COM-202 - Signal Processing

Solutions for Homework 1

Exercise 1. Digital signals

Say which of the following signals are analog and which are digital:

(a) Music recorded on a CD.

(b) Music listened by the audience at a live concert.

(c) Music recorded on a LP record (vinyl).

(d) Photo recorded using a photographic film.

(e) Photo recorded using a CCD sensor.

(f) A page on a book.

(g) The image of a book page on a Kindle.

Solution:

(a) Music recorded on a CD : ***digital***

(b) Music listened by the audience at a live concert : ***analog***

(c) Music recorded on a LP record (vinyl) : ***analog***

(d) Photo recorded using a photographic film : ***analog***

(e) Photo recorded using a CCD sensor : ***digital***

(f) A page on a book : ***analog***

(g) The image of a book page on a Kindle : ***digital***

Exercise 2. Sampling music

A music song recorded in a studio is stored as a digital sequence on a CD. The analog signal representing the music is 2 minutes long and is sampled at a frequency $f_s = 44100\ s^{-1}$. How many samples should be stored on the CD?

Solution: *To compute the number of samples N, we need to multiply the length in seconds of the signal by the sampling frequency, i.e., the number of samples per second:*

$$N = 44100 \times 2 \times 60 = 5,292,000$$

This assumes that the audio is mono; for stereo data, there are two independent channels so the number of samples is double.

Exercise 3. Basic signal operations

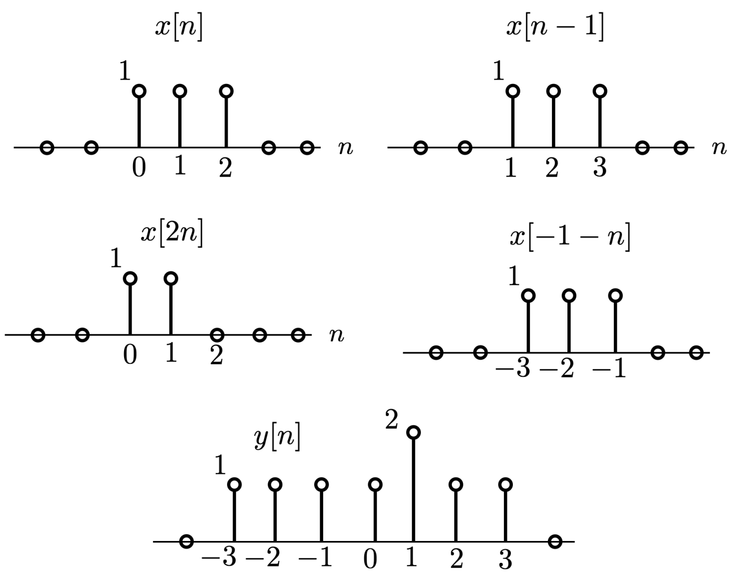
Consider the discrete-time signal:

$$x[n] \quad = \quad \begin{cases} 1, & 0 \le n \le 2, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Sketch $y[n] = x[n-1] + x[2n] + x[-1-n]$ and carefully label both axes in the plot.

Solution:

(b)



Exercise 4. Periodicity

For each of the following signals, determine if the signal is periodic and, if so, find its period.

(a) $x_1[n] = \cos\left(\frac{6\pi}{7}n + 1\right)$

(b) $x_2[n] = \sin\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

(c) $x_3[n] = \cos\left(\frac{n}{8} - \pi\right)$

Solution:

(a) The signal is periodic with period $N = 7$. Indeed we can write $\cos\left(\frac{6\pi}{7}n + 1\right) = \cos\left(\frac{6\pi}{7}(n+N) + 1\right)$; noting that the cosine has a period of 2π, we obtain $\frac{6\pi}{7}N = 2\pi m$ for some integer m. The smallest integer solution is $N = 7$ for $m = 3$.

(b) The signal is periodic with period $N = 8$. Note that $\sin\left(\frac{\pi}{2}n\right)$ is periodic with period $N_1 = 4$ and $\cos\left(\frac{\pi}{4}n\right)$ is periodic with period $N_2 = 8$. The overall period is the least common multiple of N_1 and N_2.

(c) The signal is not periodic since the equation $\frac{N}{8} = 2\pi$ does not have integer solutions.

Exercise 5. Energy and power

For each of the following signals, determine if the signal is an energy signal or a power signal and, accodingly, compute its energy or its power:

(a) $x_1[n] = \begin{cases} n & 0 \le n \le 6 \\ 2 & 6 < n \le 8 \\ 0 & \text{otherwise} \end{cases}$

(b) $x_2[n] = (1 + |n|)^{-1}$

(c) $x_3[n] = e^{jn}$

Solution:

(a) The signal is non-zero only for $n \in \{0, \ldots, 8\}$ and so

$$E_{x_1} = \sum_{n=-\infty}^{+\infty} |x_1[n]|^2 = \sum_{n\in\{0,\ldots,6\}} |n|^2 + \sum_{n\in\{7,8\}} |2|^2 = 0^2 + 1^2 + \cdots + 6^2 + 2\cdot 2^2 = 99,$$

and since this value is finite, $y[n]$ is an energy *signal.*

(b) *The signal is an energy signal since*

$$\begin{aligned}
E_{x_2} &= \sum_{n=-\infty}^{\infty} |x_2[n]|^{-2} \\
&= \sum_{n=-\infty}^{\infty} |1+|n||^{-2} \\
&= 2\sum_{n=0}^{\infty} |1+n|^{-2} - 1 \\
&= 2\sum_{n=1}^{\infty} \frac{1}{n^2} - 1 \\
&= \frac{\pi^2}{3} - 1
\end{aligned}$$

(c) *Since* $|x_3[n]| = 1$ *for all* n*, clearly this is a power signal. Its power is*

$$P_{x_3} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_3[n]|^2 = 1$$

Exercise 6. Moving average

Consider the following signal,

$$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2].$$

Compute its moving average $y[n] = \frac{x[n]+x[n-1]}{2}$, where we call $x[n]$ the input and $y[n]$ the output.

Solution: *We compute* $y[n]$ *by substituting the expression for* $x[n]$ *into* $y[n] = (x[n] + x[n-1])/2$*:*

$$y[n] = \frac{\delta[n]+2\delta[n-1]+3\delta[n-2]+\delta[n-1]+2\delta[n-2]+3\delta[n-3]}{2}$$

By collecting the delta signals that have the same delay we finally obtain:

$$\begin{aligned}
y[n] &= 0.5\delta[n] + 1.5\delta[n-1] + 2.5\delta[n-2] + 1.5\delta[n-3] \\
&= \begin{cases} 0.5 & n=0 \\ 1.5 & n=1 \\ 2.5 & n=2 \\ 1.5 & n=3 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$