EPFL

Prof. M. Gastpar

Quiz 6 (Homeworks 12 & 13)

Due on Moodle

on Monday, May 26, 2025, at 23:59.

Quiz 6

SCIPER : **111111**

- This quiz is to be solved individually.
- Try not to use any of the course materials other than the formula collection on a first attempt.
- Once you are done, enter your answers into Moodle. Moodle will give you feedback. You can update your answers as many times as you want before the deadline.
- For each question there is **exactly one** correct answer. We assign **negative points** to the **wrong answers** in such a way that a person who chooses a wrong answer loses **25 %** of the points given for that question.

Question 1

[0 points] *Note: This is an* **open** *question. In the real exam, we will grade your arguments. Here for the quiz, we do not have the capacity to do this. Therefore, you will merely enter your final answer into a multiple choice grid on Moodle. However, do make sure to carefully look at the solution and compare to your answer. How many points would you have given yourself?*

Let $\mathcal{C}$ be a (n, k) linear block code over $\mathbb{F}_2$ of block length n such that n is even and minimum distance $d_{min} = 3$. We construct a new code $\mathcal{C}'$ by appending onto each codeword $\vec{x} \in \mathcal{C}$ three parity bits as follows:

$x_{n+1} = x_1 \oplus x_3 \oplus x_5 \oplus \ldots \oplus x_{n-1}$,

$x_{n+2} = x_2 \oplus x_4 \oplus x_6 \oplus \ldots \oplus x_n$,

$x_{n+3} = x_1 \oplus x_2 \oplus x_3 \oplus \ldots \oplus x_n$.

The goal is to find the minimum distance d'_{min} of the new $(n+3, k)$ linear block code. Could it happen that $d'_{min} = 3$, $d'_{min} = 4$, $d'_{min} = 5$ or $d'_{min} = 6$? For each case, if it is possible, give conditions under which it can happen. If it is not possible, argue why not.

Question 2:

[5 points] Let $G_i, i \in \{1, \ldots, 8\}$, be valid generator matrices of dimensions $\mathbb{F}^{k_i \times n_i}$, all over the same field $\mathbb{F}$. Which of the following are always valid generator matrices?

Hint: recall that "valid" means that for all i, $k_i \leq n_i$ and rank$(G_i) = k_i$.

$\begin{pmatrix} G_1 \\ \hline G_2 \end{pmatrix}$ where $n_1 = n_2$ and $k_1 + k_2 \leq n_1$.

☐ TRUE ☐ FALSE

$\left(\begin{array}{c|cc} G_3 & G_4 & 0 \\ & 0 & G_5 \end{array}\right)$ where $k_3 = k_4 + k_5$.

☐ TRUE ☐ FALSE

$D_1 \cdot G_6 \cdot D_2$, where $D_1 \in \mathbb{F}^{k_6 \times k_6}$ and $D_2 \in \mathbb{F}^{n_6 \times n_6}$ are diagonal matrices with non-zero diagonal elements.

☐ TRUE ☐ FALSE

$G_7 + G_8$ with $k_7 = k_8$ and $n_7 = n_8$.

☐ TRUE ☐ FALSE

Question 3:

[5 points] Let

$$G = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

be the generator matrix of a $(6,4)$ linear code $\mathcal{C}$ over $\mathbb{F}_2$.

Answer the following true/false questions.

G admits a systematic form (i.e., it can be put into systematic form via elementary row operations).

☐ TRUE ☐ FALSE

$d_{min} = 2$.

☐ TRUE ☐ FALSE

If one substitutes the last row of G by $(1,0,0,1,1,1)$, the thereby obtained matrix generates the same code $\mathcal{C}$.

☐ TRUE ☐ FALSE

Performing an arbitrary column permutation on G yields a generator matrix of a linear code with the same parameters n, k, d_{min}.

☐ TRUE ☐ FALSE

Question 4:

[5 points] Let $\mathcal{C}_1$ be a linear code over $\mathbb{F}_3^n$, and let $\mathcal{C}_2$ be a linear code over $\mathbb{F}_2^n$. Answer the following true/false questions.

$\mathcal{C}_1 \cup \mathcal{C}_2$ is necessarily a linear code over $\mathbb{F}_3^n$.

☐ TRUE ☐ FALSE

$\mathcal{C}_1 \cap \mathcal{C}_2$ is necessarily a linear code over $\mathbb{F}_2^n$.

☐ TRUE ☐ FALSE

Question 5

[5 points] Let $\mathcal{C}_1$ be a (n_1, k) linear block code over $\mathbb{F}_p$ with p prime and $|\mathcal{C}_1| = 27$. Let $\mathcal{C}_2$ be a (n_2, k) linear block code over $\mathbb{F}_2$ of the same dimension k. Which of the following is true?

☐ $|\mathcal{C}_2| = 27$

☐ $|\mathcal{C}_2| = 8$

☐ $|\mathcal{C}_2| = 21$

☐ $|\mathcal{C}_2| = 16$