COM-202 - Signal Processing

Homework 3

Please submit your answer to Exercise 5 by Mar 13, 2025

Exercise 1. DFT of elementary signals

Derive the formula for the DFT of the length-N signal

$$x[n] = \cos((2\pi/N)Ln + \phi).$$

Exercise 2. DFT by inspection

The signal $\mathbf{x}$ shown at the top of the next page is the sum of the three 64-periodic signals plotted in the bottom panels of the figure, i.e.

$$\mathbf{x} = \mathbf{a} + \mathbf{b} + \mathbf{c}.$$

By looking at the plots, determine the expressions for $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and use the result to compute the DFT coefficients $X[k], k = 0, 1, \ldots, 63$.

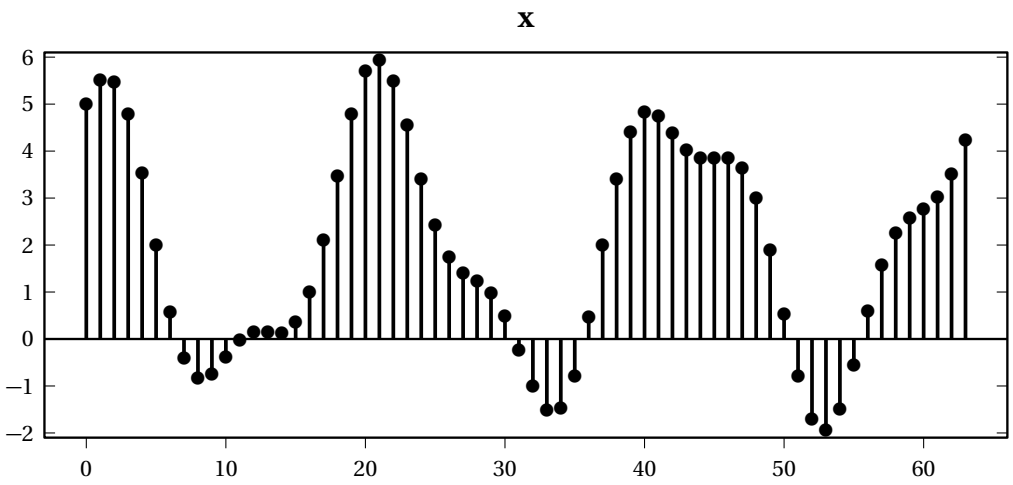
Exercise 3. DFT of a short signal

Compute the 8 DFT coefficients of the signal $\mathbf{x} = \begin{bmatrix} -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{bmatrix}^T$.

Exercise 4. Structure of the DFT formulas

The DFT and IDFT formulas are similar, but not identical. Given a length-N signal $\mathbf{x}$, write an expression for the signal $\mathbf{y}$ obtained by applying the DFT twice in a row to $\mathbf{x}$:

$$\mathbf{y} = \text{DFT}\{\text{DFT}\{\mathbf{x}\}\}$$


x
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5
4
3
2
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0
−1
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10
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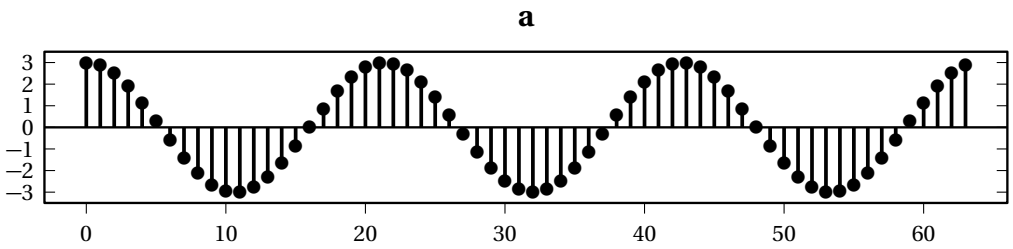
30
40
50
60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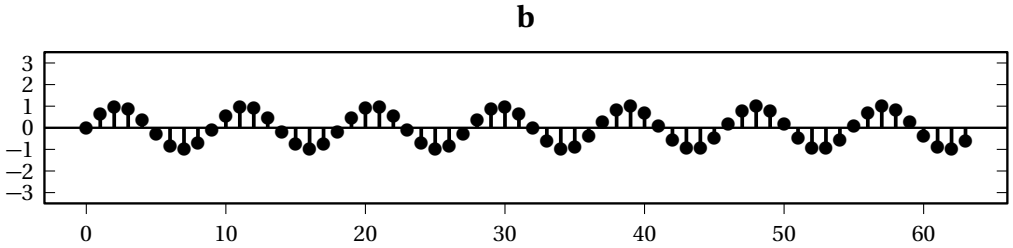
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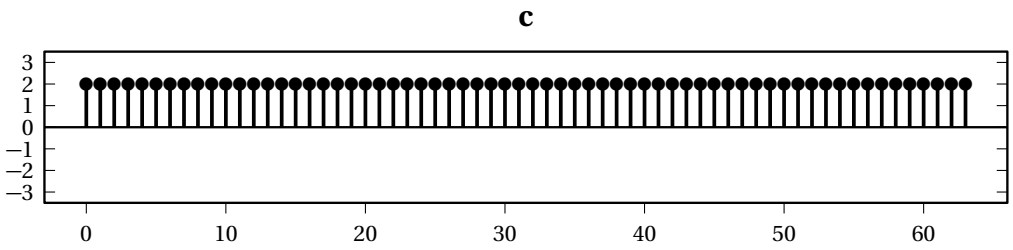
c
3
2
1
0
−1
−2
−3
0
10
20
30
40
50
60

Exercise 5. DFT of repeated signals

Take $\mathbf{x} \in \mathbb{C}^N$ and its DFT $\mathbf{X}$. Now build a signal of length $2N$ by duplicating each element of $\mathbf{x}$:

$$\mathbf{y} = \begin{bmatrix} x[0] & x[0] & x[1] & x[1] & x[2] & x[2] & \dots & x[N-1] & x[N-1] \end{bmatrix}^T$$

Determine the $2N$-point DFT $\mathbf{Y}$ in terms of the N DFT coefficients in $\mathbf{X}$.

Exercise 6. DFT in matrix form

Consider the DFT expressed as a as a matrix/vector multiplication and call $\mathbf{W}$ the $N \times N$ DFT matrix. Is $\mathbf{W}$ Hermitian-symmetric for all values of N?

Exercise 7. Implementing the DFT

You have been asked to implement a five-point DFT on a microprocessor that can only perform real-valued additions, subtractions and multiplications (in other words, there is no mathematical library with which to compute trigonometric functions, nor native support for complex numbers). In your code, you can store the values of just the two following numerical constants:

$$C = \cos(2\pi/5) \approx 0.309$$
$$S = \sin(2\pi/5) \approx 0.951$$

Write an algorithm that, for a real-valued input data vector $[x_0, x_1, x_2, x_3, x_4]$, computes the real and imaginary parts of its 5-point DFT using only additions, multiplications and the constants C, S. You can use any notation you prefer but try to be as clear as possible in your derivation.
