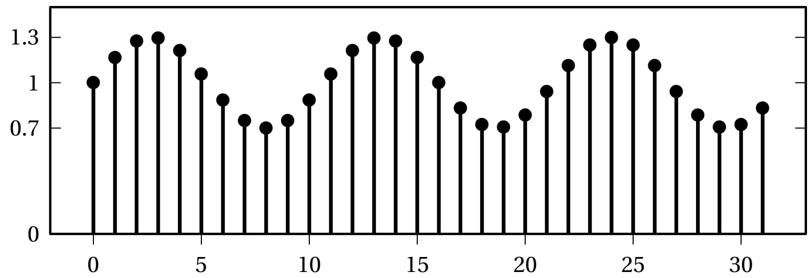
COM 202 - Signal Processing

Sample Midterm

Multiple Choice Questions: each multiple choice question is worth 4 points.

Exercise 1.

Consider the finite-length discrete-time signal $\mathbf{x} \in \mathbb{C}^{32}$ shown in this figure:



What is the DFT of this signal?

a)

$$X(k) = \begin{cases} 32, & k = 0, \\ \frac{24}{5} e^{-j\pi/2}, & k = 3, \\ \frac{24}{5} e^{j\pi/2}, & k = 29, \\ 0, & \text{otherwise} \end{cases}$$

b)

$$X(k) = \begin{cases} \frac{24}{5}, & k = 3 \text{ and } k = 29, \\ 0, & \text{otherwise} \end{cases}$$

c)

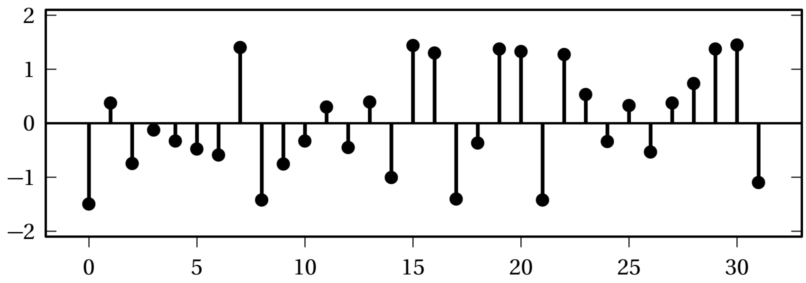
$$X(k) = \begin{cases} \frac{128}{5}, & k = 0, \\ \frac{48}{5} e^{-j\pi}, & k = 3, \\ \frac{24}{5} e^{j\pi/4}, & k = 29, \\ 0, & \text{otherwise} \end{cases}$$

d)

$$X(k) = \begin{cases} 32, & k = 1, \\ 24, & k = 7, \\ 16, & k = 30, \\ 0, & \text{otherwise} \end{cases}$$

Exercise 2.

The 32 samples of the discrete-time signal $\mathbf{x}$ shown in the figure below were obtained using a random number generator.



Say if it is possible to determine 32 complex numbers $C_0, C_1, \ldots, C_{31}$ so that the signal in the plot can be expressed as

$$x[n] = \sum_{k=0}^{31} C_k e^{j\frac{\pi}{16}nk}.$$

a) Yes, $C_k = X[k]/N$, where $\mathbf{X} = \text{DFT}\{\mathbf{x}\}$.

b) No, $\mathbf{x}$ doesn't look like a linear combination of complex exponentials.

c) No, because $\mathbf{x}$ is real-valued.

d) Yes, $C_k = X[k]$, where $\mathbf{X} = \text{DFT}\{\mathbf{x}\}$.

Exercise 3.

Given the matrix $\mathbf{H}$ below, for what values of c_1, c_2, c_3, c_4 do the rows of $\mathbf{H}$ form an *orthonormal* basis in $\mathbb{C}^4$?

$$\mathbf{H} = \begin{bmatrix} c_1 & c_1 & c_1 & c_1 \\ c_2 & c_2 & -c_2 & -c_2 \\ c_3 & -c_3 & 0 & 0 \\ 0 & 0 & c_4 & -c_4 \end{bmatrix}$$

a) $c_1 = c_2 = \frac{1}{2}$ and $c_3 = c_4 = \frac{1}{\sqrt{2}}$

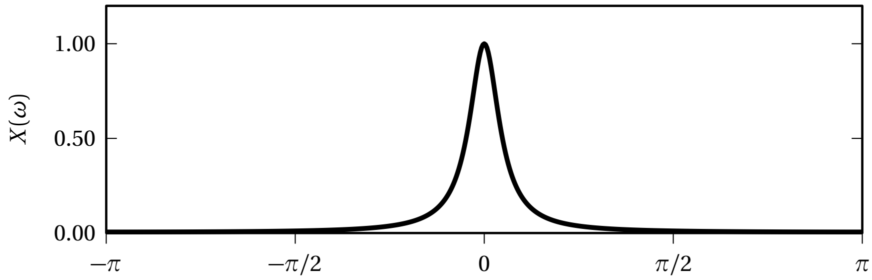
b) None.

c) $c_1 = c_2 = \frac{1}{4}$ and $c_3 = c_4 = \frac{1}{2}$

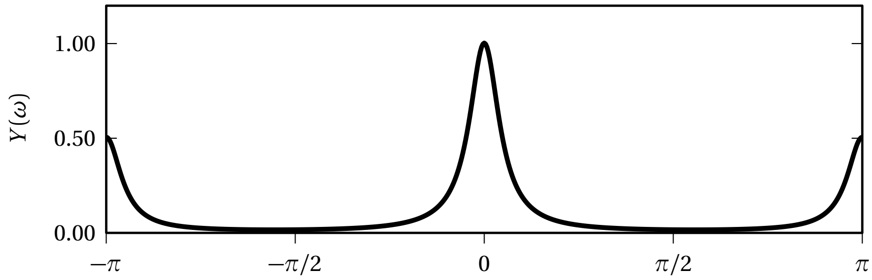
d) $c_1 = c_2 = c_3 = c_4 = 1$

Exercise 4.

Consider the infinite-length signal $\mathbf{x}$ whose DTFT $X(\omega)$ is shown in the following picture:



Consider now the following plot showing $Y(\omega)$, the DTFT of the infinite-length signal $\mathbf{y}$:



What is the correct definition of $\mathbf{y}$?

a) $y[n] = x[n] + \frac{1}{2}e^{j\pi n}x[n]$

b) $y[n] = x[n] + \frac{1}{2}x[n - \pi n]$

c) $y[n] = \frac{1}{2}e^{-j\pi n}x[n] + \frac{1}{2}e^{j\pi n}x[n]$

d) $y[n] = x[n]\left(1 + \left(\frac{1}{2}\right)^{|n|}\right)$

Exercise 5.

Consider a causal filter with transfer function

$$H(z) = \frac{1+7z^{-4}}{4+z^{-2}}$$

Which of the following statements is correct?

a) The region of convergence of the transfer function includes the unit circle

b) The filter is FIR

c) The filter is unstable

d) The filter has four poles

Exercise 6.

What is the output obtained after running the following Python code?

```
import numpy as np

N = 4
w = np.pi / N
a = np.arange(N)
print( np.dot(np.exp(-2j * w * np.outer(a, a)), np.ones(N)) )
```

a) `[1.0 1.0 1.0 1.0]`

b) `[3.14159265359j 0.0 0.0 -3.14159265359j]`

c) `[4.0 0.0 0.0 0.0]`

d) `[1.0 1.0j -1.0 -1.0j]`

Exercise 7.

A discrete-time system is defined by an input-output relationship $y[n] = x[-n]$. Which of the following claims about the system is true?

a) The system is linear

b) The system is time-invariant

c) The system is causal

d) The system is LTI

Exercise 8.

Which of the following claims about properties of infinite-length discrete-time signals is false?

- The signal $x[n] = \sin(\frac{n}{2}) + \cos(\frac{n}{3})$ is periodic
- If $x[n]$ has finite energy, then $\lim_{n\to\infty} |x[n]| = 0$
- A non-zero bounded periodic discrete-time signal must have finite power
- Given a real $a > 0$, the signal $x[n] = e^{-an} u[n]$ is in ℓ_2

Exercise 9.

LTI Systems: This question is worth 20 points.

An LTI system is given by the following difference equation

$$y[n] = \lambda_1 y[n-1] + \lambda_2 x[n]$$

where $\lambda_1, \lambda_2 > 0$ are real numbers.

(a) Assume that the system satisfies the condition of initial rest. That is, if $x[n] = 0$ for all $n < n_0$, then $y[n] = 0$ for all $n < n_0$. Find the impulse response of the system. Is this system an FIR or an IIR filter?

(b) For which values of λ_1 and λ_2 is this system stable? Justify your answer.

(c) Let $\lambda_1 = \lambda_2 = \frac{1}{2}$. Given the input signal $x[n] = \left(\frac{1}{4}\right)^n u[n]$ to the system, compute the output $y[n]$.

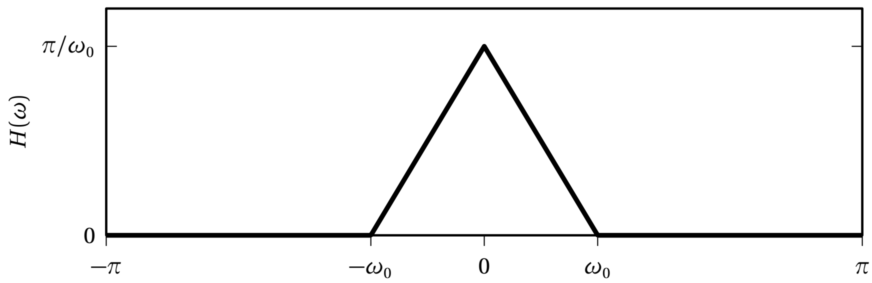
Exercise 10.

DTFT. This question is worth 22 points.

Consider an LTI system with frequency response

$$H(\omega) = \begin{cases} \frac{\pi}{\omega_0}\left(1 - \left|\frac{\omega}{\omega_0}\right|\right), & |\omega| \leq \omega_0, \\ 0, & \text{otherwise}, \end{cases}$$

for some value $0 \leq \omega_0 < \frac{\pi}{2}$. The real-valued frequency response has thus a triangular shape as in the following illustration:



(a) Is the system a low-pass, a high-pass, or a band-pass filter?

(b) Is the system causal?

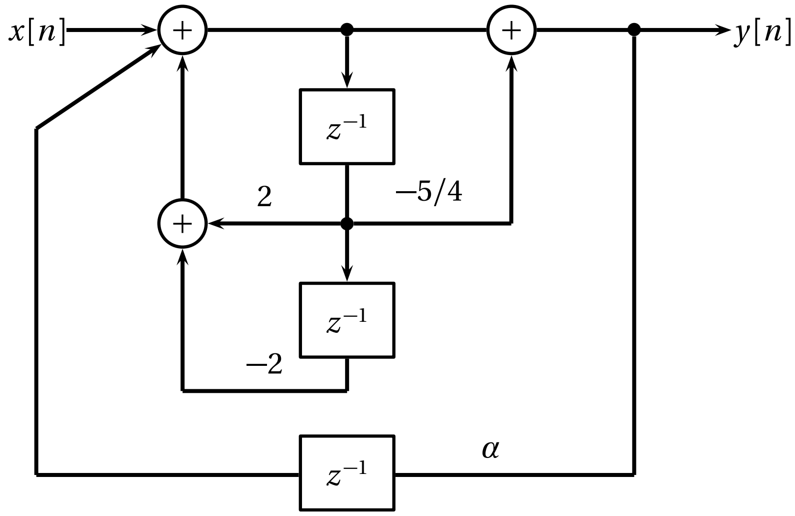
(c) Find the system's response to the input signal $x[n] = 1 + \cos\left(\frac{\omega_0}{2}n\right) + \cos(2\omega_0 n)$.

(d) Let $\mathbf{h}$ be the impulse response of the system. Show that $\sum_{n\in\mathbb{Z}} |h[n]|^2 = \frac{\pi}{3\omega_0}$.

Exercise 11.

System analysis. This question is worth 26 points.

Consider the causal LTI system shown in the following block diagram and let $\alpha = -2$:



(a) Compute the system's transfer function $H(z)$

(b) Plot the system's poles and zeros on the complex plane

(c) Sketch the magnitude of the system's frequency response $|H(\omega)|$

(d) Find the transfer function of a *stable* filter $G(z)$ so that $|H(z)G(z)| = 1$

(e) Would the system be stable if we removed the lowest branch (i.e. if we set $\alpha = 0$)?