CIVIL-557
Decision-Aid Methodologies in Transportation
Lecture V
Metaheuristics II

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Outline

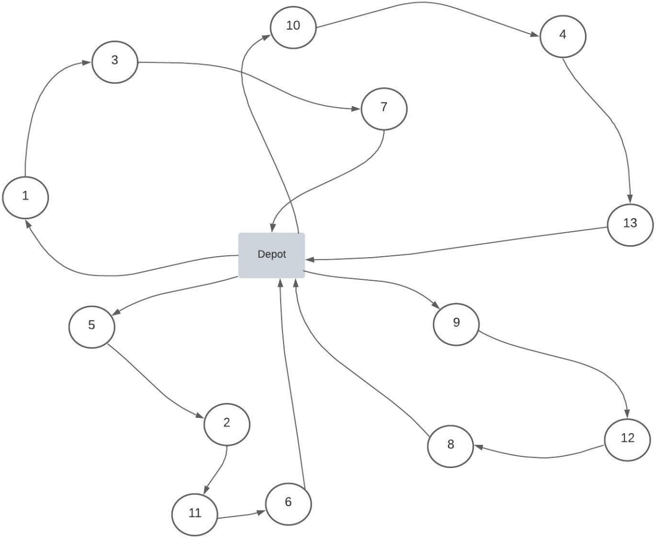
1. Large Neighborhood Search
2. ALNS
3. Matheuristics
   - Column Generation (CG)

Outline

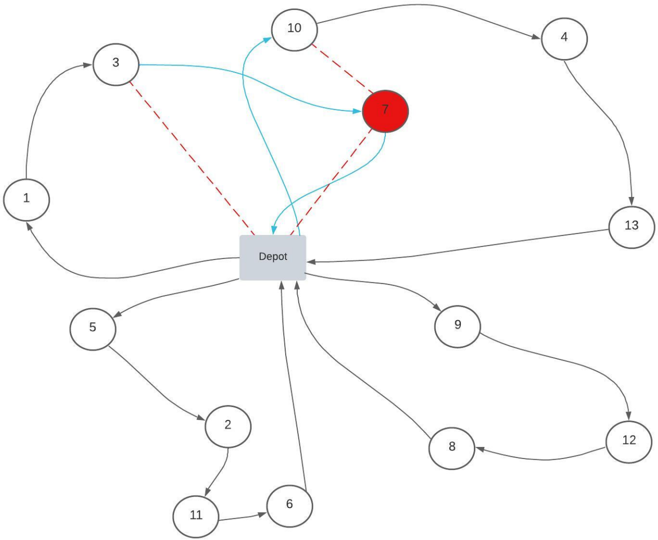
Neighborhood Search/Local Search

- The neighborhood (N_k) of a solution 'w' in the 2-opt neighborhood is the set of solutions that can be reached from 'w' by deleting two edges in 'w' and adding two other edges in order to reconnect the tour.
- Simple example of a neighborhood for the CVRP is the **relocate** neighborhood.
- In this neighborhood, N(w) is defined as the set of solutions that can be created from w by relocating a single customer. The customer can be moved to another position in its current route or to another route.

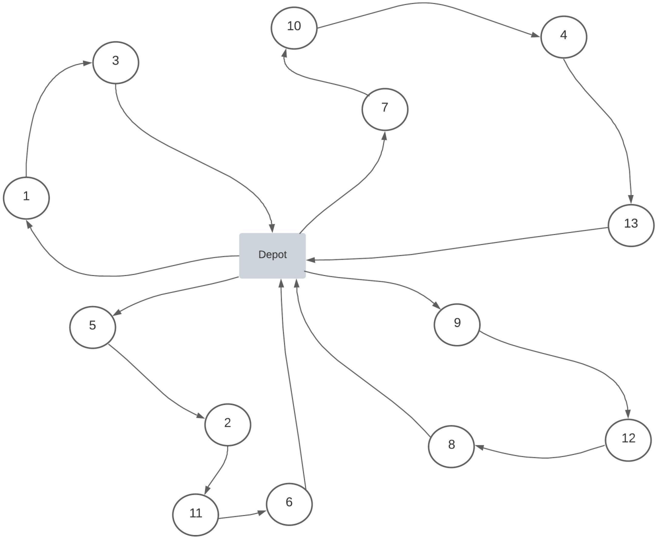
Relocate heuristic



Relocate heuristic



Relocate heuristic



Neighborhood Size:

- If we take the 2-opt algorithm for the TSP as an example, the total number of solutions that are possible from a given solution ‘w’ are:

$$\mathcal{O}(n^2)$$

- For the **relocate** algorithm for the VRP, the total number of different solutions that are possible from a given solution ‘w’ are also:

$$\mathcal{O}(n^2)$$

- These “possible” solutions are the Neighborhood of a given solution ‘w’.

Definition: A small neighborhood is a neighborhood that has a size of $\mathcal{O}(n^k)$, where $k \leq 3$.

Large Neighborhood Search (LNS)

- LNS is based on the **destroy and recreate** idea;
- Some authors use different terms, e.g., ruin and rebuild, destroy and repair, etc. However, the main idea is the same;
- A **destroy heuristic** destroys a part of the current solution;
- A **rebuild heuristic** repairs the destroyed solution.

Destroy heuristic:

- Can remove a number of customers from a current solution.

Rebuild heuristic:

- Inserts the removed customers to form a new solution.

Large Neighborhood Search (LNS)

- Local Search heuristics get stuck in local optima;
- By destroying a solution, LNS can escape local optima and go to a different neighborhood.

Neighborhood size In a VRP with 100 customers, if LNS removes 15% of the customers, the number of possible solutions is the following:

$$\binom{100}{15} = \frac{100!}{15! \times 85!} = 2.5 \times 10^{17}$$

That is the reason why this is called **Large Neighborhood Search**

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LNS: Notation

- x^* Initial solution;
- x_b Current solution;
- $f(x)$ The objective function;
- $accept(x_1, x_2)$ Criteria to accept a new solution as a current solution;

LNS

Algorithm 1: Large Neighborhood Search Algorithm

Input: Initial solution x^*

$x_b \leftarrow x^*$; Incumbent solution ;

$x \leftarrow x^*$; Current solution ;

repeat

  $x_k \leftarrow rebuild(destroy(x))$;

  if *accept*(x_k, x) **then**

    $x \leftarrow x_k$;

  if $f(x_k) < f(x_b)$ **then**

    $x_b \leftarrow x_k$;

until *stopping criterion is met*;

return x_b;

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LNS: Acceptance criteria

The acceptance criteria $accept(x_k, w)$: There are different strategies to select an acceptance criteria.

- Hill-climber, if LNS accepts only improving solutions.
- Threshold accepting. Similar to Simulated annealing, select a threshold $\mathcal{T}_0$, if a the difference $f(x_k) - f(w)$ is smaller than $\mathcal{T}_0$ then we accept the solution.
- Simulated annealing criteria. Variations of SA have been considered where the temperature '$\mathcal{T}$' decreases linearly.

LNS: Destroy method

The **"degree of destruction"** is the most important choice when implementing the destroy method.

- **Destroy too little:** If LNS removes only one customer from the solution, then LNS would be the same as the relocate heuristic.
  - The LNS algorithm is reduced to a small neighborhood algorithm.
  - LNS will not be able to escape local optima.
- **Destroy too much:** If LNS destroys all the solution, then LNS is a constructive heuristic, e.g., random insertion.
  - We are rebuilding the solution from scratch.
- Balance is required so that the degree of destruction is not too small or too large. Here are some strategies:
  - Increase the degree of destruction gradually.
  - Chose the degree of destruction randomly, from a range that is not too small or too large.

LNS: Rebuild method

- **Exact Method**
  - An exact method can reconstruct the partially destroyed solution in the best possible way;
  - It could lead to higher quality solutions;
  - In some applications it can take too long;

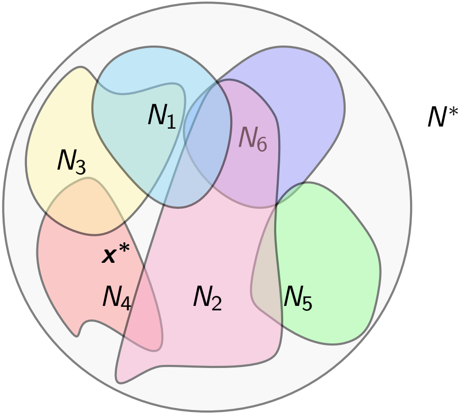
- **Heuristic**
  - An exact rebuild method can be bad for the diversification of the current solution.
  - Heuristic can escape local optima more often since it does not always find the same optima solution.
  - Heuristics are fast!

Outline

Adaptive Large Neighborhood Search (ALNS)

- ALNS extends the LNS heuristic, by allowing multiple **destroy** and multiple **rebuild** methods.
- Each destroy/repair method is assigned a weight that controls how often that particular method is attempted during the search.
- The weights are adjusted dynamically as the search progresses so that the heuristic adapts to the instance at hand and to the state of the search.

Adaptive Large Neighborhood Search (ALNS)



Algorithm 2: Adaptive Large Neighborhood Search

Input: Initial solution x^*

$\rho^- \leftarrow (1, ..., 1)$: $\rho^+ \leftarrow (1, ..., 1)$ Initial weights set to 1;

$x_b \leftarrow x^*$; Incumbent solution ;

$x \leftarrow x^*$; Current solution ;

repeat

select destroy and repair methods $d \in \Omega^-$ and $r \in \Omega^+$ using ρ^- and ρ^+;

$x_k \leftarrow r(d(x))$; Destroy and rebuild ;

if *accept*(x_k, x) **then**

$x \leftarrow x_k$;

if $f(x_k) < f(x_b)$ **then**

$x_b \leftarrow x_k$;

update ρ^- and ρ^+;

until *stopping criterion is met*;

return x_b;

ALNS: Probability of selection

- Ω^{-}Set of all destroy methods;
- Ω^{+}Set of all rebuild methods;
- ϕ_j^{-} Probability of choosing destroy method "j";
- ρ_j^{-} Weight of destroy method "j";
- ϕ_j^{+} Probability of choosing rebuild method "j";
- ρ_j^{+} Weight of rebuild method "j";

The probability to choose the "j" destroy method is the following:

$$\phi_j^{-} = \frac{\rho_j^{-}}{\sum_{k=1}^{|\Omega^{-}|} \rho_k^{-}}$$

ALNS: Reward and Update

Calculate the reward of each operator used:

$$\psi = max \begin{cases} \omega_1 & \text{if the new solution is a new global best,} \\ \omega_2 & \text{if the new solution is better than the current one,} \\ \omega_3 & \text{if the new solution is accepted,} \\ \omega_4 & \text{if the new solution is rejected,} \end{cases}$$

$$\omega_1 \geq \omega_2 \geq \omega_3 \geq \omega_4 \geq 0$$

Update weights:

$$\rho_j^- = \lambda \rho_j^- + (1 - \lambda)\psi, \quad \rho_i^+ = \lambda \rho_i^+ + (1 - \lambda)\psi$$

Where λ is the decay parameter that controls how sensitive the weights are to changes to the performance of the heuristics.

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ALNS: Rewards

- The weights of the heuristics that were not used remain unchanged.
- The goal is to select heuristics that work well for the problem being solved.
- Coupled neighborhoods: Some remove heuristics and rebuild heuristics go well together, or might be incompatible together. In such cases, one may define a set of rebuild heuristics that can be used with a specific destroy heuristic.

ALNS: Time vs quality

- If a heuristic h_1 if faster than a heuristic h_2, it can be used for hundreds of iterations for a single iteration of the slower heuristic h_2.
- The rewards in ALNS favor heuristics that find higher quality solutions, even if they require a large number of iterations in comparison to faster heuristics that require less iterations.
- The quality of the solution quality has to be evaluated with time. For example, if 100 iterations of h_1, on average, finds better solutions than a single iteration of h_2, and 100 iterations takes the same time as 1 iteration of h_2, then, h_1 is better.
- If all heuristics take the same time then it is not a problem.
- Otherwise, one can adjust the score ψ with a measure of time consumption.

Outline

Matheuristics

Matheuristic are a family of metaheuristics that combine powerful exact methods like MIP and metahusristics.
Examples:

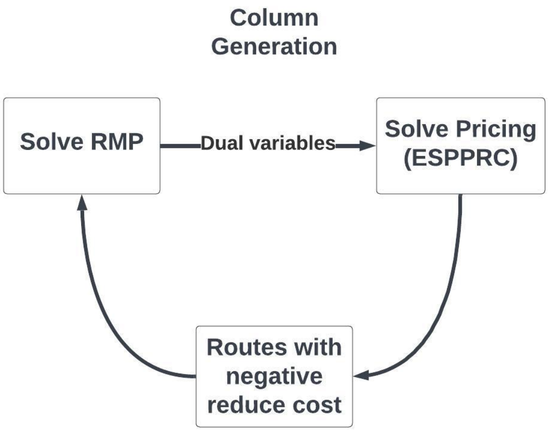
- LNS, can use an exact method to rebuild the partialy destroyed solution.
- Variable fixing: In a Branch-and-Bound algorithm, find a variable that is fractional and set to one, resolve without branching and fix another fractional value to one, until a feasible solution is found.
- Column generation: Solve the pricing problem and convert the relaxation problem to an MIP and solve.

- The **Set-covering formulation** is solved efficiently with commercial solvers, e.g., Gurobi or CPLEX.

Restricted Master problem (RMP)

$$
\begin{aligned}
\textit{Minimize} \sum_{r \in \bar{\Omega}} c_r \lambda_r & \\
\textit{s.t.} \sum_{r \in \bar{\Omega}} a_{ir} \lambda_r \geq 1 & \qquad \forall i \in N, \\
\lambda_r \in \{0, 1\} & \qquad \forall r \in \bar{\Omega}.
\end{aligned}
$$

Column Generation



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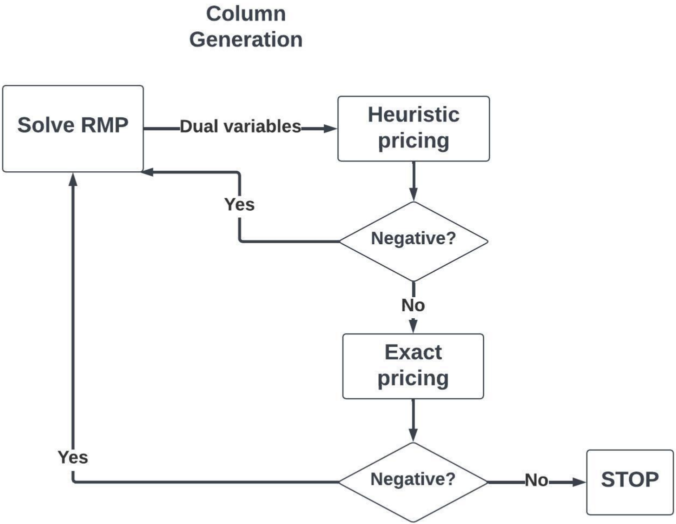
Heuristic Pricing

It is not necessary to produce the shortest path in every iteration of CG. All that is needed is a path with a negative reduced cost.

- The exact method to find the shortest path can be time consuming;
- Dual values can be unstable at the start of the algorithm taking many iterations to stabilize;
- Fast heuristics can be used to find a negative reduced cost route;
- The exact method for the pricing problem only needs to be used to prove optimality of the master problem.

Column Generation



CG: Pricing Heuristic

Recall the dominance rules **and** Feasibility check for the exact method of the ESPPRC

Dominance rules: Label $\mathcal{L}_1 = (i_1, c_1, T_1, Q_1, V_1, L_1)$ dominates label $\mathcal{L}_2 = (i_2, c_2, T_2, Q_2, V_2, L_2)$ if the following is true:

1. $i_1 = i_2$
2. $c_1 \leq c_2$
3. $T_1 \leq T_2$
4. $Q_1 \leq Q_2$
5. $V_1 \subseteq V_2 \leftarrow$ complicating rules!

Feasible extension

- **if** $j^n \in V^e$, **Then** the extension is not feasible.
  or
- **if** $T^e + t_{(i^e, j^n)} > b_{j^n}$, **Then** the extension is not feasible.
  or
- **if** $Q^e + q_{j^n} > Q$, **Then** the extension is not feasible.

CG: Pricing Heuristic

Change dominance rules and **keep** feasibility checks

1. $i_1 = i_2$
2. $c_1 \leq c_2$
3. $T_1 \leq T_2$
4. $Q_1 \leq Q_2$

- **if** $j^n \in V^e$, **Then** the extension is not feasible.
  or
- **if** $T^e + t_{(i^e, j^n)} > b_{j^n}$, **Then** the extension is not feasible.
  or
- **if** $Q^e + q_{j^n} > Q$, **Then** the extension is not feasible.

- Eliminating complicating dominance rules makes the algorithm terminate in pseudo-polynomial time. However, there is no guarantee that the solution is optimal.
- Keeping all feasibility checks guarantee that the solution (if found) will be feasible.
- If more dominance rules are eliminated the algorithm will terminate faster.

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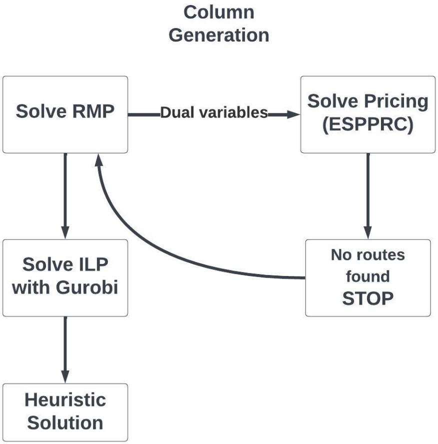
CG:Pricing heuristic

Strategy: Develop multiple heuristics and order them from fastest to slowest. The fast heuristics produce solutions of smaller quality while the slower ones will be able to find a negative column more easily.

- Start by using the fastest heuristic to produce columns.
- If the fastest heuristic found a negative column, add to and resolve the LRMP.
- If the fastest heuristic fails to find a negative column, move to the second fastest, etc.
- If all heuristics have failed to produce a negative column then use the exact method.

Column Generation heuristic

Commercial solvers, e.g., Gurobi, can solve set partitioning models quickly.



Column Generation heuristic

The solution is **not necessarily** optimal. To find the optimal solution we must use Branch-and-Price, i.e., every time we branch we must solve the pricing problem again.

- The RMP with a restricted number of routes will most likely not have the optimal routes in the subset of columns.
- Commercial solvers do not have the feature of adding columns in the middle of branching.
- If the exact method was used in the last iteration of CG then we at least get a lower bound for the problem.

Column Generation: Variable fixing

Another strategy widely used is to fix fractional variables to an integer, instead of branching.

1. Solve the MP, find a fractional variable close to 1, e.g., 0.95, and set equal to 1;
2. Solve pricing problem until no negative column is found.
3. Select another fractional variable that is close to 1 and set equal to 1.
4. Repeat until you find a feasible solution.

At any point we can stop the algorithm and turn the RMP into an IP and solve the problem with a commercial solver. The set of routes contained in the restricted set of routes will be larger, and increase the probability of finding better solutions.

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References

- Wolsey, L. A. (1998). *Integer programming*. Wiley.
- Gendreau, M., & Potvin, J.-Y. (Eds.). (2010). *Handbook of metaheuristics*. Springer.