



Prof. M. Gastpar

Quiz 6 (Homeworks 12 & 13)













Due on Moodle

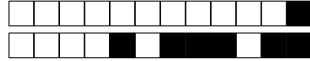
on Monday, May 26, 2025, at 23:59.

Quiz 6

SCIPER: 111111

- This quiz is to be solved individually.
- Try not to use any of the course materials other than the formula collection on a first attempt.
- Once you are done, enter your answers into Moodle. Moodle will give you feedback. You can update your answers as many times as you want before the deadline.
- For each question there is **exactly one** correct answer. We assign **negative points** to the **wrong answers** in such a way that a person who chooses a wrong answer loses **25 %** of the points given for that question.

Respectez les consignes suivantes Observe this guidelines Beachten Sie bitte die unten stehenden Richtlinien		
choisir une réponse select an answer Antwort auswählen	ne PAS choisir une réponse NOT select an answer NICHT Antwort auswählen	Corriger une réponse Correct an answer Antwort korrigieren
  		 
ce qu'il ne faut PAS faire what should NOT be done was man NICHT tun sollte		
     		



Question 1

[0 points] *Note: This is an **open** question. In the real exam, we will grade your arguments. Here for the quiz, we do not have the capacity to do this. Therefore, you will merely enter your final answer into a multiple choice grid on Moodle. However, do make sure to carefully look at the solution and compare to your answer. How many points would you have given yourself?*

Let \mathcal{C} be a (n, k) linear block code over \mathbb{F}_2 of block length n such that n is even and minimum distance $d_{\min} = 3$. We construct a new code \mathcal{C}' by appending onto each codeword $\vec{x} \in \mathcal{C}$ three parity bits as follows:

$$x_{n+1} = x_1 \oplus x_3 \oplus x_5 \oplus \dots \oplus x_{n-1},$$

$$x_{n+2} = x_2 \oplus x_4 \oplus x_6 \oplus \dots \oplus x_n,$$

$$x_{n+3} = x_1 \oplus x_2 \oplus x_3 \oplus \dots \oplus x_n.$$

The goal is to find the minimum distance d'_{\min} of the new $(n+3, k)$ linear block code. Could it happen that $d'_{\min} = 3$, $d'_{\min} = 4$, $d'_{\min} = 5$ or $d'_{\min} = 6$? For each case, if it is possible, give conditions under which it can happen. If it is not possible, argue why not.

Question 2:

[5 points] Let $G_i, i \in \{1, \dots, 8\}$, be valid generator matrices of dimensions $\mathbb{F}^{k_i \times n_i}$, all over the same field \mathbb{F} . Which of the following are always valid generator matrices?

Hint: recall that "valid" means that for all i , $k_i \leq n_i$ and $\text{rank}(G_i) = k_i$.

$$\begin{pmatrix} G_1 \\ G_2 \end{pmatrix} \text{ where } n_1 = n_2 \text{ and } k_1 + k_2 \leq n_1.$$

☐ TRUE ☐ FALSE

$$\left(G_3 \mid \begin{array}{c|c} G_4 & 0 \\ \hline 0 & G_5 \end{array} \right) \text{ where } k_3 = k_4 + k_5.$$

☐ TRUE ☐ FALSE

$D_1 \cdot G_6 \cdot D_2$, where $D_1 \in \mathbb{F}^{k_6 \times k_6}$ and $D_2 \in \mathbb{F}^{n_6 \times n_6}$ are diagonal matrices with non-zero diagonal elements.

☐ TRUE ☐ FALSE

$G_7 + G_8$ with $k_7 = k_8$ and $n_7 = n_8$.

☐ TRUE ☐ FALSE

**Question 3:**

[5 points] Let

$$G = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

be the generator matrix of a $(6, 4)$ linear code \mathcal{C} over \mathbb{F}_2 .

Answer the following true/false questions.

 G admits a systematic form (i.e., it can be put into systematic form via elementary row operations).☐ TRUE ☐ FALSE $d_{\min} = 2$.☐ TRUE ☐ FALSEIf one substitutes the last row of G by $(1, 0, 0, 1, 1, 1)$, the thereby obtained matrix generates the same code \mathcal{C} .☐ TRUE ☐ FALSEPerforming an arbitrary column permutation on G yields a generator matrix of a linear code with the same parameters n, k, d_{\min} .☐ TRUE ☐ FALSE**Question 4:**[5 points] Let \mathcal{C}_1 be a linear code over \mathbb{F}_3^n , and let \mathcal{C}_2 be a linear code over \mathbb{F}_2^n . Answer the following true/false questions. $\mathcal{C}_1 \cup \mathcal{C}_2$ is necessarily a linear code over \mathbb{F}_3^n .☐ TRUE ☐ FALSE $\mathcal{C}_1 \cap \mathcal{C}_2$ is necessarily a linear code over \mathbb{F}_2^n .☐ TRUE ☐ FALSE**Question 5**[5 points] Let \mathcal{C}_1 be a (n_1, k) linear block code over \mathbb{F}_p with p prime and $|\mathcal{C}_1| = 27$. Let \mathcal{C}_2 be a (n_2, k) linear block code over \mathbb{F}_2 of the same dimension k . Which of the following is true?☐ $|\mathcal{C}_2| = 27$ ☐ $|\mathcal{C}_2| = 8$ ☐ $|\mathcal{C}_2| = 21$ ☐ $|\mathcal{C}_2| = 16$