Appendix A. Discrete Fourier Transform (DFT)

used for:	finite-length signals, $x[n] \in \mathbb{C}^N$
analysis formula:	$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}, \quad k = 0, \ldots, N-1$
synthesis formula:	$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}nk}, \quad n = 0, \ldots, N-1$
time and frequency reversal:	$x[-n \mod N] \overset{\text{DFT}}{\longleftrightarrow} X[-k \mod N]$
	$x^*[n] \overset{\text{DFT}}{\longleftrightarrow} X^*[-k \mod N]$
circular shifts:	$x[(n-n_0) \mod N] \overset{\text{DFT}}{\longleftrightarrow} e^{-j\frac{2\pi}{N}n_0 k} X[k]$
	$e^{j\frac{2\pi}{N}nk_0} x[n] \overset{\text{DFT}}{\longleftrightarrow} X[(k-k_0) \mod N]$
energy conservation:	$\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \frac{1}{N} \sum_{k=0}^{N-1} \lvert X[k]\rvert^2$

Appendix B. Notable DFT Pairs

$(n, k = 0, 1, \ldots, N-1)$

$x[n] = \delta[n-L]$	$X[k] = e^{-j\frac{2\pi}{N}Lk}$
$x[n] = 1$	$X[k] = N\delta[k]$
$x[n] = e^{j\frac{2\pi}{N}Ln}$	$X[k] = N\delta[k-L]$
$x[n] = \cos\left(\frac{2\pi}{N}Ln + \phi\right)$	$X[k] = (N/2)[e^{j\phi}\delta[k-L] + e^{-j\phi}\delta[k-N+L]]$
$x[n] = \sin\left(\frac{2\pi}{N}Ln + \phi\right)$	$X[k] = (-jN/2)[e^{j\phi}\delta[k-L] - e^{-j\phi}\delta[k-N+L]]$
$x[n] = \begin{cases} 1 & n < M \\ 0 & M \le n < N \end{cases}$	$X[k] = \frac{\sin((\pi/N)Mk)}{\sin((\pi/N)k)} e^{-j\frac{\pi}{N}(M-1)k}$

Appendix C. Discrete-Time Fourier Transform (DTFT)

used for:	finite-energy, infinite-length signals ($x[n] \in \ell_2(\mathbb{Z})$)
analysis formula:	$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$
synthesis formula:	$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$
symmetries:	$x[-n] \overset{\text{DTFT}}{\longleftrightarrow} X(-\omega)$
	$x^*[n] \overset{\text{DTFT}}{\longleftrightarrow} X^*(-\omega)$
shifts:	$x[n-n_0] \overset{\text{DTFT}}{\longleftrightarrow} e^{-j\omega n_0} X(\omega)$
	$e^{j\omega_0 n} x[n] \overset{\text{DTFT}}{\longleftrightarrow} X(\omega - \omega_0)$
convolution:	$(x * y)[n] \overset{\text{DTFT}}{\longleftrightarrow} X(\omega) Y(\omega)$
modulation:	$x[n] y[n] \overset{\text{DTFT}}{\longleftrightarrow} \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\theta) Y(\omega - \theta) d\theta$
differentiation:	$n x[n] \overset{\text{DTFT}}{\longleftrightarrow} j \frac{d}{d\omega} X(\omega)$
Parseval's Relation:	$\sum_{n=-\infty}^{\infty} \lvert x[n] \rvert^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \lvert X(\omega) \rvert^2 d\omega$

Appendix D. Notable DTFT Pairs:

$x[n] = \delta[n-k]$	$X(\omega) = e^{-j\omega k}$
$x[n] = 1$	$X(\omega) = \tilde{\delta}(\omega)$
	where $\tilde{\delta}(\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$x[n] = u[n]$	$X(\omega) = \dfrac{1}{1-e^{-j\omega}} + \dfrac{1}{2}\tilde{\delta}(\omega)$
$x[n] = a^n u[n] \quad \lvert a\rvert < 1$	$X(\omega) = \dfrac{1}{1-ae^{-j\omega}}$
$x[n] = e^{j\omega_0 n}$	$X(\omega) = \tilde{\delta}(\omega - \omega_0)$
$x[n] = \cos(\omega_0 n + \phi)$	$X(\omega) = (1/2)[e^{j\phi}\tilde{\delta}(\omega-\omega_0) + e^{-j\phi}\tilde{\delta}(\omega+\omega_0)]$
$x[n] = \sin(\omega_0 n + \phi)$	$X(\omega) = (-j/2)[e^{j\phi}\tilde{\delta}(\omega-\omega_0) - e^{-j\phi}\tilde{\delta}(\omega+\omega_0)]$
$x[n] = \begin{cases} 1 & 0 \le n < M \\ 0 & \text{otherwise} \end{cases}$	$X(\omega) = \dfrac{\sin((M/2)\omega)}{\sin(\omega/2)} e^{-j\frac{M-1}{2}\omega}$
$x[n] = \frac{\omega_c}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c}{\pi} n\right)$	$X(\omega) = \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right)$ for $-\pi \le \omega \le \pi$
where $\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$	and $\mathrm{rect}(\omega) = \begin{cases} 1 & \lvert\omega\rvert < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

Appendix E. Continuous-Time Fourier Transform (CTFT)

used for:	finite-energy, infinite-length signals ($x(t) \in L_2(\mathbb{R})$)
analysis formula:	$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}\,dt$
synthesis formula:	$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t}\,df$
scaling:	$x(at) \stackrel{\text{CTFT}}{\longleftrightarrow} \frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
symmetries:	$x(-t) \stackrel{\text{CTFT}}{\longleftrightarrow} X(-f)$
	$x^*(t) \stackrel{\text{CTFT}}{\longleftrightarrow} X^*(-f)$
shifts:	$x(t-t_0) \stackrel{\text{CTFT}}{\longleftrightarrow} e^{-j2\pi f t_0} X(f)$
	$e^{j2\pi f_0 t} x(t) \stackrel{\text{CTFT}}{\longleftrightarrow} X(f-f_0)$
convolution:	$(x * y)(t) \stackrel{\text{CTFT}}{\longleftrightarrow} X(f)Y(f)$
modulation:	$x(t)y(t) \stackrel{\text{CTFT}}{\longleftrightarrow} (X * Y)(f)$
Parseval's Relation:	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \int_{-\infty}^{\infty} \lvert X(f)\rvert^2\,df$

Appendix F. Notable CTFT Pairs:

$x(t) = \delta(t)$	$X(f) = 1$
$x(t) = \delta(t - t_0)$	$X(f) = e^{-j2\pi f t_0}$
$x(t) = 1$	$X(f) = \delta(f)$
$x(t) = e^{j2\pi f_0 t}$	$X(\omega) = \delta(f - f_0)$
$x(t) = \cos(2\pi f_0 t)$	$X(f) = \frac{1}{2}\big(\delta(f - f_0) + \delta(f + f_0)\big)$
$x(t) = \sin(2\pi f_0 t)$	$X(f) = \frac{1}{2j}\big(\delta(f - f_0) - \delta(f + f_0)\big)$
$x(t) = e^{-at}u(t)$	$X(f) = \frac{1}{a + j2\pi f}$
where $Re(a) > 0$	
$x(t) = \mathrm{rect}(t)$	$X(f) = \mathrm{sinc}\big(f\big)$
$x(t) = \mathrm{sinc}(t)$	$X(f) = \mathrm{rect}\big(f\big)$
where $\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$	and $\mathrm{rect}(x) = \begin{cases} 1 & \lvert x \rvert < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

Appendix G. z-Transform

formula: $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$x[n] \overset{Z}{\longleftrightarrow} X(z)$ with ROC R

linearity: $x_1[n] + x_2[n] \overset{Z}{\longleftrightarrow} X_1(z) + X_1(z)$ with ROC at least $R_1 \cap R_2$

scaling: $z_0^n x[n] \overset{Z}{\longleftrightarrow} X\left(\frac{z}{z_0}\right)$ with ROC $|z_0|R$

symmetries: $x[-n] \overset{Z}{\longleftrightarrow} X(z^{-1})$ with ROC R^{-1}

$x^*[n] \overset{Z}{\longleftrightarrow} X^*(z^*)$ with ROC R^*

shift: $x[n-n_0] \overset{Z}{\longleftrightarrow} z^{-n_0} X(z)$ with ROC R

(except for the possible addition or deletion of the origin)

convolution: $(x_1 * x_2)[n] \overset{Z}{\longleftrightarrow} X_1(z) X_2(z)$ with ROC at least $R_1 \cap R_2$

Appendix H. Notable z-Transform Pairs:

$x[n] = \delta[n]$	$X(z) = 1$ with ROC all z
$x[n] = \alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$ with ROC $\lvert z\rvert > \lvert\alpha\rvert$
$x[n] = -\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$ with ROC $\lvert z\rvert < \lvert\alpha\rvert$