CIVIL-557
Decision-Aid Methodologies in Transportation
Lecture III
Exercises: SPPs

Prunelle Vogler, Tom Haering

Transport and Mobility Laboratory TRANSP-OR
École Polytechnique Fédérale de Lausanne EPFL

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Outline

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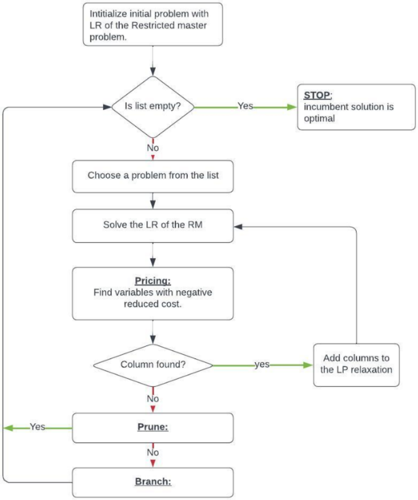
Summary of Branch-and-Price

1. Solve the Restricted Master problem (our case: VRP restricted problem) $\Rightarrow$ Get dual prices π_i.
2. Solve the Pricing problem (our case: ESPPRC) $\Rightarrow$ Find new columns (routes).
3. If negative reduced cost $\Rightarrow$ Add column to RM.
4. Repeat until no improving column exists.

What is the Pricing Problem?



- A subproblem in the column generation phase of Branch-and-Price.
- Identifies new variables (columns) with **negative reduced cost** to add them to the Restricted Master Problem (RM).

Mathematical Formulation

Reduced Cost:

$$\text{Reduced Cost} = c_r - \sum_{i \in V} \pi_i a_{ir} \tag{1}$$

where:

- c_r = Cost of route r.
- π_i = Dual price of customer i.
- a_{ir} = 1 if customer i is visited in route r, 0 otherwise.

Pricing problem: minimize the reduced cost over all feasible paths.

$\rightarrow$ We change this formulation to use the variables x_{ij}.

Pricing Problem for VRP

Objective:

$$\min \sum_{i\in V}\sum_{j\in V}(c_{ij}-\pi_i)x_{ij} = \sum_{i\in V}\sum_{j\in V}\hat{c}_{ij}x_{ij} \tag{2}$$

Constraints:

- Flow conservation: Each customer is visited exactly once.
- Vehicle capacity constraint: $\sum_i q_i x_{ij} \leq Q$.
- Time-windows constraints.

It is an **Elementary Shortest Path Problem with Resource Constraints (ESPPRC)**. NP-hard problem. $\rightarrow$ solve by relaxation of the "elementarity" (solve SSPRC with labeling algorithm and eliminate cycles).

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Exercise 1

Use the code provided to solve the Shortest Path Problem (no resources). In this case, $L =$ (current node, parent node, current cost of the path).

Answer the following questions:

1. Is there a cycle in the solution? if so which nodes are cycling?
2. Is the solution a feasible solution? Explain!
3. Is the solution the optimal solution? Explain!
4. If we are using branch and price to solve the VRP and this SPP was the relaxation of the pricing problem (ESPPRC), can we add this path to the RMP (Restricted Master Problem) as a column? Explain!

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Exercise 1

Use the code provided to solve the Shortest Path Problem (no resources).
Answer the following questions:

1. Is there a cycle in the solution? if so which nodes are cycling?
   Yes, there is a cycle. From 9 to 19.
2. Is the solution a feasible solution? Explain!
   The solution is feasible because it does not violate any constraints.
3. Is the solution the optimal solution? Explain!
   The solution is unbounded. It contains a negative cost cycle, hence, it could cycle forever.
4. If we are using branch and price to solve the VRP and this SPP was the relaxation of the pricing problem (ESPPRC), can we add this path to the RMP (Restricted Master Problem) as a column? Explain!
   Yes. If there are no capacity or time windows constraints, we can add this column as a negative cost column. This solution will be elimintaed in the branching process.

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Exercise 2.1

Solve the Shortest Path Problem with Resource Constraints by **adding time constraints to the model**.

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Solve the Shortest Path Problem with Resource Constraints by **adding time constraints to the model**. In this case,

$$L = (\text{node, parent node, cost of the path, time reaching the node})$$

Modify the class Label, the resource_extension function (time of the next node?), the feasible function (time infeasibility check) and the dominance function (earlier is better).

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Solve the Shortest Path Problem with Resource Constraints by **adding time constraints to the model**. In this case,

$$L = (\text{node, parent node, cost of the path, time reaching the node})$$

Modify the class Label, the resource_extension function (time of the next node?), the feasible function (time infeasibility check) and the dominance function (earlier is better).

Answer the following questions:

- Why did the cost of the solution increase from the solution of the SPP in exercise 1?
- Identify all cycles in the solution.
- Why is the path leaving negative cost cycles and not continuing to cycle as in exercise 1?
- Is the solution feasible? Explain!
- Is the solution the optimal solution? Explain!

Exercise 2.1

Solve the Shortest Path Problem with Resource Constraints by **adding time constraints to the model**. Answer the following questions:

- Why did the cost of the solution increase from the solution of the SPP in exercise 1?
  The solution increased because we added time window constraints that made the previous solution unfeasible.
- Identify all cycles in the solution.
  [29, 16], [14, 27, 20, 27, 14], [27, 20], [14, 7], [40, 18], [47, 4], [21, 45]
- Why is the path leaving negative cost cycles and not continuing to cycle as in exercise 1? It would violate the time windows constraints.
- Is the solution feasible? Explain! Yes, it satisfies all the constraints.
- Is the solution the optimal solution? Explain! Yes, the solution is optimal for the SPP with time windows. The algorithm uses the principal of optimality to solve this problem.

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Exercise 2.2

Add capacity constraints to the code.

- Identify all cycles in the solution. [29, 16], [40, 18]
- Why did the cost increase with respect to the previous solution value of the SPPRC (time windows) We added capacity constraints the eliminated the solution of the previous exercise. By adding constraints the solution can only increase in a minimization problem.

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Exercise 3

Use the Ng-route relaxation to solve the problem.

$$L = (\text{node, parent, cost, time, capacity, unreachable nodes})$$

Hint : change the extension (given an array Ng) and the feasibility function.

- Identify all cycles
- Is this solution also the optimal solution of the ESPPRC? Explain!
- Increase the value of the size of the Ng_set, i.e., Ng_v, from Ng_v = 2, Ng_v = 5, Ng_v = 10, Ng_v = 15, and Ng_v = 20.
  (Warning: if your computer takes longer then a minute, stop!)
  - At what point are the cycles eliminated?
  - Why does the algorithm take more time as you increase Ng_v? Explain!

Exercise 3

Use the Ng-route relaxation to solve the problem.

- Identify all cycles
  There are no cycles the path is elementary.
- Is this solution also the optimal solution of the ESPPRC? Explain!
  Yes, because the SPPRC is a relaxation of the ESPPRC and therefore it provides a lower bound for the optimal solution of the ESPPRC. Since it is feasible for the ESPPRC, the solution is both an upper bound and a lower bound, thus, it is optimal.
- Increase the value of the size of the Ng_set, i.e., Ng_v, from Ng_v = 2, Ng_v = 5, Ng_v = 10, Ng_v = 15, and Ng_v = 20.
  (Warning: if your computer takes longer then a minute, stop!)
  - At what point are the cycles eliminated? When Ng_v $\geq$ 3.
  - Why does the algorithm take more time as you increase Ng_v? Explain!
    The set of labels grows, since it becomes more difficult to delete labels through dominance rules.