

CIVIL-557

# Decision aid methodologies in transportation

Lab I:  
Using a mathematical solver (GUROBI)

Tom Haering, Prunelle Vogler

Transport and Mobility Laboratory (TRANSP-OR)  
École Polytechnique Fédérale de Lausanne (EPFL)

# Overview

- Using a mathematical solver (GUROBI)
  - What is a mathematical solver
  - What is GUROBI?
  - Installing GUROBI
  - Using GUROBI
  - Example
- Container storage problem
- Transportation problem

# Using a mathematical solver

# What is a mathematical solver?

- **Input** = mathematical formulation of an optimization problem
- **Output** = optimal solution to the problem
- **Options** for optimality gap, time limits, numerical precision etc.
- Applies several **optimization techniques** in order to find the optimal solution(s).
- Includes **simplex** + many other methods, such as **branch and bound**, cutting planes, probing, **pre-processing**, and heuristics.

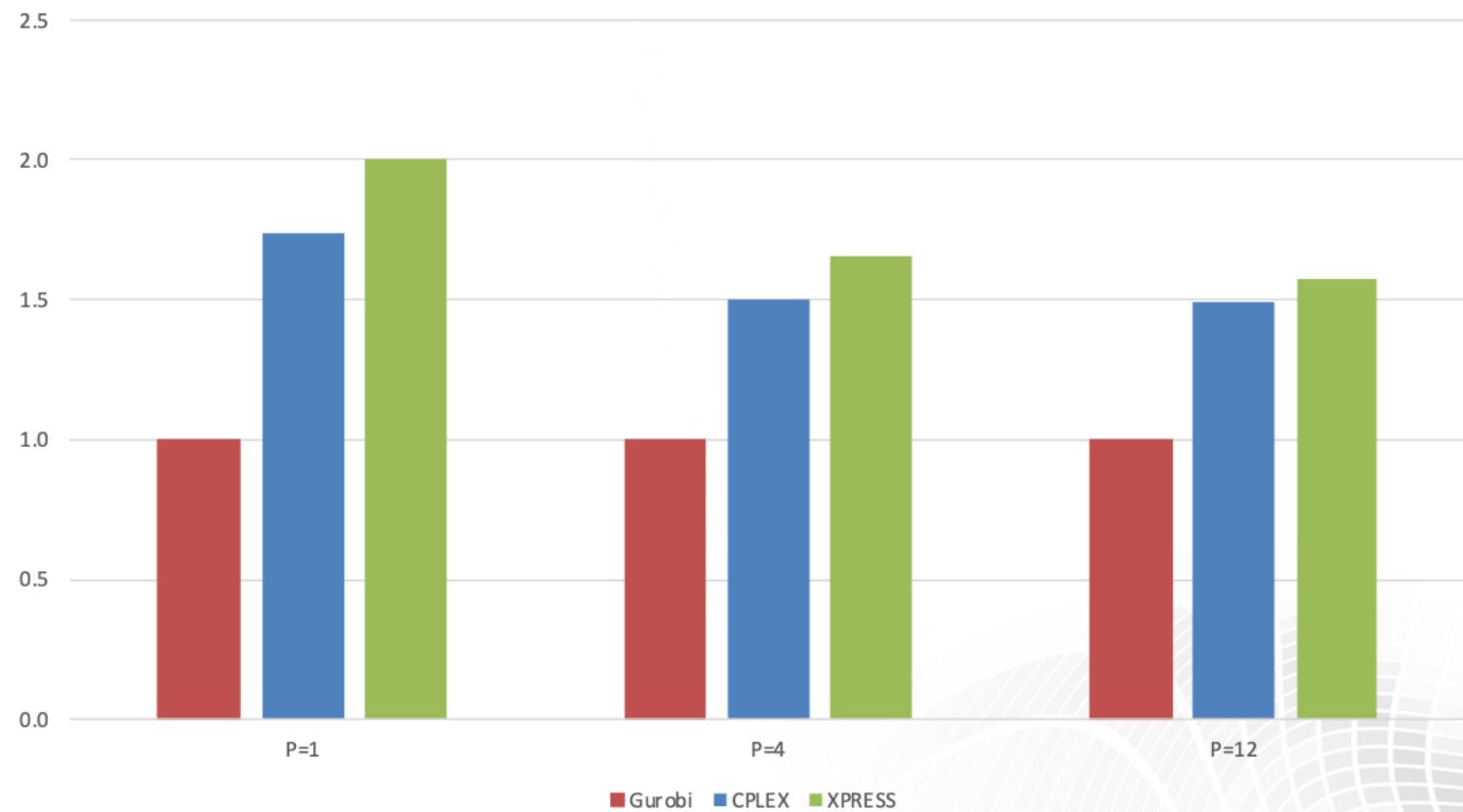
# What is GUROBI?



Gurobi Optimizer — The State-of-the-Art  
Mathematical Programming Solver



# What is GUROBI?



For example, on time to optimality benchmark (87 models) using 4 threads (P=4), CPLEX was 50% slower (1.50) and XPRESS was 66% slower (1.66) than Gurobi.

# What is GUROBI?

- **Free for academic use**

<https://www.gurobi.com/academia/academic-program-and-licenses/>

- **No local installation** of optimization suite necessary

<https://support.gurobi.com/hc/en-us/articles/360044290292-How-do-I-install-Gurobi-for-Python>

- Latest version (12.0.1) **compatible with Python 3.9+**

- Easy to use with **Pandas** etc.

# Installing GUROBI

1. Create **Gurobi account** using epfl email
2. Download **academic license**
3. pip install gurobipy

- Use the python coding environment you're **most comfortable** with
- **Jupyter notebooks**: great to play around with
- To use jupyter notebooks we recommend using **anaconda**, convenient to set up environments etc.  
<https://docs.anaconda.com/anaconda/install/>
- Alternatively you could use VS Code

# Installing GUROBI

- Let's take some minutes until everyone can do this in some way:

```
import gurobipy as gp

m = gp.Model()
m.optimize()

Set parameter Username
Academic license - for non-commercial use only - expires 2026-02-17
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[x86] - Darwin 23.6.0 23G93)

CPU model: Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz
Thread count: 6 physical cores, 12 logical processors, using up to 12 threads

Optimize a model with 0 rows, 0 columns and 0 nonzeros
Model fingerprint: 0xf9715da1
Coefficient statistics:
  Matrix range      [0e+00, 0e+00]
  Objective range   [0e+00, 0e+00]
  Bounds range      [0e+00, 0e+00]
  RHS range         [0e+00, 0e+00]
Presolve time: 0.02s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.    Time
          0    0.0000000e+00  0.000000e+00  0.000000e+00    0s

Solved in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective  0.000000000e+00
```

# Using GUROBI (example)

max  $x + y + 2z$   
 s.t.  $x + 2y + 3z \leq 5$   
 $x + y \geq 1$   
 $x, y, z \in \{0,1\}$

```

from gurobipy import GRB

# Create a new model
m = gp.Model("mip1")

# Create variables
x = m.addVar(vtype=GRB.BINARY, name="x")
y = m.addVar(vtype=GRB.BINARY, name="y")
z = m.addVar(vtype=GRB.BINARY, name="z")

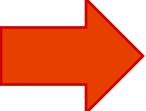
# Set objective
m.setObjective(x + y + 2 * z, GRB.MAXIMIZE)

# Add constraint: x + 2 y + 3 z <= 4
m.addConstr(x + 2 * y + 3 * z <= 4, "c0")

# Add constraint: x + y >= 1
m.addConstr(x + y >= 1, "c1")

# Optimize model
m.optimize()
    
```

# First example to solve

$\min 3x + 2y$   
subject to  $x - y \geq 5$    
 $3x + 2y \geq 10$



**Hint:** `inf = GRB.INFINITY`  
`x = m.addVar(lb = -inf, vtype=GRB.CONTINUOUS)`

# Container storage problem

# Exercise

- Solve the **container storage problem** (balanced load allocation)

**Storage blocks**  $i \in \{1, \dots, B\}$

## Parameters

- $a_i$ : Initial number of stored containers in block  $i$
- $N$ : Number of new containers expected to arrive for storage in this period
- $B$ : Total number of blocks in the storage yard
- $A$ : Number of storage positions per block
- $F$ : The fill-ratio in the whole yard at the end of this period  $((N + \sum_i a_i) / AB)$

$$\min \sum_{i=1}^B |a_i + x_i - AF|$$

$$s. t. \sum_{i=1}^B x_i = N$$

$$x_i \in \mathbb{N}_0 \quad \forall i$$

## Decision variables

- $x_i$ : Number of arriving containers in this period to be stored to block  $i$

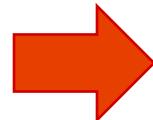
# Exercise

- How can we model an absolute value in a linear program?

$$\min \sum_{i=1}^B |a_i + x_i - AF|$$

$$s.t. \sum_{i=1}^B x_i = N$$

$$x_i \in \mathbb{N}_0 \quad \forall i$$



?

# Exercise

- Set  $N=15166$ ,  $B=100$ ,  $A=600$ ,  $F = \frac{N + \sum_i a_i}{AB}$
- $a$  can be read from the numpy file on the moodle (Lab1\_a.npy)

```
import numpy as np
```

```
a = np.load("Lab1_a.npy")
```

- Hint 1: use **dictionaries** for variables

```
x = {i: m.addVar(vtype=GRB.INTEGER) for i in range(B)}
```

- Hint 2: for sums use **gp.quicksum()**

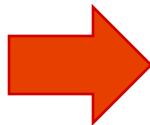
$$\sum_{i=1}^B x_i = \text{gp.quicksum}(x[i] \text{ for } i \text{ in range}(B))$$

# Surprise exercise!

# Surprise exercise

- How can we model logical constraints linearly?

$$c_i = \begin{cases} d_i & \text{if } d_i \geq 5 \\ e_i & \text{else} \end{cases}$$



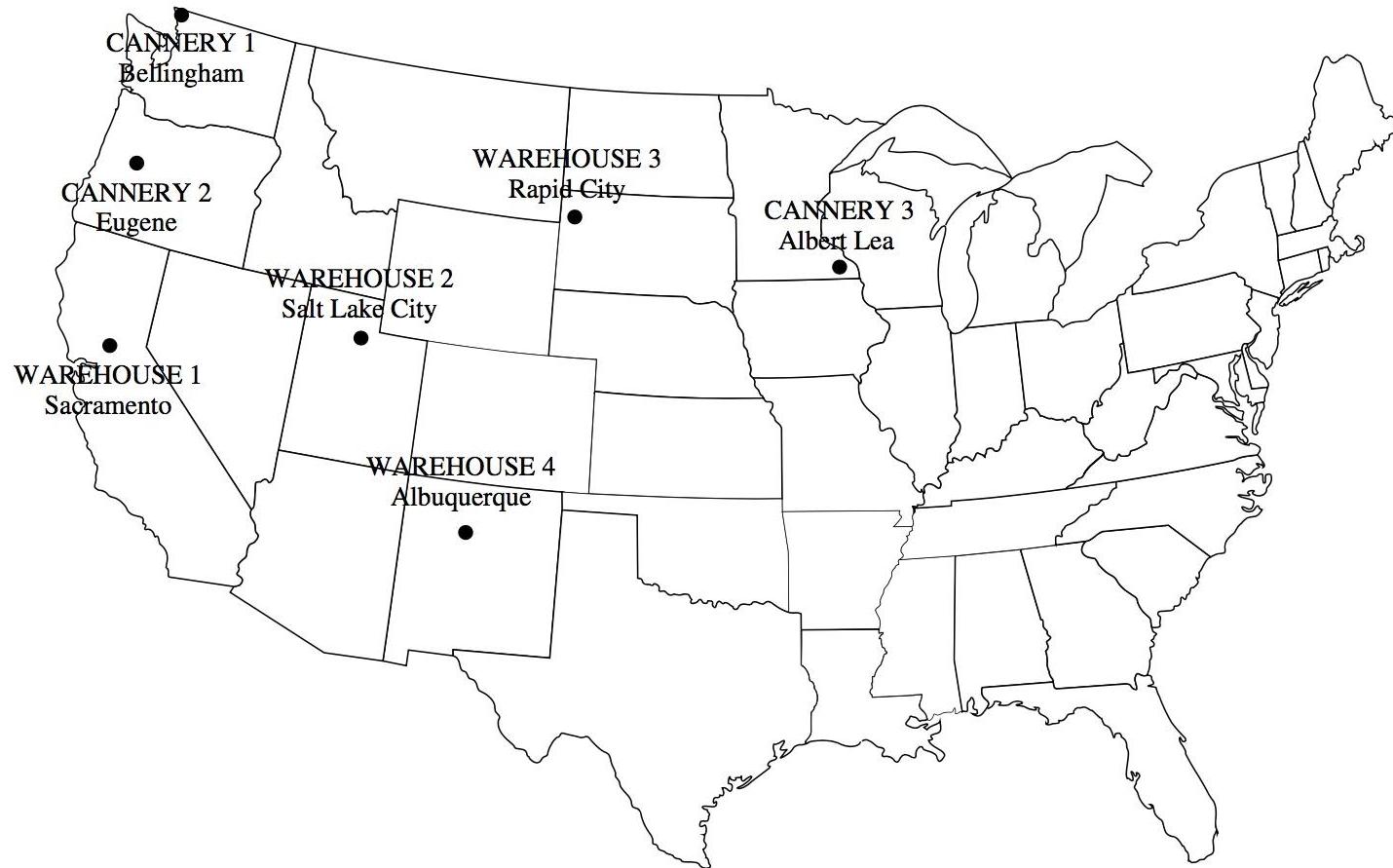
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# Transportation Problem (Bonus exercise)

# Transportation Problem

One of the main products of the P&T COMPANY is canned peas. The **peas are prepared at three canneries** (near Bellingham, Washington; Eugene, Oregon; and Albert Lea, Minnesota) and then **shipped by truck to four distributing warehouses** in the western United States (Sacramento, California; Salt Lake City, Utah; Rapid City, South Dakota; and Albuquerque, New Mexico). Because the **shipping costs are a major expense**, management is initiating a study to **reduce them as much as possible**. For the upcoming season, an estimate has been made of the **output from each cannery**, and **each warehouse has been allocated a certain amount from the total supply of peas**. This information (in units of truckloads), along with the shipping cost per truckload for each cannery-warehouse combination, is given in Table 8.2 (and is provided as data). Thus, there are a **total of 300 truckloads to be shipped**. The problem now is to determine which plan for **assigning these shipments to the various cannery-warehouse combinations** would **minimize the total shipping cost**.

# Transportation Problem – Layout



# Transportation Problem – Table 8.2

**TABLE 8.2** Shipping data for P & T Co.

		Shipping Cost (\$) per Truckload				Output	
		Warehouse					
		1	2	3	4		
Cannery	1	464	513	654	867	75	
	2	352	416	690	791	125	
	3	995	682	388	685	100	
Allocation		80	65	70	85		

# Transportation Problem

- Formulate the problem mathematically
- Implement the problem and report the optimal solution
- Solve the problem for a large instance (use  $N= 20$ ,  $M = 30$ , and files `Lab1_cost.npy`, `Lab1_output.npy` and `Lab1_allocation.npy`).