
Appendix A. Discrete Fourier Transform (DFT)

used for: finite-length signals, $x[n] \in \mathbb{C}^N$

analysis formula:
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} nk}, \quad k = 0, \dots, N-1$$

synthesis formula:
$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi}{N} nk}, \quad n = 0, \dots, N-1$$

time and frequency reversal:
$$x[-n \bmod N] \xleftrightarrow{\text{DFT}} X[-k \bmod N]$$

$$x^*[n] \xleftrightarrow{\text{DFT}} X^*[-k \bmod N]$$

circular shifts:
$$x[(n - n_0) \bmod N] \xleftrightarrow{\text{DFT}} e^{-j \frac{2\pi}{N} n_0 k} X[k]$$

$$e^{j \frac{2\pi}{N} n k_0} x[n] \xleftrightarrow{\text{DFT}} X[(k - k_0) \bmod N]$$

energy conservation:
$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

Appendix B. Notable DFT Pairs

$(n, k = 0, 1, \dots, N-1)$

$$x[n] = \delta[n - L] \quad X[k] = e^{-j \frac{2\pi}{N} Lk}$$

$$x[n] = 1 \quad X[k] = N \delta[k]$$

$$x[n] = e^{j \frac{2\pi}{N} Ln} \quad X[k] = N \delta[k - L]$$

$$x[n] = \cos\left(\frac{2\pi}{N} Ln + \phi\right) \quad X[k] = (N/2)[e^{j\phi} \delta[k - L] + e^{-j\phi} \delta[k - N + L]]$$

$$x[n] = \sin\left(\frac{2\pi}{N} Ln + \phi\right) \quad X[k] = (-jN/2)[e^{j\phi} \delta[k - L] - e^{-j\phi} \delta[k - N + L]]$$

$$x[n] = \begin{cases} 1 & n < M \\ 0 & M \leq n < N \end{cases} \quad X[k] = \frac{\sin((\pi/N)Mk)}{\sin((\pi/N)k)} e^{-j \frac{\pi}{N}(M-1)k}$$

Appendix C. Discrete-Time Fourier Transform (DTFT)

used for: finite-energy, infinite-length signals ($x[n] \in \ell_2(\mathbb{Z})$)

analysis formula: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

synthesis formula: $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega$

symmetries: $x[-n] \xleftrightarrow{\text{DTFT}} X(e^{-j\omega})$

$$x^*[n] \xleftrightarrow{\text{DTFT}} X^*(e^{-j\omega})$$

shifts: $x[n - n_0] \xleftrightarrow{\text{DTFT}} e^{-j\omega n_0} X(e^{j\omega})$

$$e^{j\omega_0 n} x[n] \xleftrightarrow{\text{DTFT}} X(e^{j(\omega - \omega_0)})$$

convolution: $(x * y)[n] \xleftrightarrow{\text{DTFT}} X(e^{j\omega})Y(e^{j\omega})$

modulation: $x[n]y[n] \xleftrightarrow{\text{DTFT}} \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$

differentiation: $nx[n] \xleftrightarrow{\text{DTFT}} j \frac{d}{d\omega} X(e^{j\omega})$

Parseval's Relation: $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

Appendix D. Notable DTFT Pairs:

$$x[n] = \delta[n - k] \quad X(e^{j\omega}) = e^{-j\omega k}$$

$$x[n] = 1 \quad X(e^{j\omega}) = \tilde{\delta}(\omega)$$

$$\text{where } \tilde{\delta}(\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

$$x[n] = u[n] \quad X(e^{j\omega}) = \frac{1}{1 - e^{-j\omega}} + \frac{1}{2} \tilde{\delta}(\omega)$$

$$x[n] = a^n u[n] \quad |a| < 1 \quad X(e^{j\omega}) = \frac{1}{1 - a e^{-j\omega}}$$

$$x[n] = e^{j\omega_0 n} \quad X(e^{j\omega}) = \tilde{\delta}(\omega - \omega_0)$$

$$x[n] = \cos(\omega_0 n + \phi) \quad X(e^{j\omega}) = (1/2)[e^{j\phi} \tilde{\delta}(\omega - \omega_0) + e^{-j\phi} \tilde{\delta}(\omega + \omega_0)]$$

$$x[n] = \sin(\omega_0 n + \phi) \quad X(e^{j\omega}) = (-j/2)[e^{j\phi} \tilde{\delta}(\omega - \omega_0) - e^{-j\phi} \tilde{\delta}(\omega + \omega_0)]$$

$$x[n] = \begin{cases} 1 & 0 \leq n < M \\ 0 & \text{otherwise} \end{cases} \quad X(e^{j\omega}) = \frac{\sin((M/2)\omega)}{\sin(\omega/2)} e^{-j\frac{M-1}{2}\omega}$$

$$x[n] = \frac{\omega_c}{\pi} \text{sinc}\left(\frac{\omega_c}{\pi} n\right) \quad X(e^{j\omega}) = \text{rect}\left(\frac{\omega}{2\omega_c}\right) \text{ for } -\pi \leq \omega \leq \pi$$

$$\text{where } \text{sinc}(x) = \frac{\sin \pi x}{\pi x} \quad \text{and } \text{rect}(\omega) = \begin{cases} 1 & |\omega| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Appendix G. z -Transform

formula: $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

$$x[n] \xleftrightarrow{Z} X(z) \text{ with ROC } R$$

linearity: $x_1[n] + x_2[n] \xleftrightarrow{Z} X_1(z) + X_2(z)$ with ROC at least $R_1 \cap R_2$

scaling: $z_0^n x[n] \xleftrightarrow{Z} X\left(\frac{z}{z_0}\right)$ with ROC $|z_0|R$

symmetries: $x[-n] \xleftrightarrow{Z} X(z^{-1})$ with ROC R^{-1}

$$x^*[n] \xleftrightarrow{Z} X^*(z^*) \text{ with ROC } R^*$$

shift: $x[n - n_0] \xleftrightarrow{Z} z^{-n_0} X(z)$ with ROC R

(except for the possible addition or deletion of the origin)

convolution: $(x_1 * x_2)[n] \xleftrightarrow{Z} X_1(z)X_2(z)$ with ROC at least $R_1 \cap R_2$

Appendix H. Notable z -Transform Pairs:

$$x[n] = \delta[n] \qquad X(z) = 1 \text{ with ROC all } z$$

$$x[n] = \alpha^n u[n] \qquad \frac{1}{1-\alpha z^{-1}} \text{ with ROC } |z| > |\alpha|$$

$$x[n] = -\alpha^n u[-n-1] \qquad \frac{1}{1-\alpha z^{-1}} \text{ with ROC } |z| < |\alpha|$$
