CIVIL-557
Decision-aid methodologies in transportation
Lecture I

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Outline

Outline

Motivation

Motivation

"The large number of real-world applications, both in North America and in Europe, have widely shown that the use of computerized procedures for the distribution process planning produces substantial savings (generally from **5% to 20%***) in the global transportation costs. It is easy to see that the impact of these savings on the global economic system is significant. Indeed, the transportation process involves all stages of the production and distribution systems and represents a relevant component (generally from* **10% to 20%***) of the final cost of the goods."*

Toth, P., & Vigo, D. (2002). An overview of vehicle routing problems. The vehicle routing problem, 1-26

Operations research

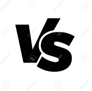
Operations research arose in second world war to optimize military operations.

Bomber Planes had to be sent in aerial structures that maximized the coverage of enemy land bombed while minimizing the damage the planes take.

Outline

Defining the problem

Real-world problems: Described in a vague, imprecise way.



Textbook examples: Described in a simple, precise way.

"It is difficult to extract a **right** answer from the **wrong** problem"

$\Rightarrow$ We need a **well-defined statement of the problem**

- What are the appropriate objectives ?
- Are there constraints ?
- What is the time horizon?
- Is there a time limit to make a decision?
- Are there parameters to the problem?

Real-world problems

Objectives

In real-world problems there might be several objectives that the company wants to optimize simultaneously. Multi-objective strategies can be used (e.g., a weighted sum of the objectives)

Constraints

Some constraints are not explained explicitly and/or they increase the difficulty to solve the model.

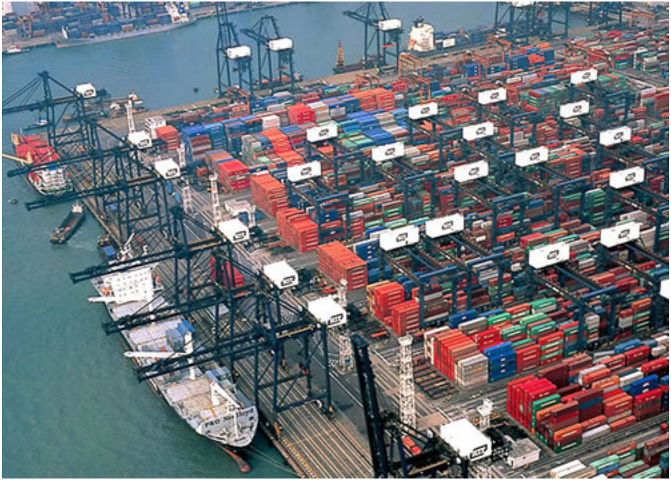
Flow of information

Some information is revealed in stages and decisions can be made in each stage. These problems are multi-stage optimization problems.

Parameters

Some parameters are unknown in advance. However, probabilistic information can be available. (Stochastic optimization)

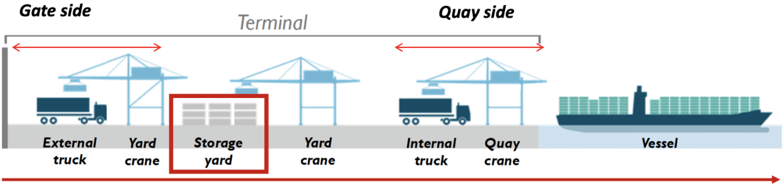
Maritime container terminals



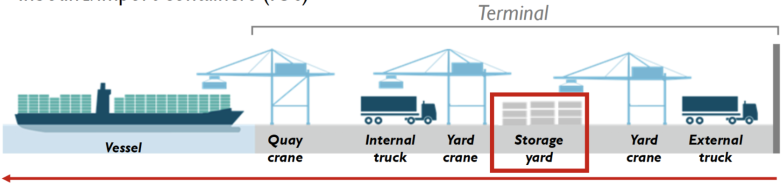
Barcelona

Maritime container terminals

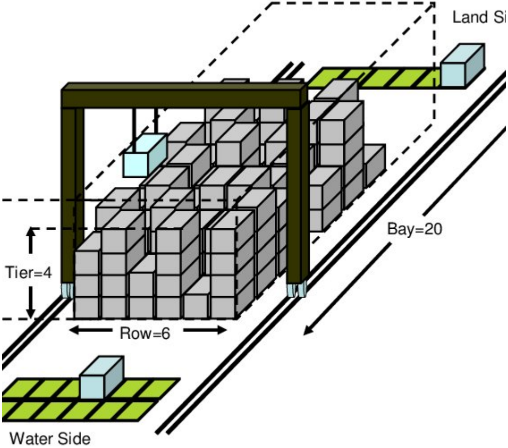
- Outbound/export containers (EC's)



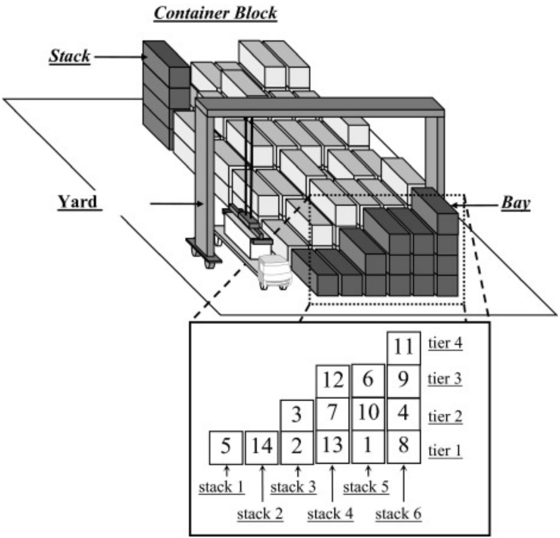
- Inbound/import containers (IC's)



Container yard



Conainer relocation problem



Mediterranean Shipping Company (MSC)

Container relocation problem

- Containers are retrieved in a specific order. Truck drivers are served on a first come first serve basis.
- Containers are stacked on top of each other. Therefore,there are blocking containers that must be relocated to retrieve the desired container.

Objective

The objective is to minimize the number of unproductive moves or relocation of containers.

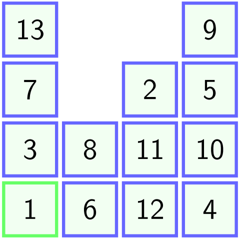


Figure: Container relocation problem

Relocations $= 1$

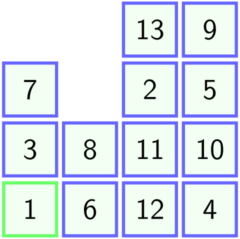


Figure: Container relocation problem

Relocations = 2

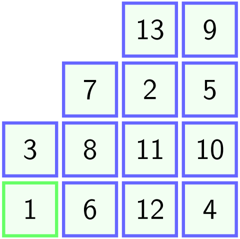


Figure: Container relocation problem

Relocations = 3

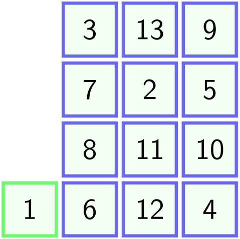


Figure: Container relocation problem

Relocations $= 4$

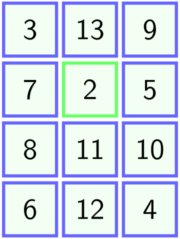


Figure: Container relocation problem

Relocations = 5

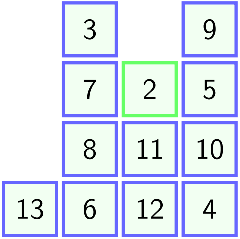


Figure: Container relocation problem

Relocations = 6

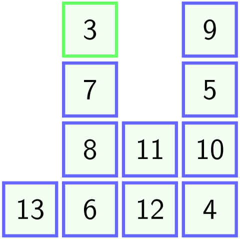


Figure: Container relocation problem

Relocations $= 7$

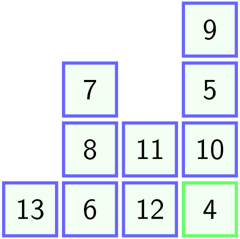


Figure: Container relocation problem

Relocations = 8

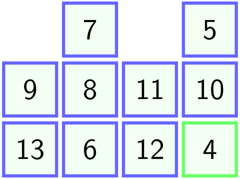


Figure: Container relocation problem

Relocations = 9

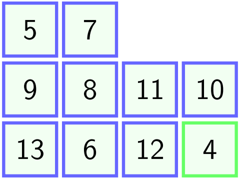


Figure: Container relocation problem

Relocations = 10

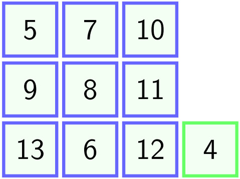


Figure: Container relocation problem

Relocations = 11

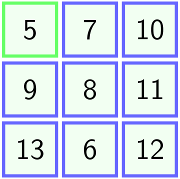


Figure: Container relocation problem

Relocations = 12

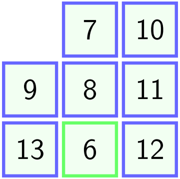


Figure: Container relocation problem

Relocations = 13

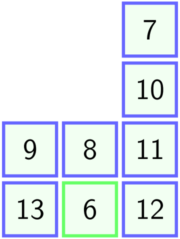


Figure: Container relocation problem

Relocations = 14

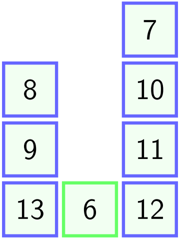


Figure: Container relocation problem

Relocations = 15

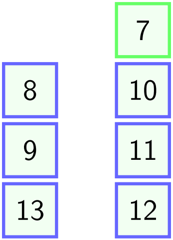


Figure: Container relocation problem

Relocations = 16

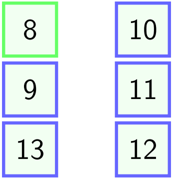


Figure: Container relocation problem

Relocations = 17

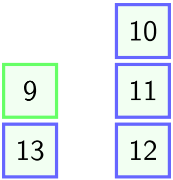


Figure: Container relocation problem

Relocations = 18

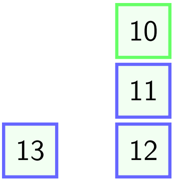


Figure: Container relocation problem

Relocations $= 19$

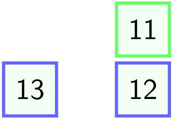


Figure: Container relocation problem

Relocations $= 20$

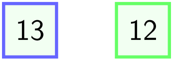


Figure: Container relocation problem

Relocations = 21

13

Figure: Container relocation problem

Relocations = 22

Figure: Container relocation problem

Simplifying assumptions

- If we consider all aspects of a problem, it can be difficult to find a solution in any reasonable amount of time.
- Hence, we relax some constraints in a way that makes the problem more tractable.
- CRP, in practice, containers come and go from the yard in real time. The order of containers retrieved is unknown before hand and changes based on the arrival of truck to the yard.
- Container arrive dynamically over time.

Elevator Dispatching Problem

Vertical transportation system.

- Transportation also happens in buildings with elevators.
- People need to be transported from an origin floor to a destination floor.
- Modern elevators are faster and have new systems that can benefit from operations research.

Destination control

Destination control

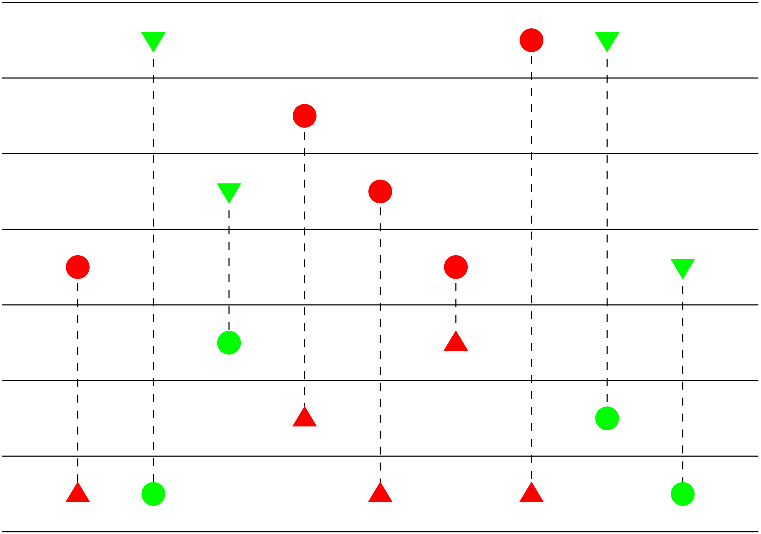


Figure: Passenger calls

Destination control

- Passengers select the destination floor instead of the direction of travel. The direction of travel is inferred from the origin floor and the destination floor.
- At each call the elevator control system has to assign the optimal elevator to the passenger within seconds.
- A mathematical algorithm that could take minutes (or hours) to find the optimal solution is not practical.
- A fast algorithm that provides a quick assignment is needed.

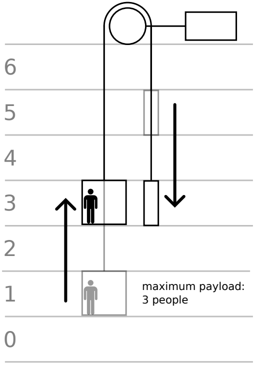
Heuristic

A Heuristic is an algorithm that finds a solution quickly without a guarantee that the solution found is optimal.

Destination control

Multiple Objectives

Minimize average destination time of passengers and minimize the energy consumption of elevators.



Constraints of elevator dispatching

Capacity Constraints:

Elevators can only hold a limited number of people at a time. Thus a bound on the total number of people inside the elevator cannot exceed a value of Q.

Assumption: All passengers weigh 75 kg.

Direction constraint

To reduce passenger inconvenience and discomfort, passengers cannot travel in the wrong direction.

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Bin packing problem

Objective

The objective is to minimize the number of bins necessary to fit all items.

Constraints

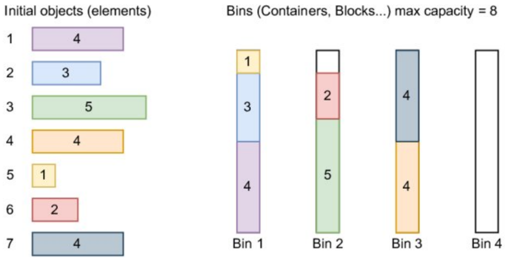
1. The capacity of the bin must not be violated.
2. All items must be placed in one bin.

Assumption: Items are one dimensional.

Applications:

1. Loading items to containers. Each container is expensive, using less is important to reduce unnecessary costs.
2. Loading packages in trucks for delivery.
3. Loading items in boxes to minimize the total boxes used to send items to customers.

Bin packing problem



Integer Linear Program (ILP)

$$\min \sum_{r \in N} c_r x_r$$

$$s.t.$$

$$\sum_{r \in N} a_{ir} x_r = 1 \qquad \forall i \in M,$$

$$x_r \in \mathbb{B} \qquad \forall r \in N.$$

Exercise Model the Bin packing problem

Model the bin packing problem as an integer program

5 minutes.

Parameters of the problem

- List of items $1, \ldots, n$.
- The weight or size of the items w_n.
- Capacity of the bins c.
- List of bins available $1, \ldots, u$.

Objective

Minimize the total number of bins used.

Constraints

- Capacity constraints.
- All items must be in one bin.

Solution

Variables:

$$y_i = \begin{cases} 1 & \text{if bin } i \text{ is used in the solution;} \\ 0 & \text{otherwise} \end{cases} \qquad (i = 1, \ldots, u),$$

$$x_{ij} = \begin{cases} 1 & \text{if item } j \text{ is packed into bin } i; \\ 0 & \text{otherwise} \end{cases} \qquad (i = 1, \ldots, u;\ j = 1, \ldots, n),$$

Solution

Model:

$$\min \quad \sum_{i=1}^{u} y_i$$

$$\text{s.t.} \quad \sum_{j=1}^{n} w_j x_{ij} \le c y_i \quad (i = 1, \dots, u),$$

$$\sum_{i=1}^{u} x_{ij} = 1 \quad (j = 1, \dots, n),$$

$$y_i \in \{0, 1\} \quad (i = 1, \dots, u),$$

$$x_{ij} \in \{0, 1\} \quad (i = 1, \dots, u;\ j = 1, \dots, n).$$

Outline

Vehicle Routing

Given: A set of transportation requests and a fleet of vehicles.

Task: Determine a set of vehicle routes to perform all(or some) transportation requests with the given vehicle fleet at minimum cost; in particular, decide which vehicle handles which requests in which sequence so that all vehicle routes can be feasibly executed.

Toth and Vigo Vehicle Routing Problems, Methods and Applications (2014)

Application to Parcel deliveries

- A company must deliver parcels to its customers.
- The company has a fleet of vehicles to fulfill delivery requests.
- The vehicles have a limited capacity to fit packages.

Objective

- Deliver all parcels to customers at the smallest cost.

Assumptions

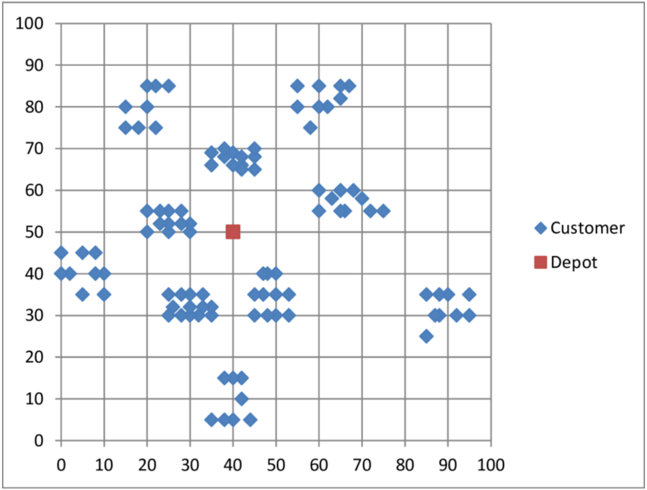
1D Capacity

The parcels are three dimensional objects that would require the solution of a 3D bin-packing problem to fit parcels on the truck. Instead, we consider one dimension that represents the weight or volume of each item.

Deterministic travel times

In practice, the time it takes to travel from one place to another is uncertain. Traffic or weather events could slow down the truck in the route.

VRP



100
90
80
70
60
50
40
30
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30
40
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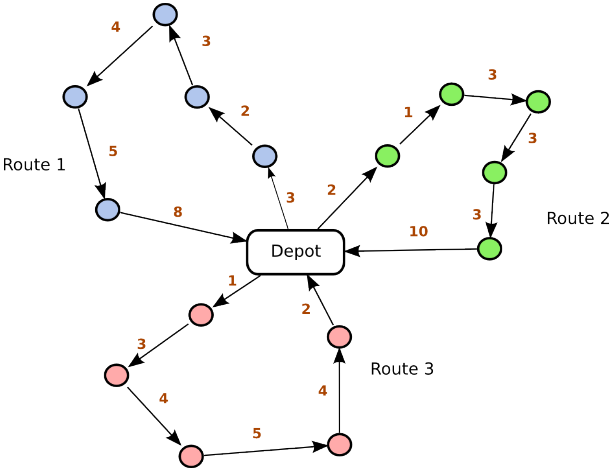
60
70
80
90
100
Customer
Depot

Routes



Notation

N Set of all customers

K Set of all vehicles

A Set of all arcs connecting customers and the depot

E Set of all edges connecting customers and the depot

q_i Demand of customer $i \in N$

Q Capacity of each vehicle

$A(S)$ Set of all arcs incident in set S

$E(S)$ Set of all edges incident in set S

$\delta^-(S) = \{(i,j) \in A : i \notin S, j \in S\}$ in arcs

$\delta^+(S) = \{(i,j) \in A : i \in S, j \notin S\}$ out arcs

$r(S) =$ Solution of a bin packing problem over set of customers S

Notation

x_{ij} Binary variable, equal to 1 if a vehicle goes from i to j, equal to 0 otherwise;

V Set with all vertices including the depot (i.e., $V = N \cup \{0\}$);

c_{ij} Cost of a vehicle going from i to j;

Model 1 (2-index Directed model)

$$\text{Minimize} \sum_{(i,j)\in A} c_{ij}x_{ij} \tag{1}$$

$$\text{s.t.} \sum_{j\in\delta^+(i)} x_{ij} = 1 \qquad \forall i \in N, \tag{2}$$

$$\sum_{i\in\delta^-(j)} x_{ij} = 1 \qquad \forall j \in N, \tag{3}$$

$$\sum_{j\in\delta^+(0)} x_{0j} = |K|, \tag{4}$$

$$\sum_{(i,j)\in\delta^+(S)} x_{ij} \geq r(S) \qquad \forall S \subseteq N, S \neq \emptyset, \tag{5}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A.$$

Vehicle constraints

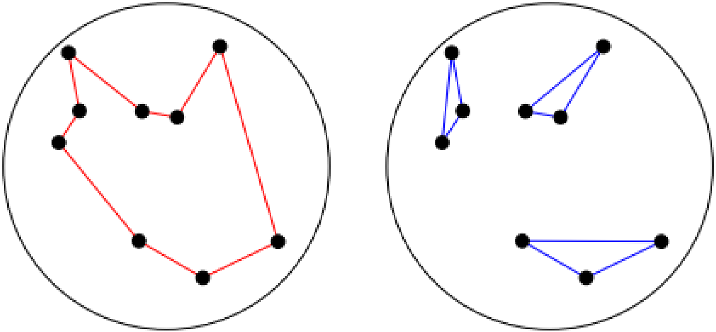
Not all vehicles are used

$$\sum_{j \in \delta^+(0)} x_{0j} \leq |K|,$$

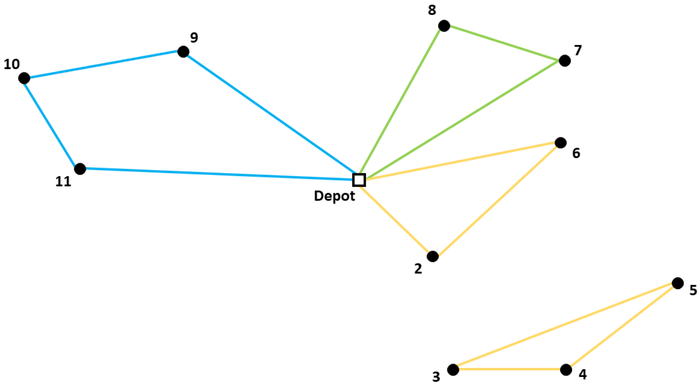
Unlimited number of vehicles

If and unlimited number of vehicles are available, then we can just ignore constraints (4)

Subtours



Subtours



Advantages and disadvantages (Model 1)

Advantage

1. Polynomial number of variables.
2. Strong lower bound from linear relaxation.
3. We can model the direction of the vehicles.

Disadvantages

1. Exponential number of constraints.
2. Route orientation can be a problem. For each route there are $2^{|K|}$ equivalent solutions. We have to find every single one to prove optimality.

$$2^{10} = 1024 \text{ equivalent solutions}$$

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Model 2 (2-index Undirected model)

$$\text{Minimize} \sum_{e \in E} c_e x_e \tag{6}$$

$$\text{s.t.} \sum_{e \in \delta(i)} x_e = 2 \qquad \forall i \in N, \tag{7}$$

$$\sum_{e \in \delta(0)} x_e = 2|K|, \tag{8}$$

$$\sum_{e \in \delta(S)} x_e \geq 2r(S) \qquad \forall S \subseteq N, S \neq \emptyset, \tag{9}$$

$$x_e \in \{0, 1\} \qquad \forall e \in E.$$

$$x_e \in \{0, 1, 2\} \qquad \forall e \in \delta(0).$$

Advantages and disadvantages (Model 2)

Advantage

1. Polynomial number of variables.
2. Strong lower bound from linear relaxation.

Disadvantages

1. Exponential number of constraints.
2. Not flexible model to include realistic constraints.
3. Requires solution of NP-hard bin packing problem.

Rounded Capacity Inequalities (RCI)

$$r(S) \approx \left\lceil \frac{\sum_{i \in S} q_i}{Q} \right\rceil$$ approximation of the bin packing problem.

$$\sum_{e \in \delta(S)} x_e \geq 2 \left\lceil \frac{\sum_{i \in S} q_i}{Q} \right\rceil \qquad \forall S \subseteq N, S \neq \emptyset,$$

$$\sum_{(i,j) \in \delta^+(S)} x_{ij} \geq \left\lceil \frac{\sum_{i \in S} q_i}{Q} \right\rceil \qquad \forall S \subseteq N, S \neq \emptyset,$$

Example

$$
\begin{aligned}
q =& \{3, 4, 5, 3, 4, 8, 11, 7, 9, 1\} \\
Q =& 30 \\
& \left\lceil \frac{\sum_{i \in S} q_i}{Q} \right\rceil \\
& \left\lceil \frac{55}{30} \right\rceil = 2
\end{aligned}
$$

We need at least 2 vehicles to satisfy the demand in this subset of customers.

Miller-Tucker-Zemlin constraints

In the directed model of the VRP, we eliminate the exponential subtour-elimination constraints by adding additional variables.

Miller, Tucker and Zemlin developed this new formulation for the TSP in 1960;

MTZ subtour elimination constraints

$$u_i - u_j + Qx_{ij} \leq Q - q_j \qquad \forall (i,j) \in A(N)$$

Capacity constraints

$$qi \leq u_i \leq Q \qquad \forall i \in N$$

Miller-Tucker-Zemlin constraints

if $x_{ij} = 1$

$$u_j \geq u_i + q_j > u_i$$

Presence of subtour not possible

Subtour $(i, j, \ldots, i)$;

Contradiction $u_i > u_j > \cdots > u_i$

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Notation

y_{ik} Binary variables equal to 1 if vehicle k visits customer i,

o Starting depot, origin of all vehicles,

d Destination of all vehicles,

x_{ijk} Binary variables equal to 1 if vehicle k visits customer i and then j immediately afterwards, it equals 0 otherwise.

Model 3 (3-index formulation)

$$\textit{Minimize} \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ijk} \tag{10}$$

$$s.t. \sum_{k\in K} y_{ik} = 1 \qquad \forall i \in N, \tag{11}$$

$$\sum_{j\in\delta^+(i)} x_{ijk} - \sum_{j\in\delta^-(i)} x_{ijk} = 0 \qquad \forall i \in N, k \in K, \tag{12}$$

$$\sum_{j\in\delta^+(o)} x_{ojk} - \sum_{j\in\delta^-(o)} x_{ojk} = 1 \qquad \forall k \in K, \tag{13}$$

$$y_{ik} = \sum_{j\in\delta^-(i)} x_{ijk} \qquad \forall i \in N \cup \{o\}, k \in K, \tag{14}$$

$$y_{dk} = \sum_{i\in\delta^-(d)} x_{idk} \qquad \forall k \in K, \tag{15}$$

$$u_{ik} - u_{jk} + Q x_{ijk} \leq Q - q_j \qquad \forall (i,j) \in A, k \in K, \tag{16}$$

$$q_i \leq u_{ik} \leq Q \qquad \forall i \in V, k \in K, \tag{17}$$

$$x_{ijk} \in \{0,1\}, y_{ik} \in \{0,1\} \qquad \forall (i,j) \in A, k \in K.$$

Advantages and disadvantages (Model 3)

Advantage

1. Polynomial number of variables and constraints.
2. Flexibility to model truck specific constraints.

Disadvantages

1. Linear relaxation obtains a weak lower bound in comparison to other models.
2. Permutations of routes causes multiple equivalent solutions.

$$|K|!$$

$$10! = 3,628,800 \text{ equivalent solutions}$$

Notation

a_{ir} Parameter equal to 1 if route r visits customer i,

Ω Set of all feasible routes

λ_r Binary variables equal to 1 if route r is chosen in the optimal solution it equals 0 otherwise.

Model 4 (Set-partitioning formulation)

$$\text{Minimize} \sum_{r \in \Omega} c_r \lambda_r \tag{18}$$

$$\text{s.t.} \sum_{r \in \Omega} a_{ir} \lambda_r = 1 \qquad \forall i \in N, \tag{19}$$

$$\sum_{r \in \Omega} \lambda_r = |K|, \tag{20}$$

$$\lambda_r \in \{0, 1\} \qquad \forall r \in \Omega.$$

Advantages and disadvantages (Model 4)

Advantage

1. Polynomial number of constraints.
2. Strongest lower bound.
3. Complex intra route constraints can be easily modeled in the definition of the set Ω
4. No symmetry problems.

Disadvantages

1. Exponential number of variables.
2. Requires advanced methods to solve.

$$
\begin{aligned}
\text{Routes } &\approx !(|N| - 1) \\
&\text{With 12 customers} \\
&\approx !(12-1) = !11 = 39{,}916{,}800
\end{aligned}
$$

EPFL

Outline

Intra-route constraints

Loading

$\sum_{i \in V} q_i y_{ik} \leq Q \quad \forall k \in K$

Route length

$\sum_{(i,j) \in A} t_{ij} x_{ijk} \leq L \quad \forall k \in K$

Intra-route constraints(VRP with Time windows)

1. Time windows per vertex, i.e., earliest and latest arrival times,
2. Waiting time, the vehicles can arrive before the earliest arrival time and wait.
3. Service time, vehicles can take some time to provide a service at the customer.

Intra-route constraints(VRPTW)

T_{ik} Time at which vehicle k visits customer i,

$[a_i, b_i]$ Earliest and latest time window for customer i

t_{ij} Time to travel from i to j

$$a_i \leq T_{ik} \leq b_i$$

$$\text{If } x_{ijk} = 1 \implies T_{ik} + t_{ij} \leq T_{jk}$$

linearize constraints with big M

$$T_{ik} - T_{jk} + Mx_{ijk} \leq M - t_{ij}$$

Intra-route constraints

Time dependent travel times

Travel times might depend on the time of the day. The time t_{ij} has to be replaced with time functions, i.e., $t_{ij}(T_i)$, where the time to travel from i to j is a function of the current time.

Soft time windows

With soft time windows, vehicles are allowed to arrive early or late at a customer. However, there is a penalty to pay for early or late arrival.

Intra-route constraints

Driving rules by European regulations

- Driving periods, maximum 4.5 hours.
- Daily driving times, 9 hours.
- Weekly driving times, max 56 hours.
- Breaks of 45 minutes to end a driving period, it could also be 30 minutes followed by 15 minutes. .
- Daily rest period of min 11 hours.

Fleet characteristics

Multiple Depot VRP

Vehicles can start and end their routes at different depots. Some depot can have a different capacity for vehicles.

Heterogeneous fleet of vehicles

Vehicles can be different. Some vehicles have a smaller capacity and different costs to travel trough an arc. A different capacity has to be checked per vehilce type Q_k.

Routing of trucks and trailers

Trailers are big, poor manoeuvrability can make it difficult to service some customers with trailers. Hence, a subset of customers are inaccessible based on the vehicle used to service them.

Types of Delivery Requests

Delivery and collection

Some problems have a mix of deliveries and collection. Therefore, the capacity of the vehicle changes based on the route. These are called pickup-and-delivery problems.

Split delivery

Instead of delivering all of a customers items in a single visit, the delivery can be split by different vehicles. Oddly, this leads to a reduction in total cost in comparison to single delivery VRP.

Stochastic demand

The demand of customers can be unknown before the vehicle arrives, and can become known only when the vehicle arrives. This problem arises when distributing fuel.

Outline

Summary

- Ubiquitous applications of operations research.
- Real world problems are vague and not well described.
- Models for VRP.
- 2-index formulation for VRP directed.
- 2-index formulation for VRP undirected.
- 3-index formulation for VRP.
- Set partitioning formulation for VRP.
- Realistic variants of VRPs.