COM-202 - Signal Processing

Homework 4

Please submit your answer to Exercise 1 by Mar 20, 2025

Exercise 1. DFT as matrix-vector multiplication

Compute the DFT of the $\mathbb{C}^4$ vector $\mathbf{x} = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T$ as a matrix-vector multiplication.

Exercise 2. Signals with imaginary DFT

In this exercise we will consider sets of finite-length signals whose DFT coefficients are purely imaginary, that is, their real part is equal to zero.

(a) consider the set of length-3 signals

$$A = \{\mathbf{x} \in \mathbb{C}^3 \big| \text{Re}\{x_0\} = 0, x_2 = -x_1^*\}$$

Show that if $\mathbf{x} \in A$ and $\mathbf{X} = \text{DFT}\{\mathbf{x}\}$ then $\text{Re}\{X[k]\} = 0$ for $k = 0, 1, 2$.

(b) Describe the set of signals in $\mathbb{C}^4$ for which the DFT coefficients are all purely imaginary.

Exercise 3. DFT and time reversal

Show that $\text{DFT}\{\mathscr{R}\mathbf{x}\} = \mathscr{R}\{\text{DFT}\{\mathbf{x}\}\}$.

Here are some pointers that you may find useful in your derivations:

- if $\mathbf{x} \in \mathbb{C}^N$ and $\mathbf{x}_r = \mathscr{R}\mathbf{x}$, then $x_r[n] = x[-n \mod N]$
- all harmonic complex exponential in $\mathbb{C}^N$ are N-periodic and so they contain an "implicit" modulo operation:

$$e^{-j\frac{2\pi}{N}nk} = e^{-j\frac{2\pi}{N}(nk \mod N)}$$

- in modular arithmetic the following equivalences always hold

$$n \mod N = (n + kN) \mod N \quad \forall k \in \mathbb{Z}$$

$$(n \mod N) \mod N = n \mod N$$

- in modular arithmetic the distributive property for multiplication is

$$(n_1 n_2) \mod N = [(n_1 \mod N)(n_2 \mod N)] \mod N$$
