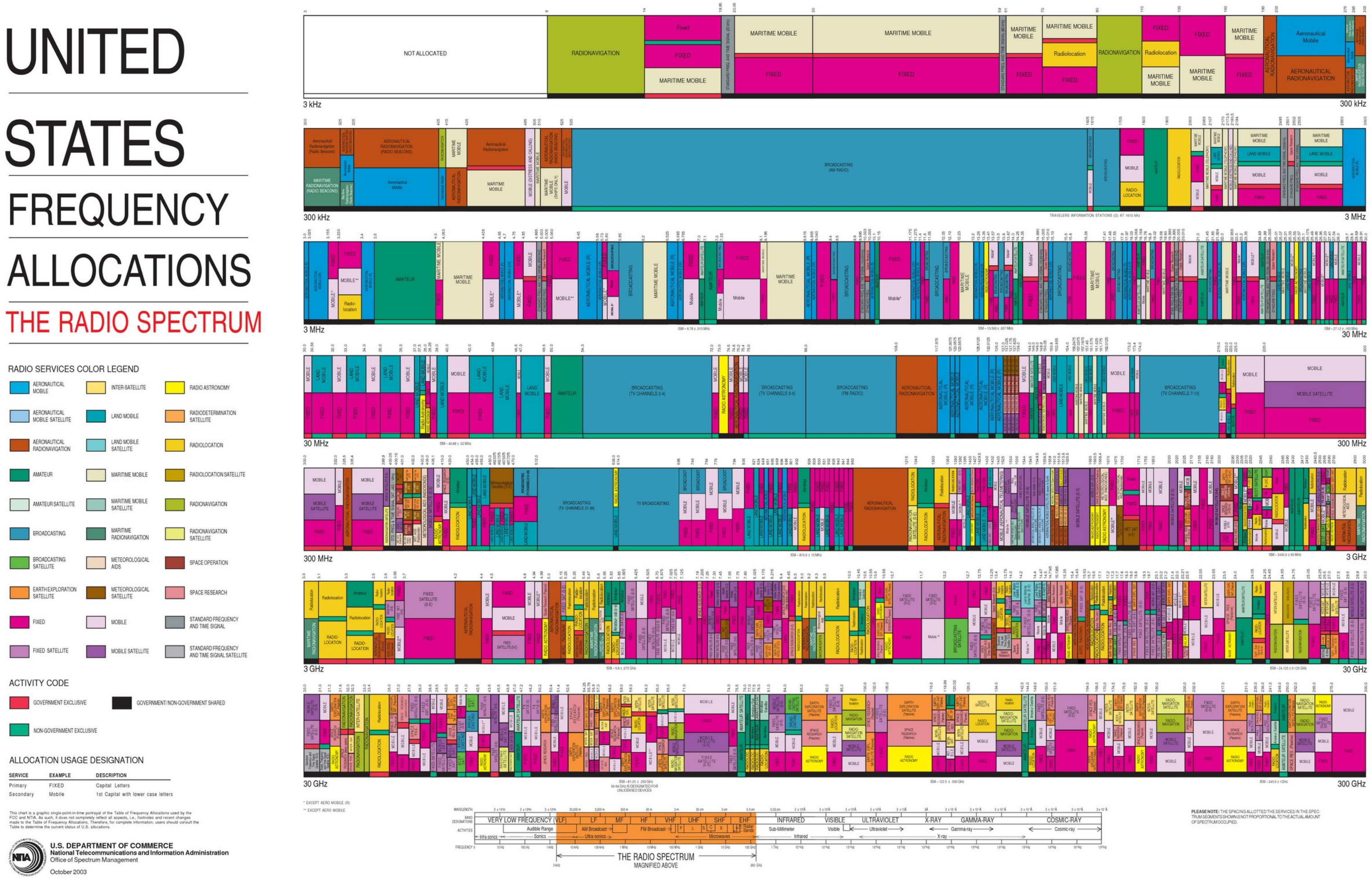
(a) DAS azimuthal sound field.
(b) DeepWave azimuthal sound field.
(c) DAS spherical sound field (resolution: 25.3° , RMS contrast: 0.78).
(d) Frequency-colour mapping.
(e) DeepWave spherical sound field (resolution: 18.5° , contrast: 0.97).
(f) Pyramic array.

The Electro-Magnetic Spectrum

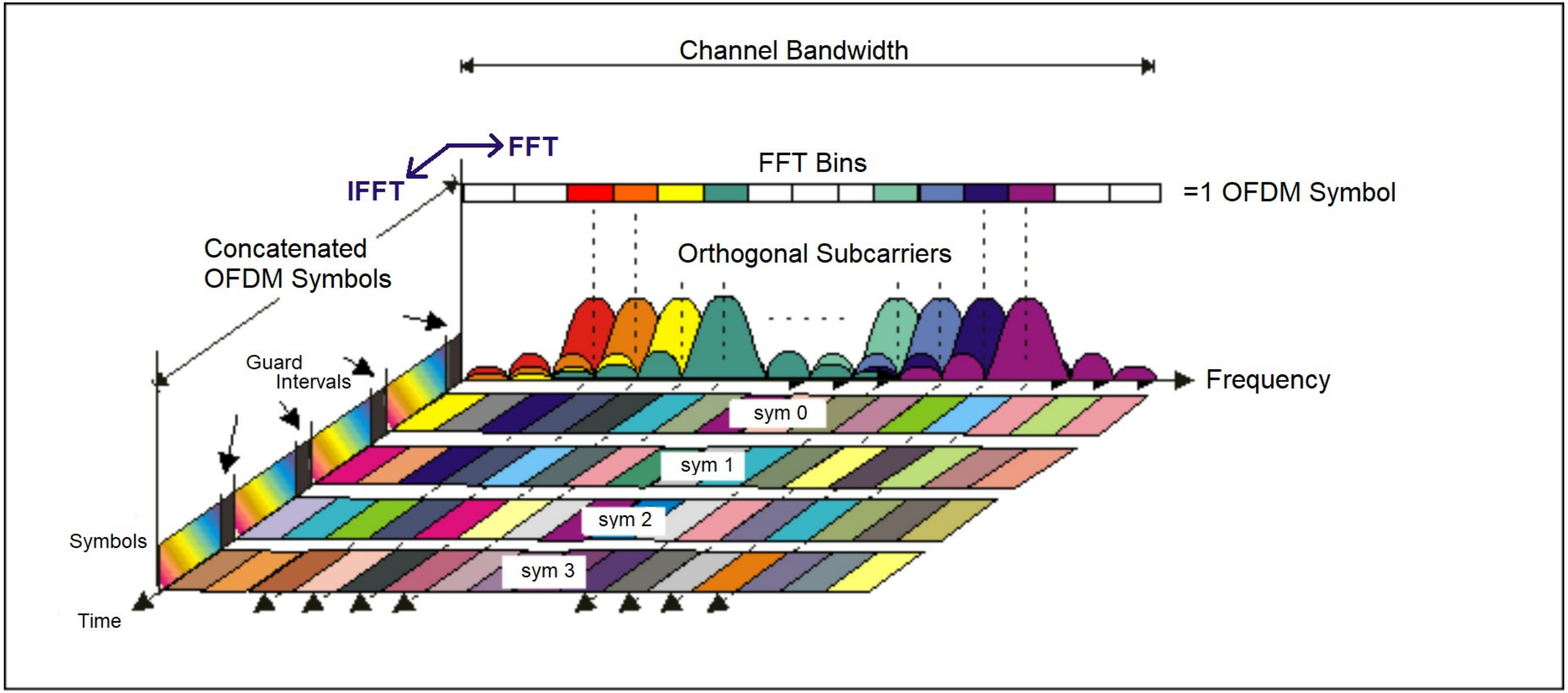


Digital Communications





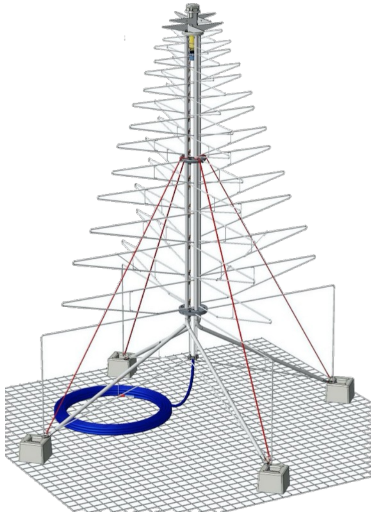
Digital Communications: 5G Coding

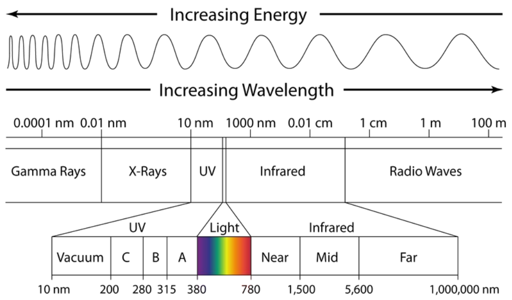


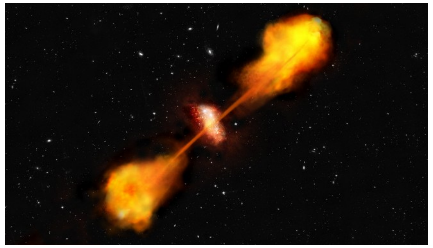
Frequency-Time Representative of an OFDM signal

Radio-Interferometry

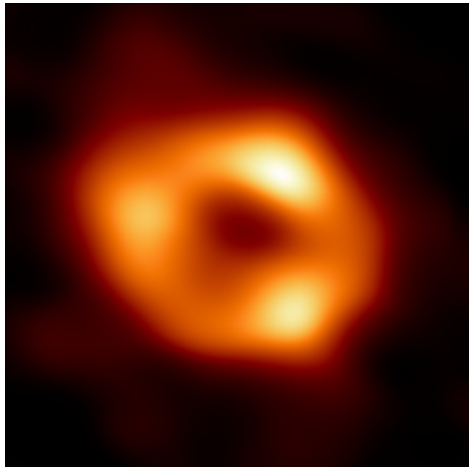
- Determine sky brightness distribution
  - Stars emit radio emissions (among other things)
  - Recorded on Earth with antennas







Sgr A^*

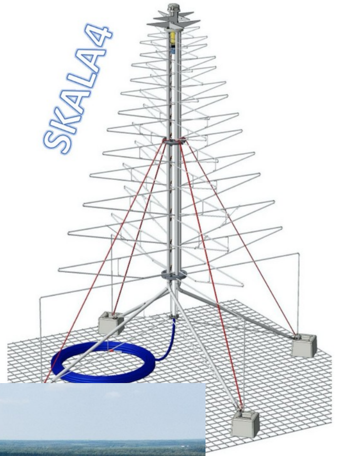


measurement →

← computation?

antenna time series

Radio-Interferometry



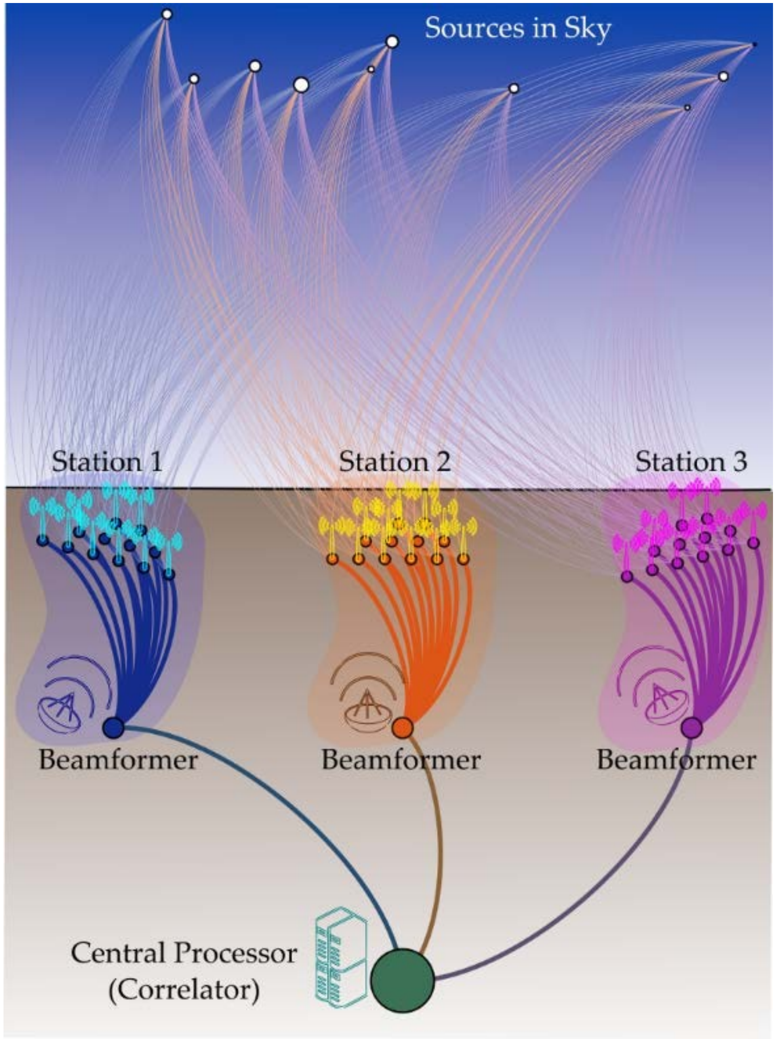
RI: Design of Modern Interferometers

- Hierarchical phased-array architecture using beamforming.
  - Station-level spatial filtering.
  - Reduce data rate from antennas.

$$\mathbf{y}_m = \int_{\mathbb{S}^2} S(\mathbf{r})\alpha_m^*(\mathbf{r})e^{-j\frac{2\pi}{\lambda}\langle \mathbf{p}_m, \mathbf{r}\rangle} d\mathbf{r}$$

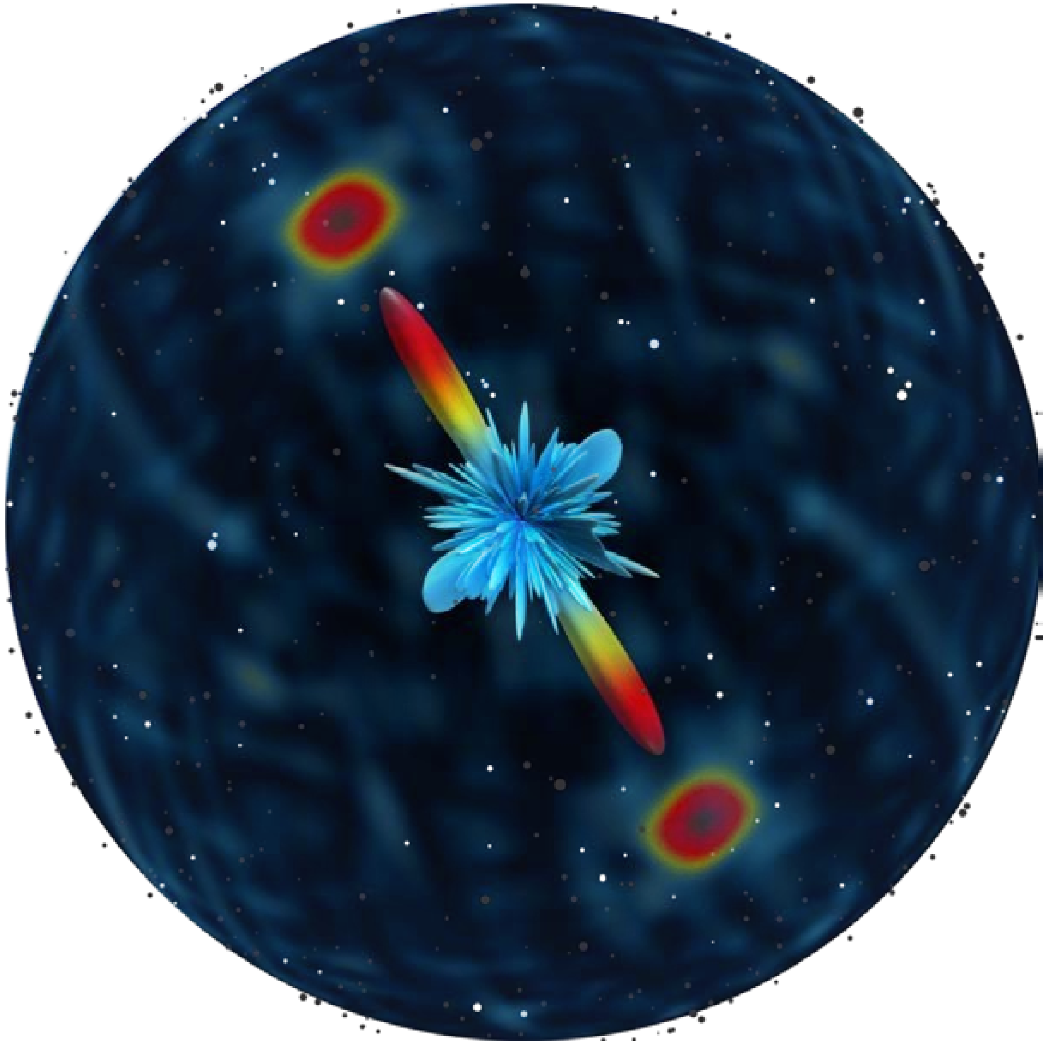
$$= \sum_{\mathbf{r}\in\theta} S(\mathbf{r})\alpha_m^*(\mathbf{r})e^{-j\frac{2\pi}{\lambda}\langle \mathbf{p}_m, \mathbf{r}\rangle}$$

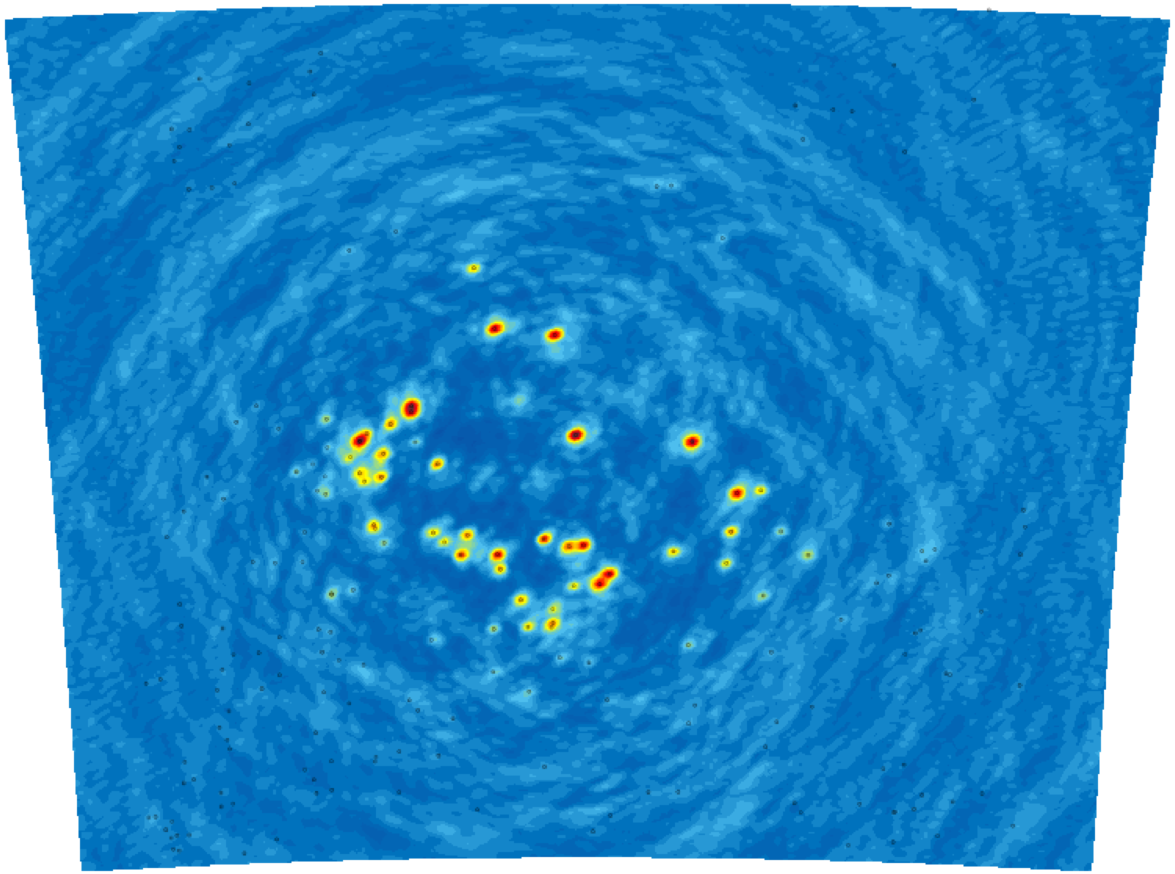
$$\mathbf{x}_l = \mathbf{w}^H \mathbf{y}$$



RI: Adaptive Beamforming

Trading Resolution for Sensitivity





RI: Adaptive Beamforming

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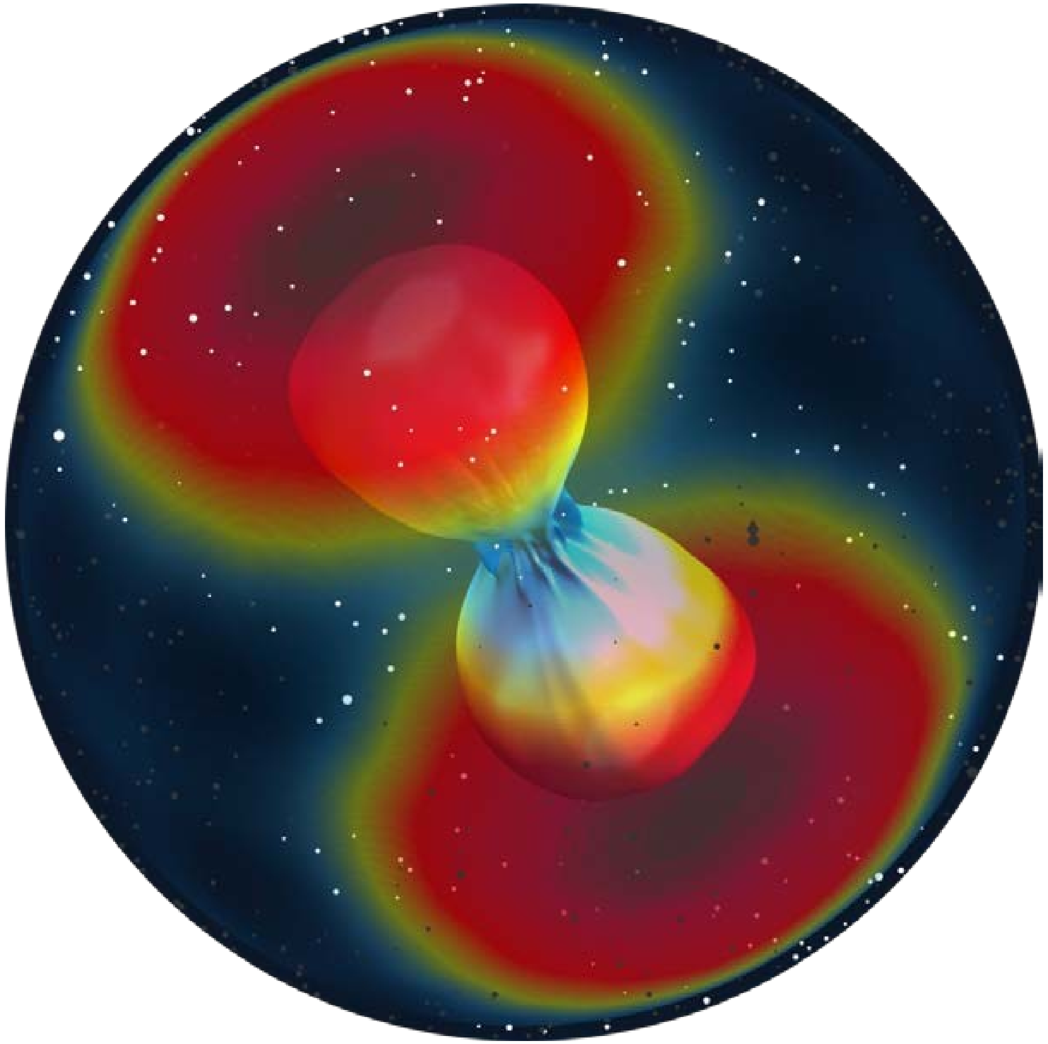
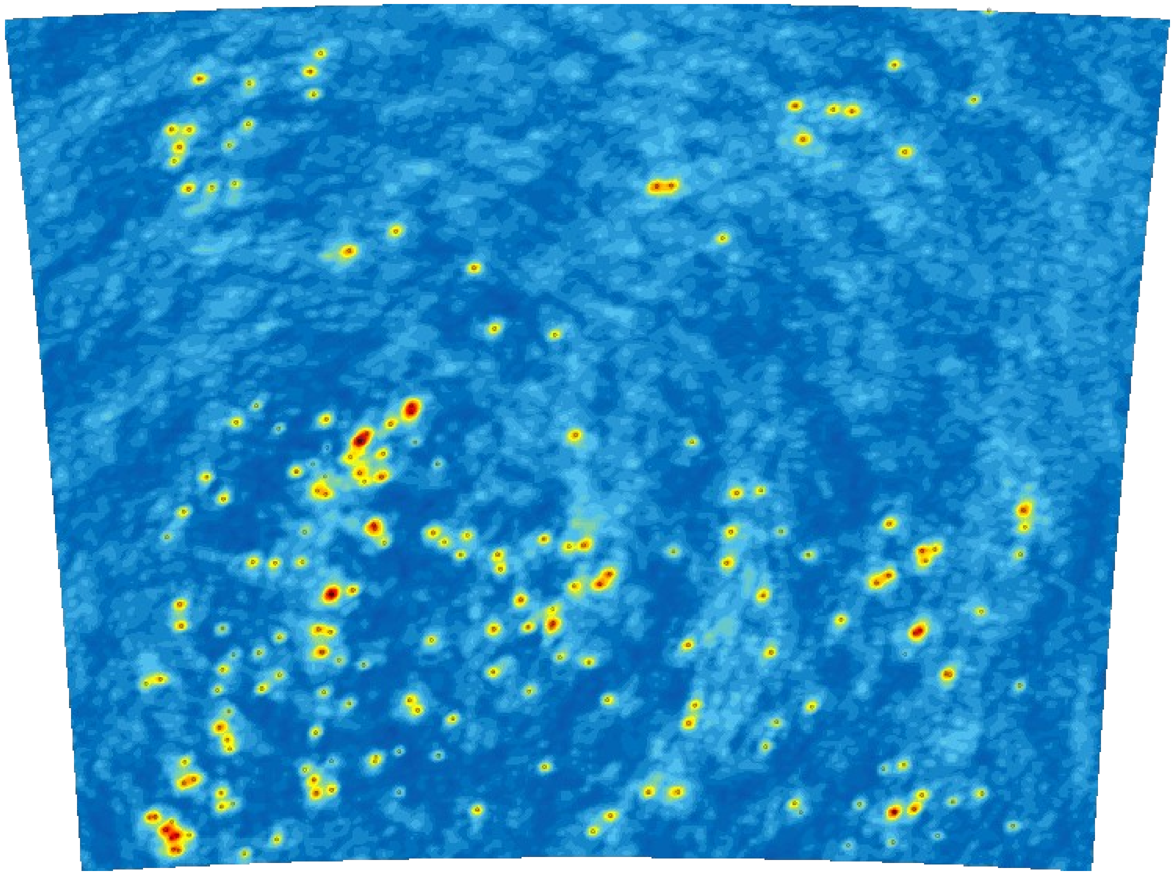
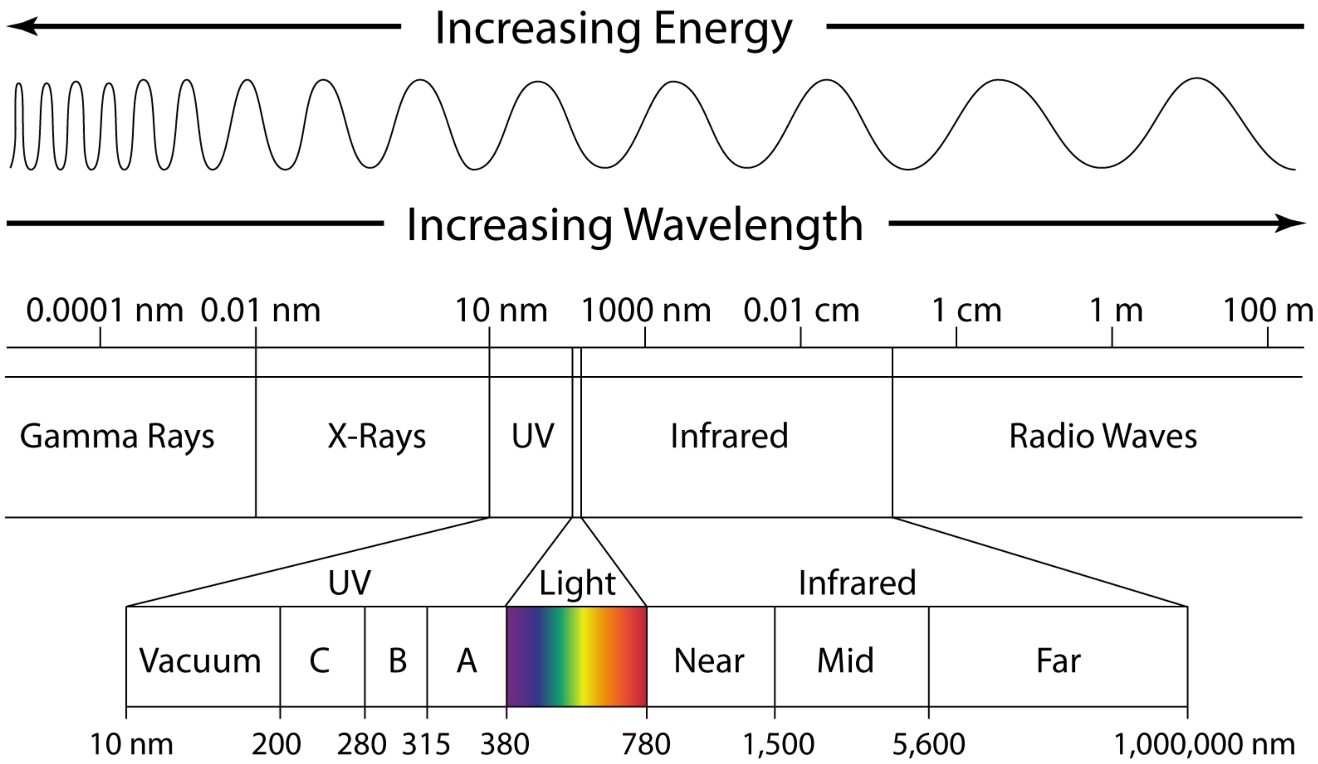


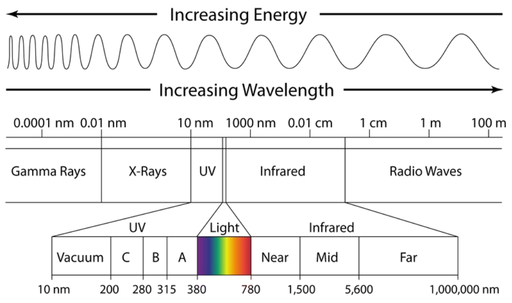


The Electro-Magnetic Spectrum



Optical Imaging

- Capture scene radiance
  - Visible light enters camera
  - Recorded on pixel detector



scene

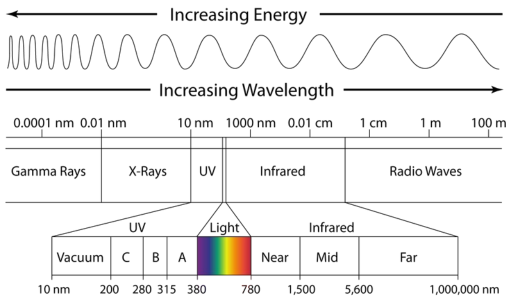
measurement

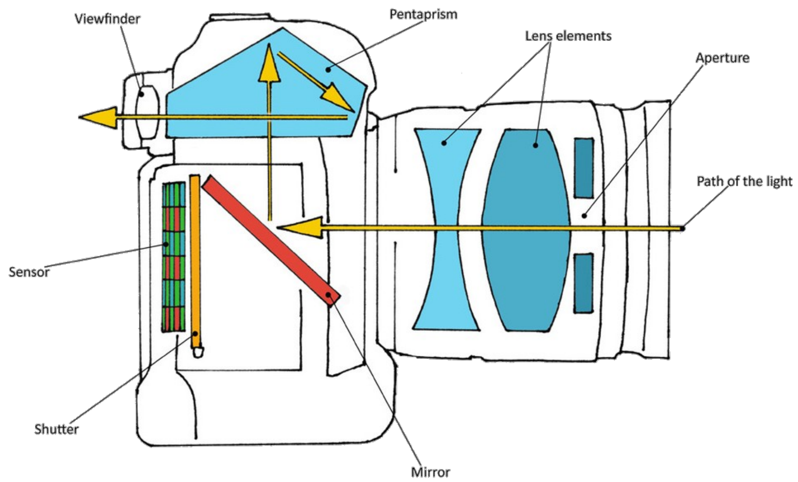
computation?

camera picture (shown to user)

Optical Imaging 2

- Camera does not capture true scene
  - Use computation to recover it





scene

measurement

computation?

sensor recording

$$\mathbf{y}(\mathbf{r}) = (\mathbf{h} * \mathbf{x})(\mathbf{r})$$

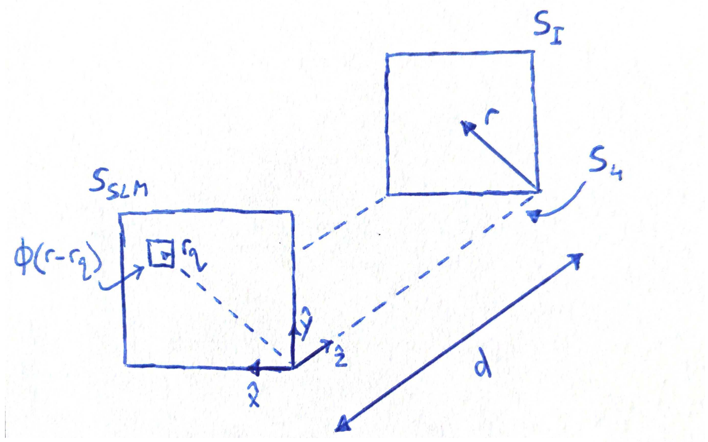
+ color channels...

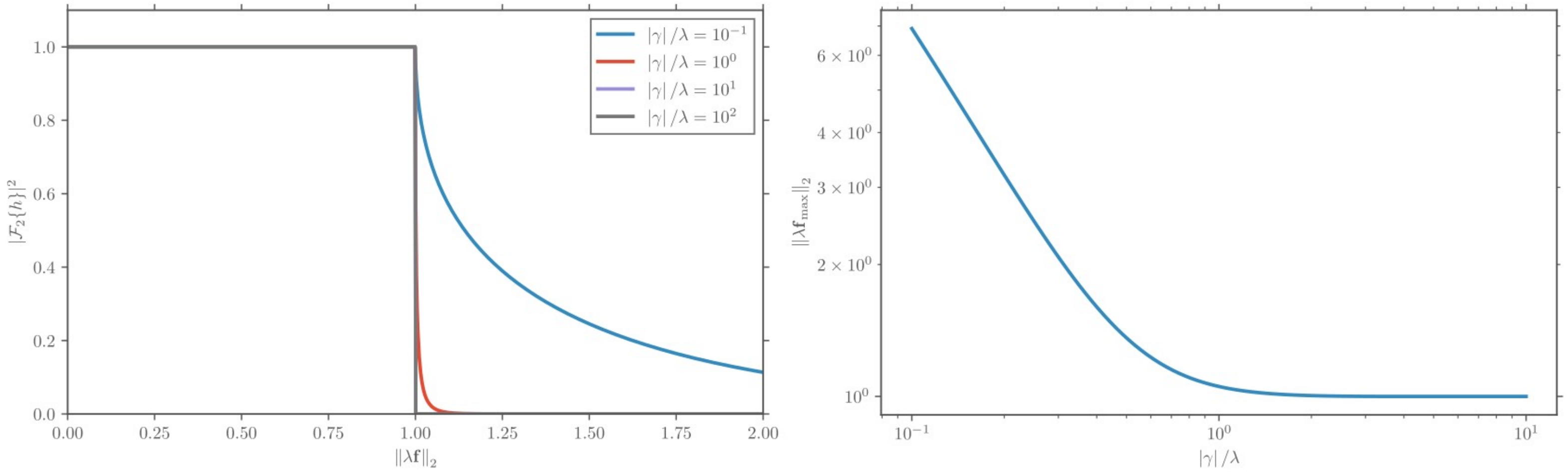
Optical SP: A 2D LTI System

$$U(\mathbf{r}) = \sum_q \alpha_q (\chi\phi \star\star \tilde{h})(\mathbf{r} - \mathbf{r}_q), \quad \mathbf{r} \in \mathbb{R}^2$$

$$\tilde{h}(\mathbf{r}; d) = \frac{-d}{4\pi\beta^2(\mathbf{r}; d)} \left[\frac{2\pi}{j\lambda} + \frac{1}{\beta(\mathbf{r}; d)} \right] \exp\left(j\frac{2\pi}{\lambda}\beta(\mathbf{r}; d) \right),$$

$$\beta(\mathbf{r}; d) = \sqrt{\|\mathbf{r}\|_2^2 + d^2}.$$

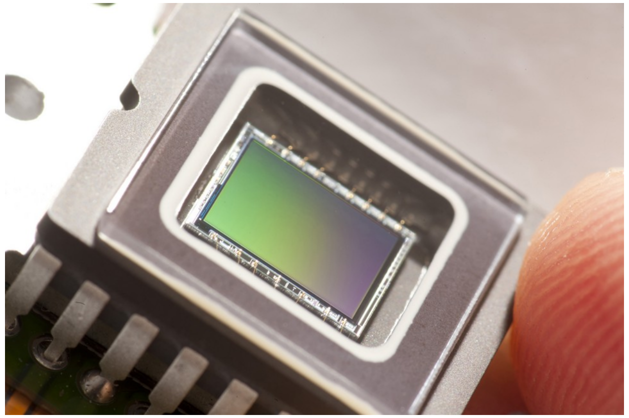


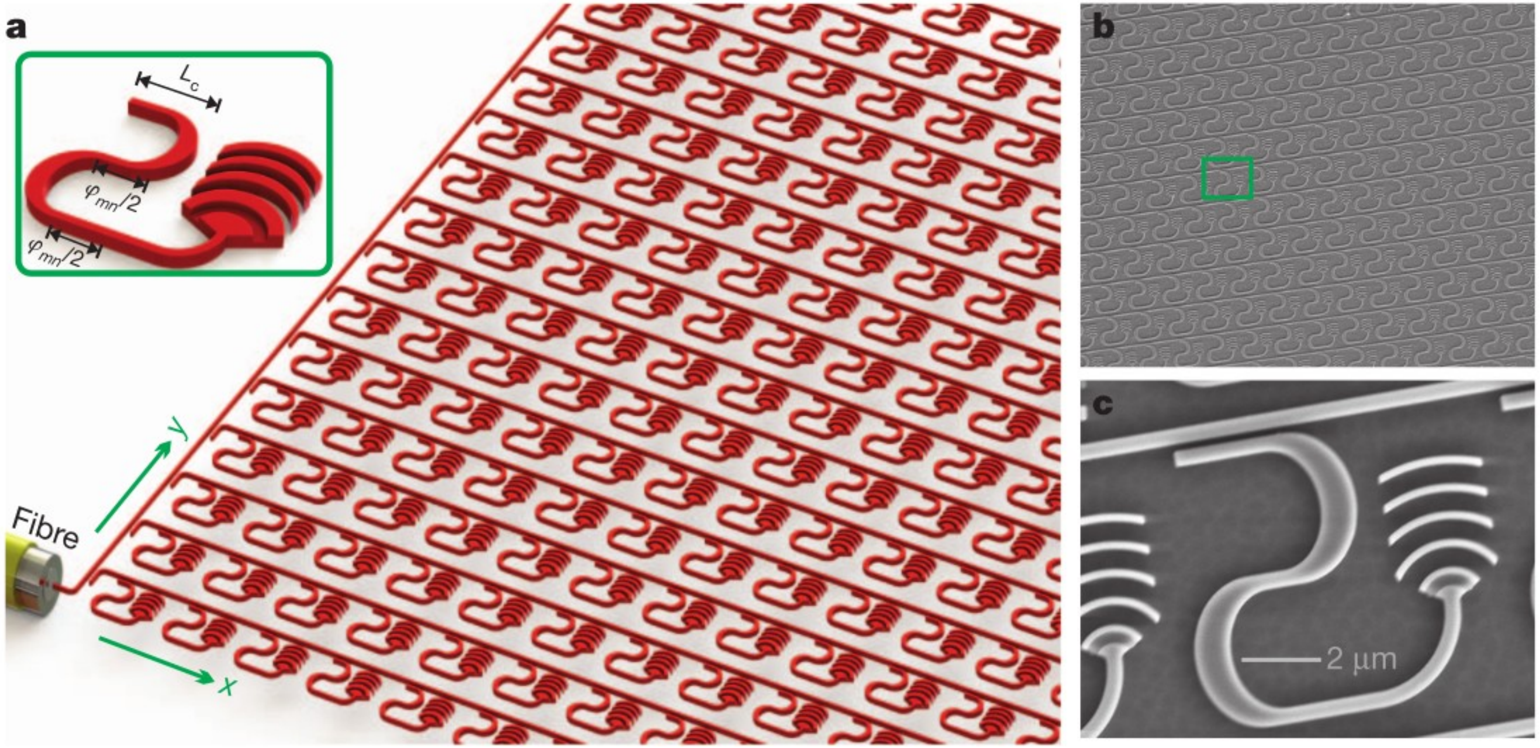


(a) Magnitude response (b) 99.9% normalized bandwidth

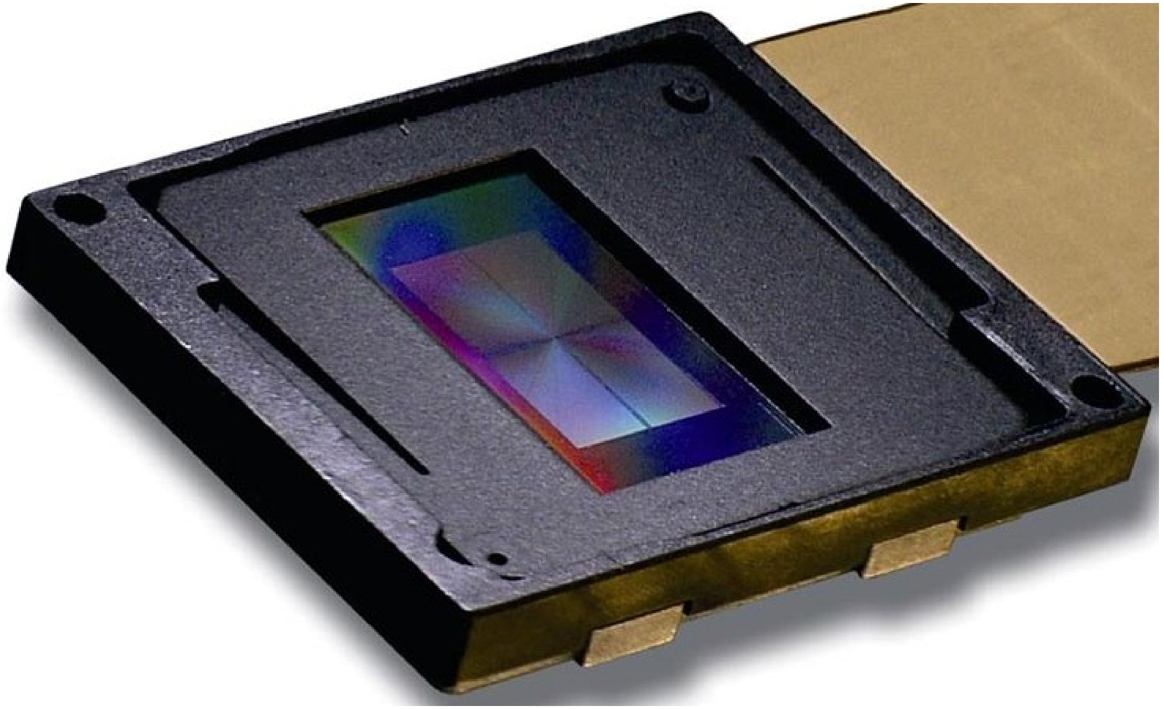
Figure 5. *Spectral properties of coherent propagation in homogeneous media. When not in the sub-wavelength regime, the channel is effectively* $\frac{1}{\lambda}$*-bandlimited.*

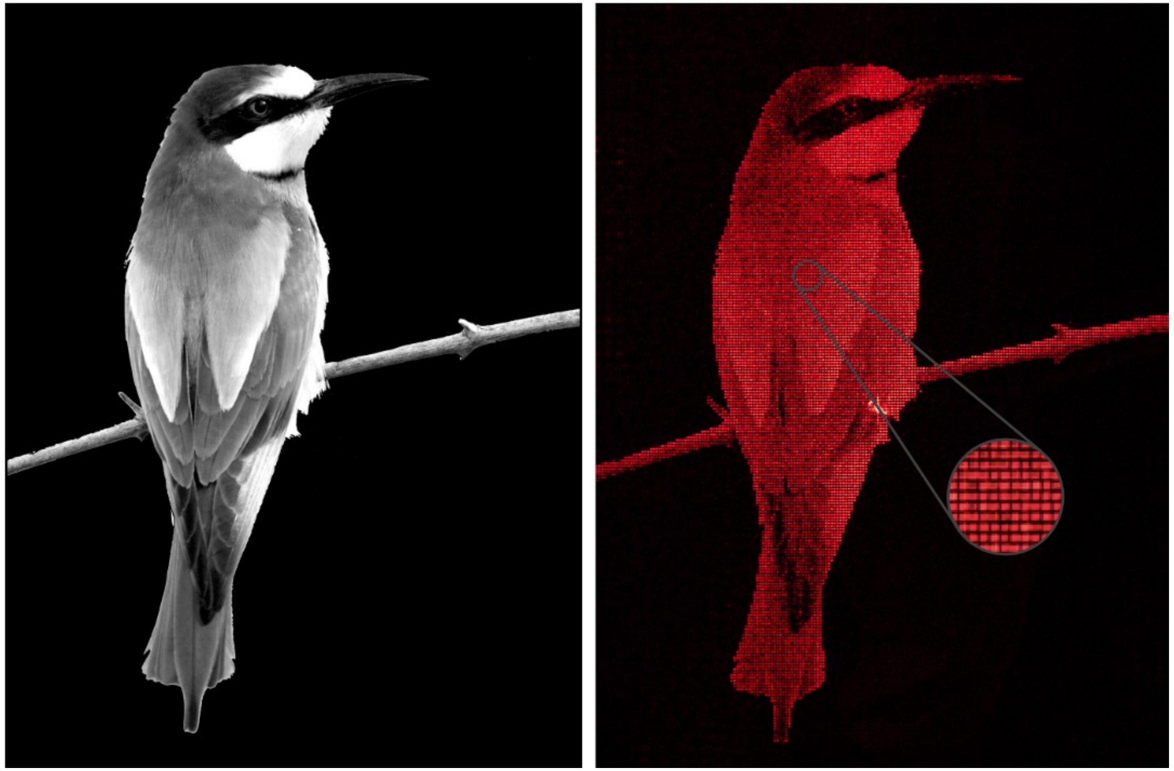
Optical SP 2





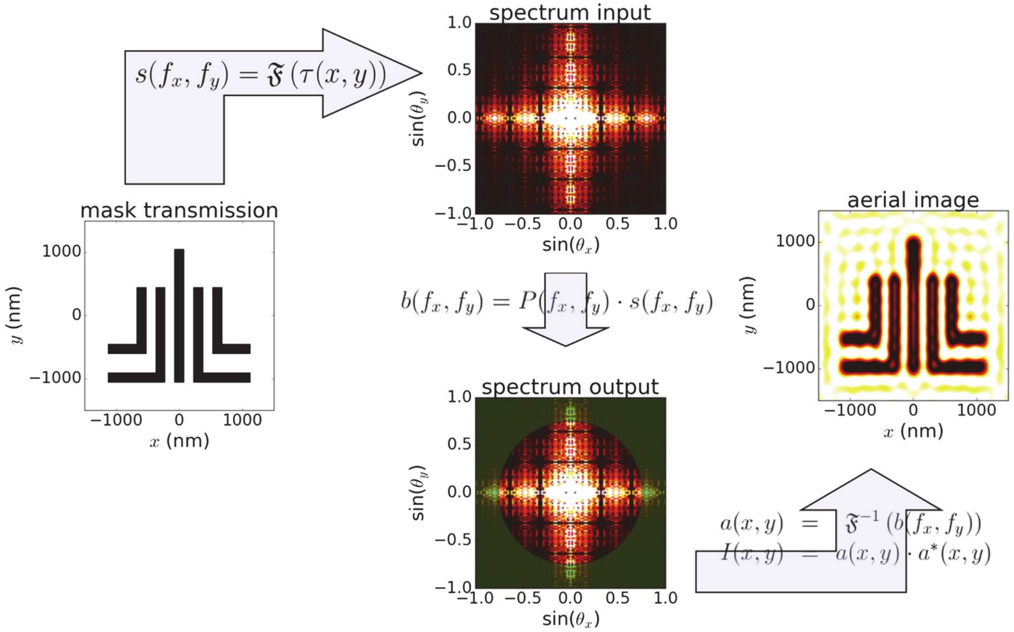
Optical SP 3

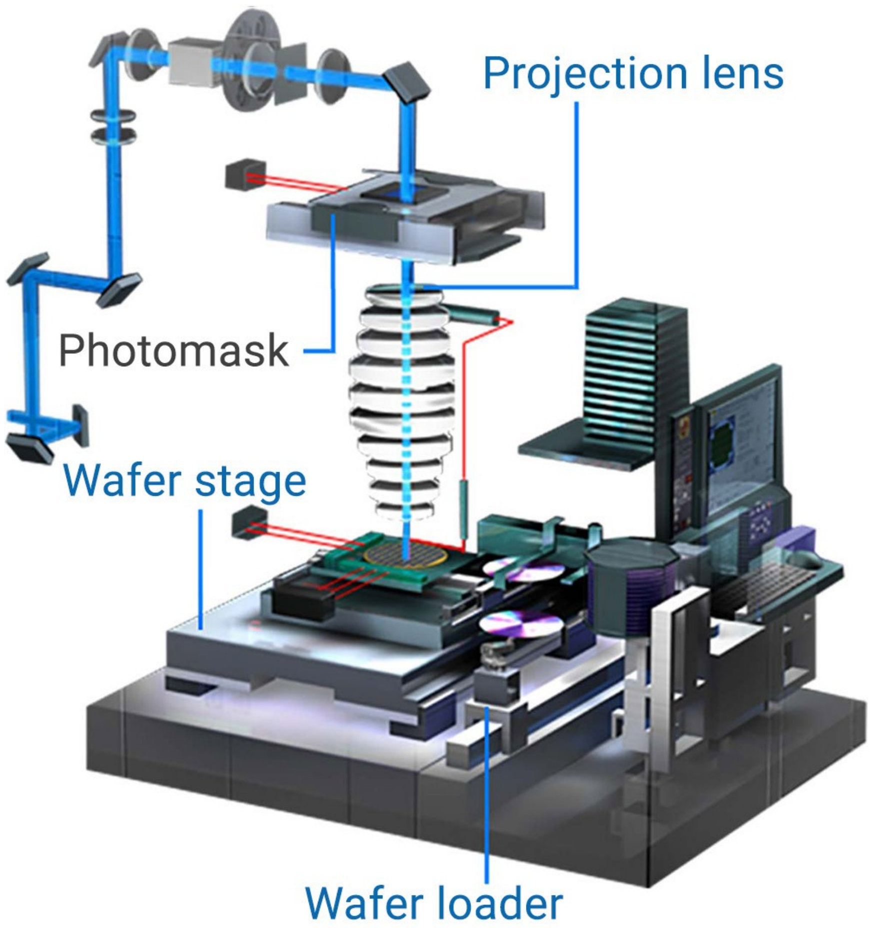


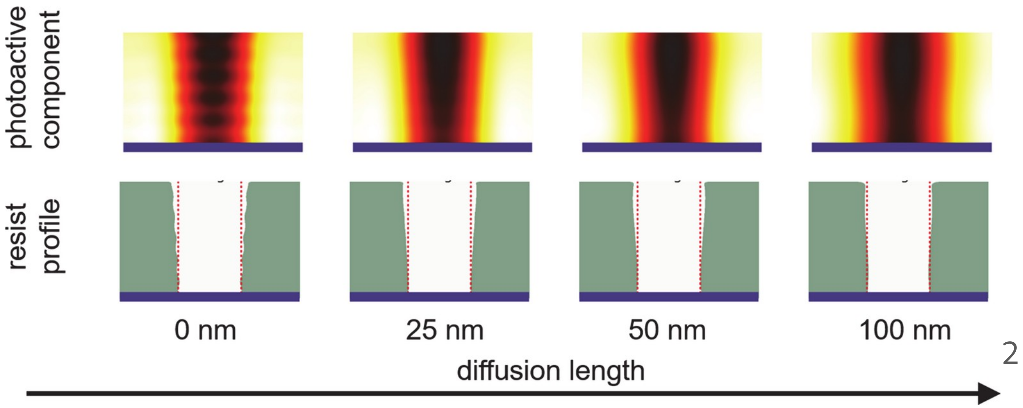


OSP: Photo-Lithography

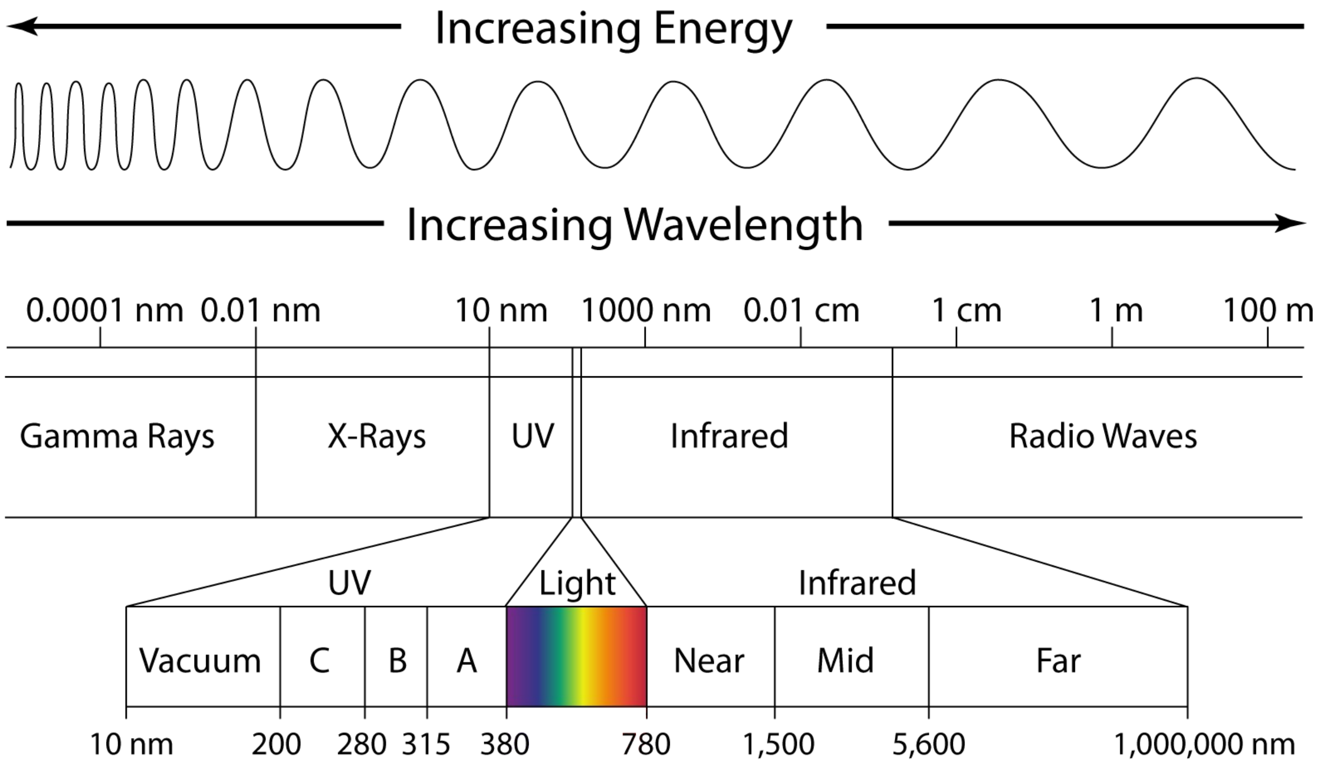
- DUV/EUV light shines onto mask/SLM
  - Free-space propagation + lens effects
  - Intensity pattern at wafer surface
    - Non-linear chemical etching.



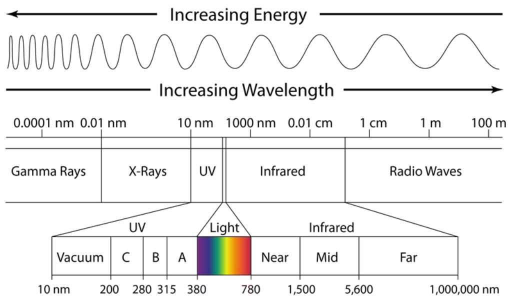




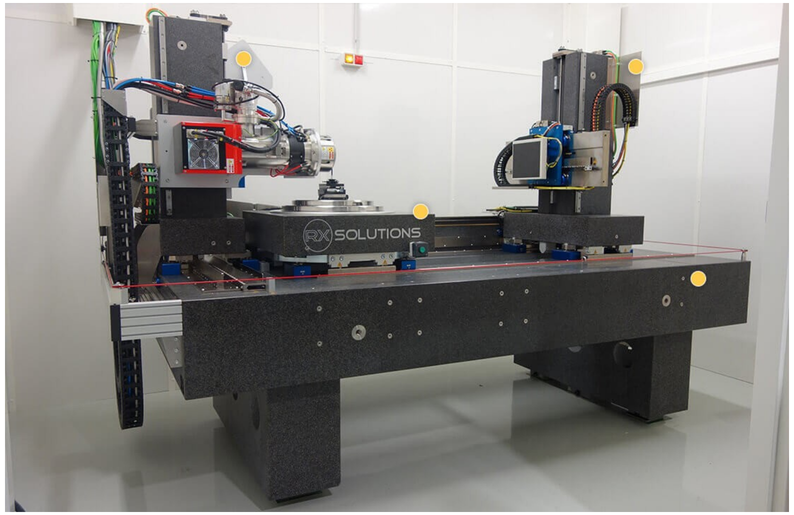
The Electro-Magnetic Spectrum



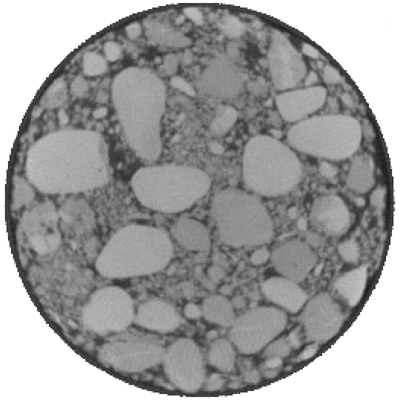
Tomography



- Determine volume absorption profile
  - Project X-rays through object
  - Record shadows from different directions



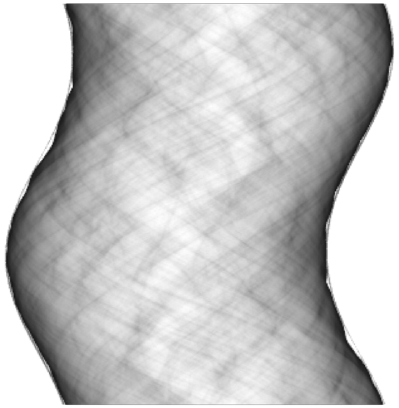
3D volume



measurement →

← computation?

2D projections

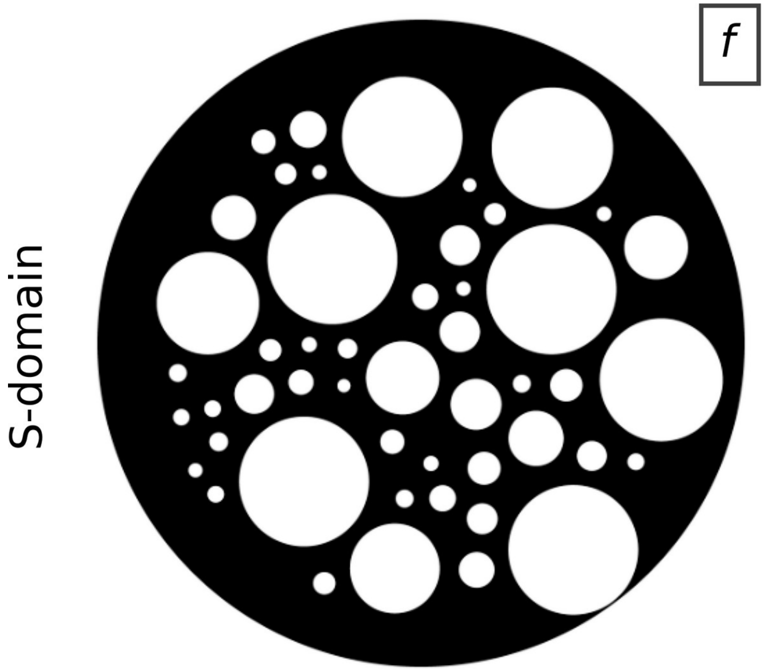


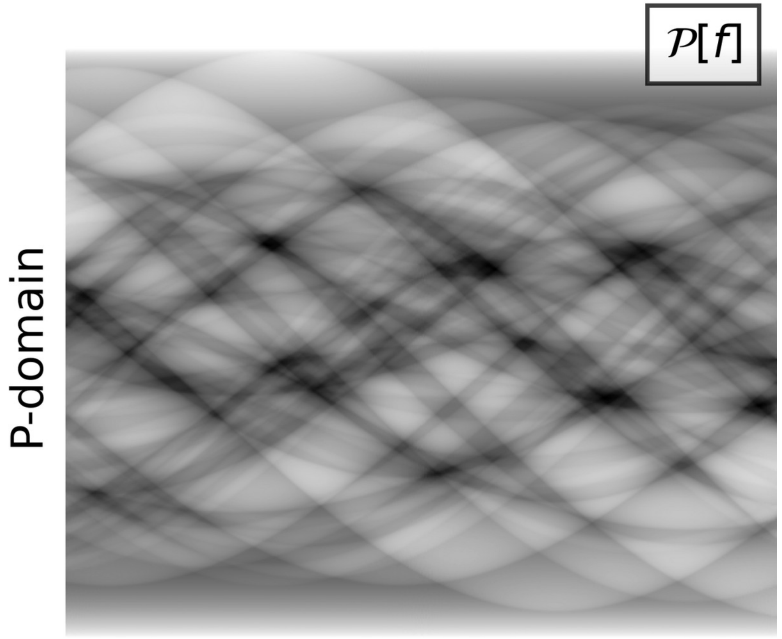
X-Ray Tomography

- Applications
  - Non-Destructive Testing (NDT)
  - Medical Imaging: X-CT
  - Material Science Research
    - Absorption CT
    - Multi-Modal Imaging: PXCT, PyXL

$$\mathrm{P}[\mathrm{f}](\mathbf{n},\mathbf{s}) = \int_{\mathbb{R}} f(\mathbf{n}\alpha + U_{\mathbf{n}^{\perp}}\mathbf{s})d\alpha$$

- Goal: recover f from samples of $g = \mathcal{P}$[f]



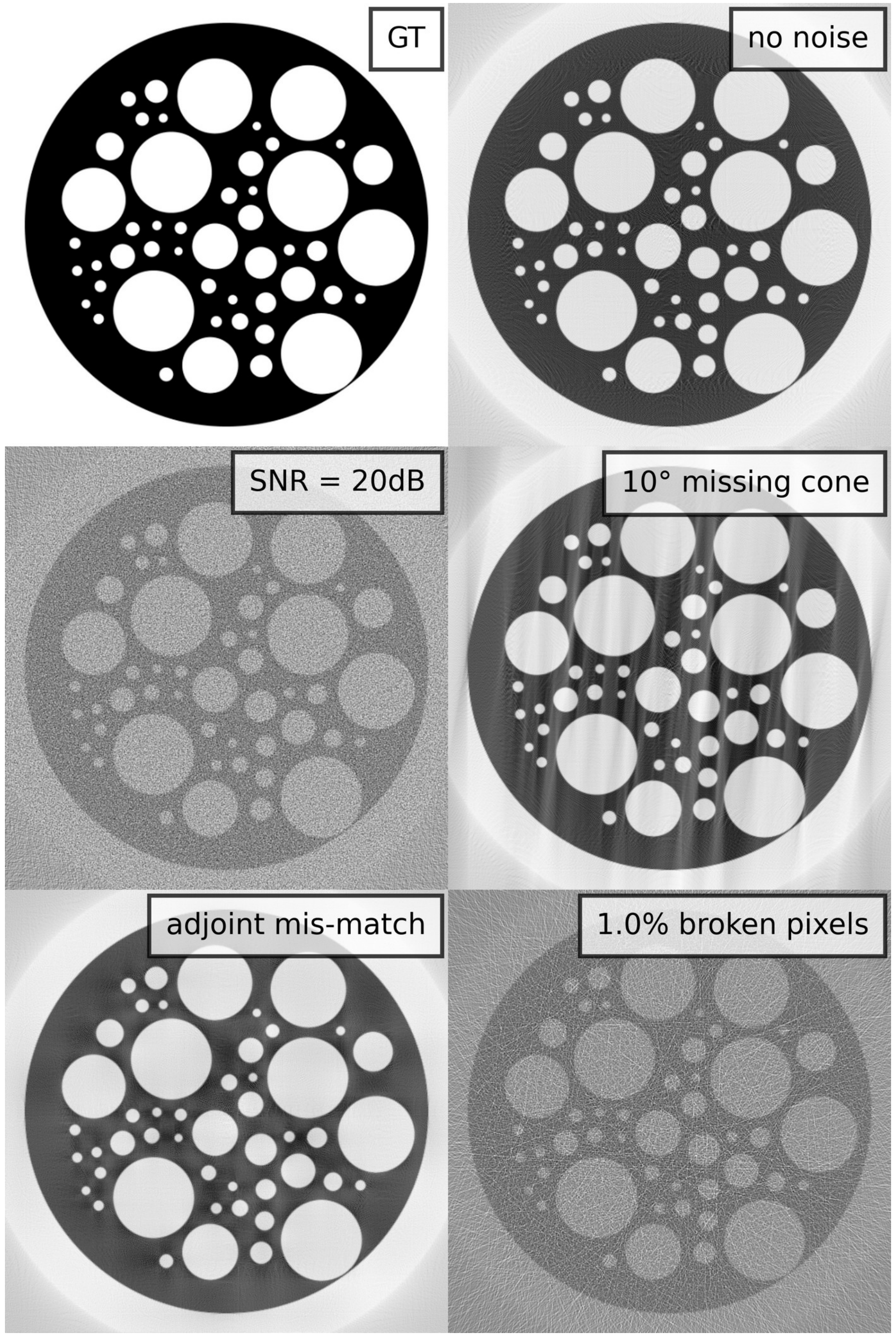


Filtered Back-Projection

- Analytic **direct** inverse

$$\hat{f} = (P^* P)^{-1} P^*[g](\mathbf{x})$$
$$= P^*[h * g](\mathbf{x})$$

- Issues
  - Dense P-space sampling
  - Model mis-match
  - Implementation details

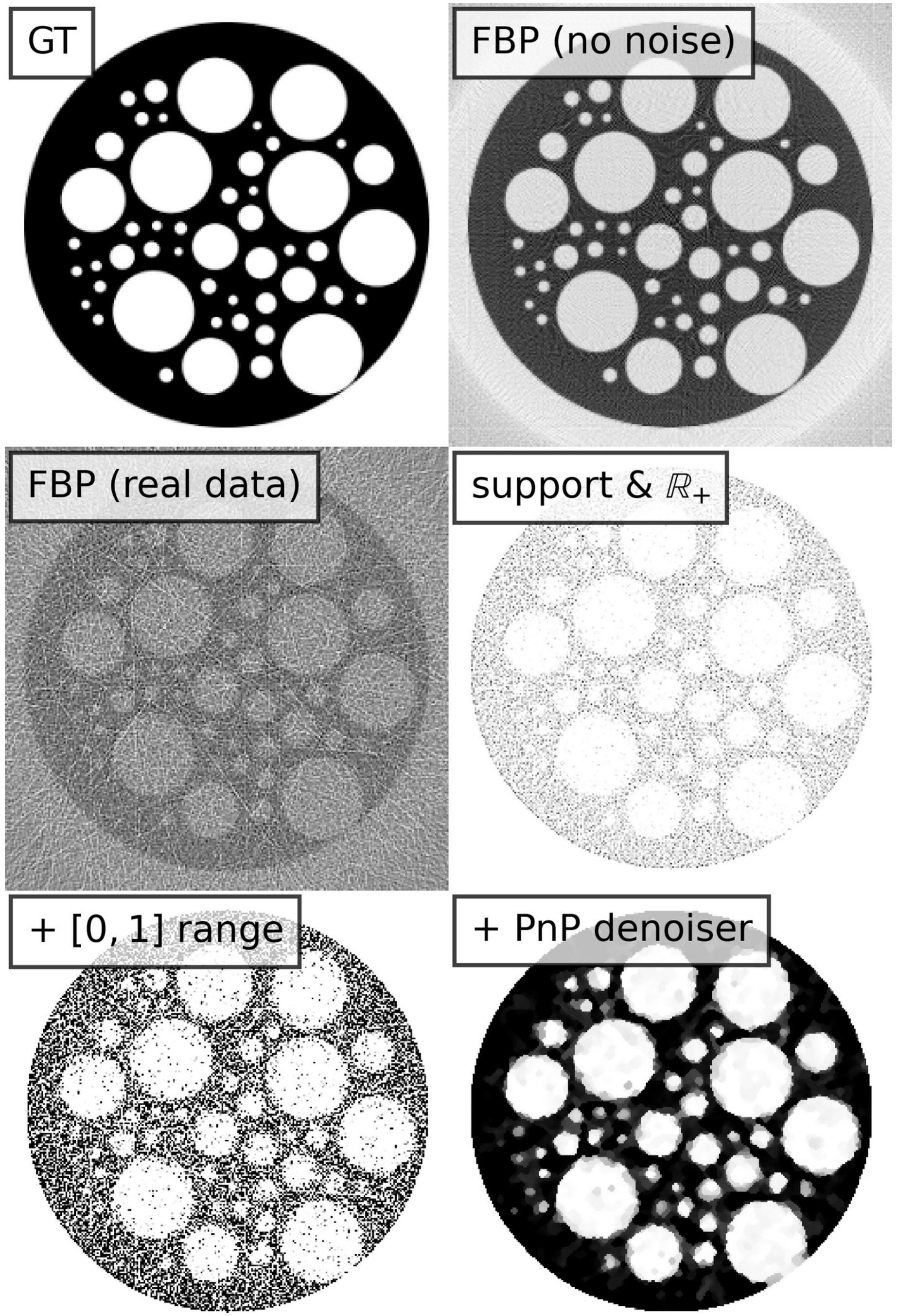


General Image Reconstruction

- Image reconstruction formulated as an inverse problem

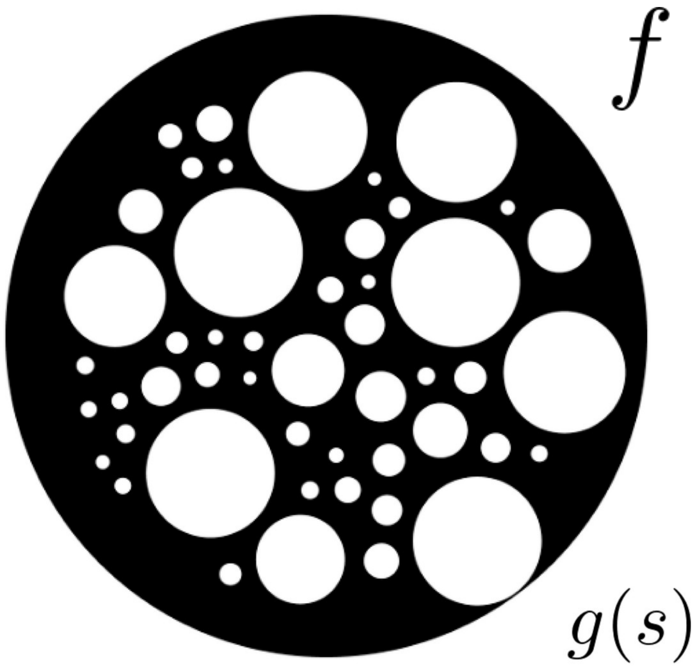
$$\hat{f} = \underset{f \in \mathcal{L}^2(\mathbb{R}^D, \mathbb{R})}{\arg\min} \; \mathcal{F}\{g - \underbrace{\Phi}_{\cdots \circ \mathcal{P} \circ \cdots} f\} + \lambda \, \mathcal{G}\{f\}$$

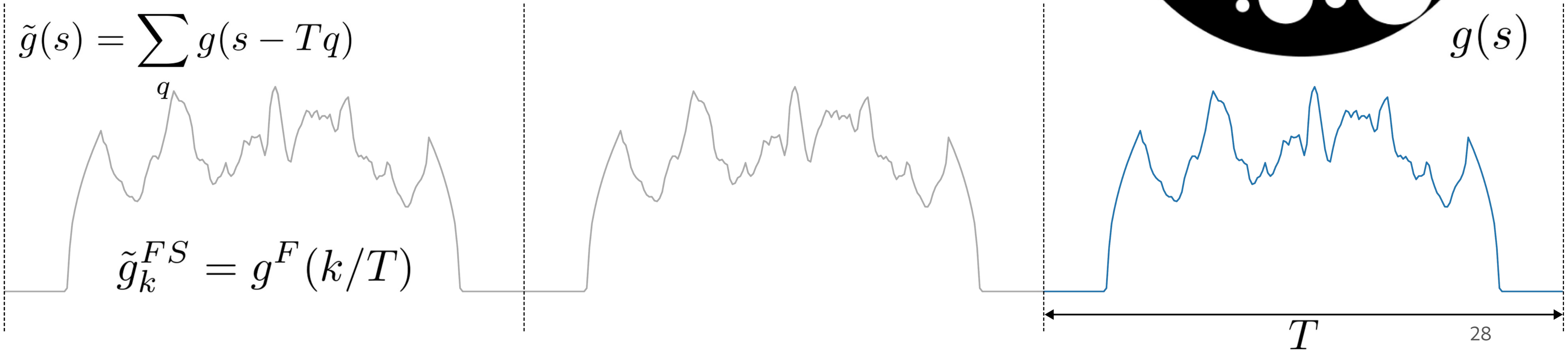
- Advantages
  - Inject prior knowledge
  - Less measurements
  - Account for instrument deviations
  - Leverage **AI** in imaging pipeline
    - Denoisers, generative priors, ...
- Solved via **iterative** 1st-order methods
  - PGD, CG, PDS, SGD, Adam, ...



FourierXRT: Idea

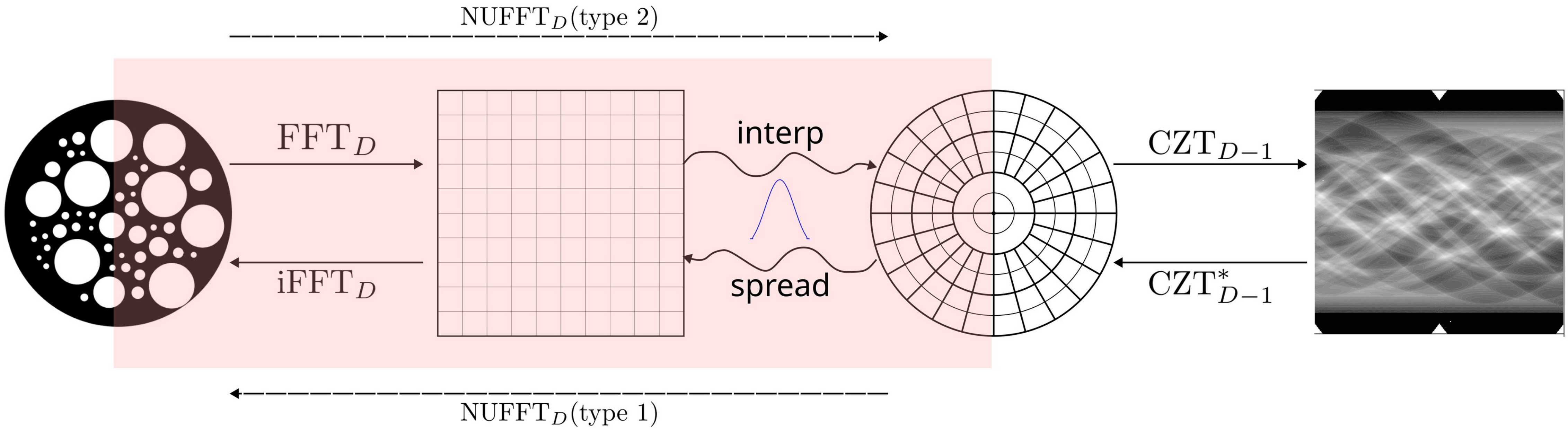
- Projections $g(s)$ are **finite-support** and **band-limited**
  - FS coefficients $\tilde{g}_k^{FS}$ can be computed exactly via the FFT
  - $g^F(v)$ obtained via $\mathrm{FS} \longleftrightarrow \mathrm{FT}$ equivalence
  - *Fourier Slice Theorem* relates $g^F(v)$ with $f^F(v)$



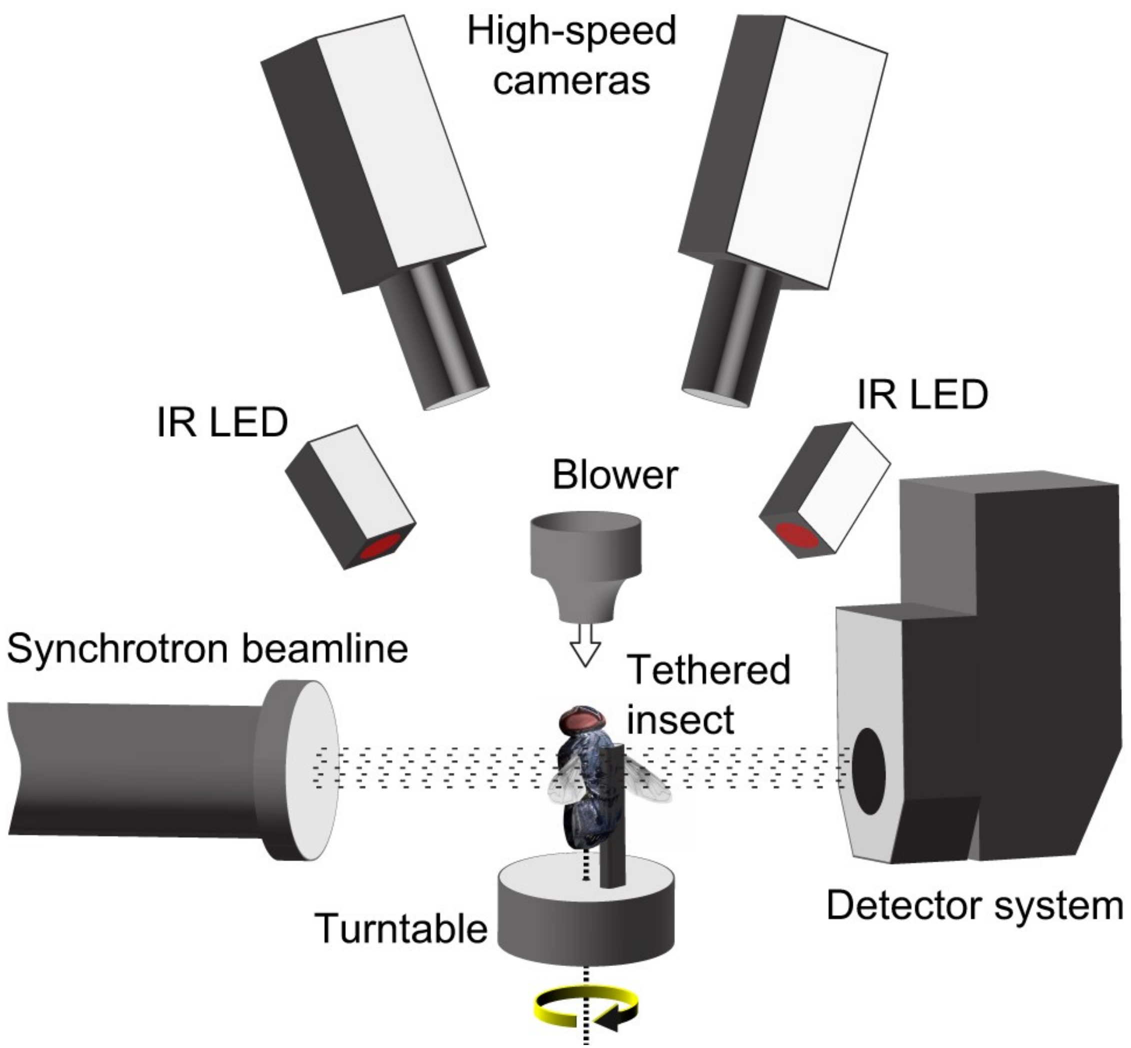


FourierXRT: Idea

$$\mathcal{P}[f](\mathbf{n},\mathbf{s}) = \frac{1}{T^{D-1}} \sum_{\mathbf{m}\in[-N,N]^{D-1}} \mathcal{F}_D[f]\left(\frac{1}{T}U_{\mathbf{n}^\perp}\mathbf{m}\right) \exp\left[j\frac{2\pi}{T}\langle\mathbf{m},\mathbf{s}\rangle\right]$$



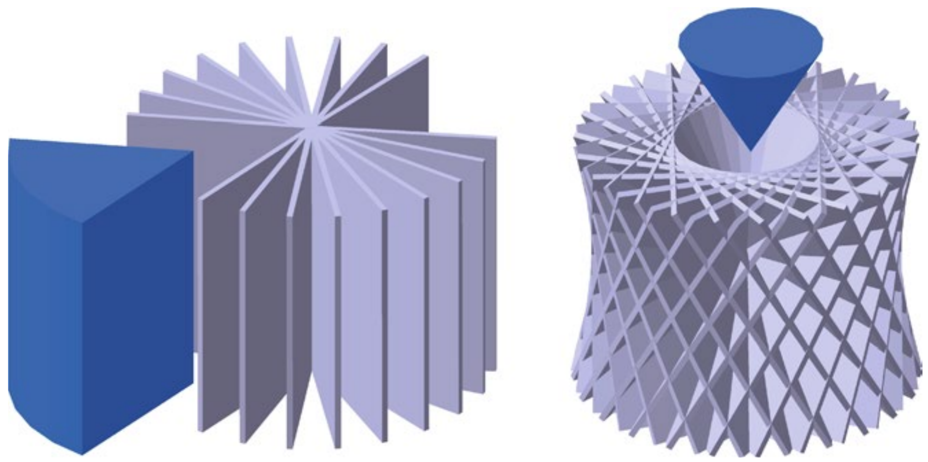
XRT: In-Vivo Imaging

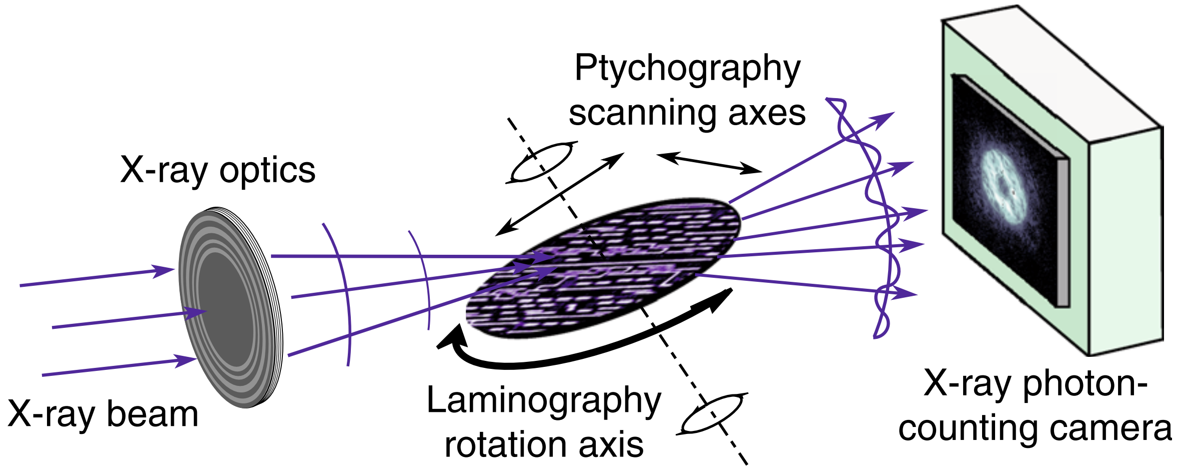


- Fly video (see link below)

In Vivo Time-Resolved Microtomography Reveals the Mechanics of the Blowfly Flight Motor

XRT: Non-Destructive Processor Inspection





- Processor 3D structure video (see link below)

High-resolution non-destructive three-dimensional imaging of integrated circuits