FAT5 EXERCISE: SIMPLIFIED FATIGUE CHECK – CORRECTION

1a. Determine for each span and for the support regions the corresponding values of the lambda factor, under the action of a bending moment.

The side spans are 60 m long and the central span is 80 m long. For a bending moment in a continuous beam, one must consider as « span length » :

- in span regions, the length L_i of the span in question,
- in the support regions, the average of the 2 adjacent spans L_i and L_j at this support.

For lambda 1, the figures of EN1993-2 are reproduced in Figure 1. Figure 2 shows these same curves in comparison with simulation results. There is a wide dispersion of the values, a poor correspondence of the shape of the curve in the support regions. Note that this justifies, for spans greater than 80 m, extrapolating using the value at 80 m.

The calculations for lambda 1 lead to the values given in Table 1 below, in the case of details where the stress (stress range) is related to bending moments.

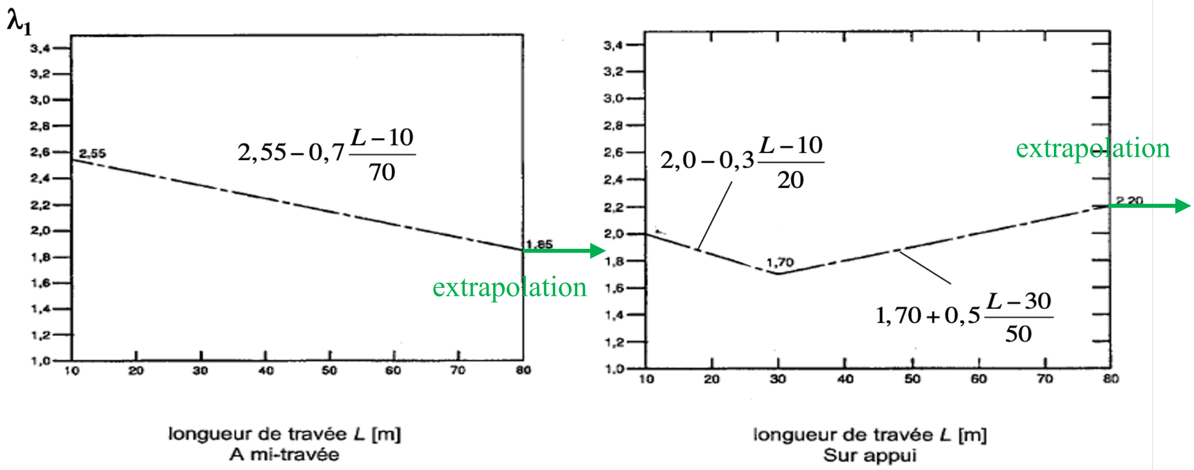

Figure 1 –λ_1 for road bridges according to EN 1993-2

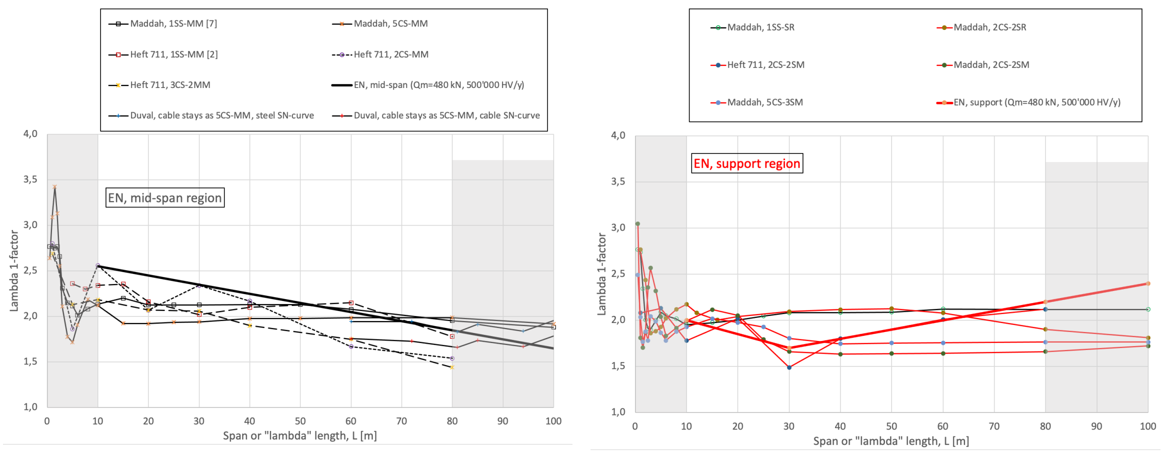

Figure 2 – Comparison between λ_1 curves and simulations (RESSLab)

Table 1 – Partial factor values λ_1

Region		Section location	L (m)	λ_1
Midspan	Support			
X		Side spans	60	2.55 – 0.7 (L-10)/70 = 2.05
	X	Close to the supports	(80+60)/2 = 70	1.7 + 0.5 (L-30)/50 = 2.10
X		Centre span	80	2.55 – 0.7 (L-10)/70 = 1.85

In the code, the traffic changes with respect to the number of cycles but not the loads ($Qm_1 = Q_0$), therefore the partial factor λ_2 is equal to:

$$\lambda_2 = \frac{Q_{m1}}{Q_0} \cdot \left[\frac{N_{obs}}{N_0}\right]^{1/m2} = \left(\frac{2000000}{500000}\right)^{1/5} = 1.32$$

There is no deviation regarding service life from the code, so the partial factor λ_3 is 1.0.

For λ_4, the bridge section is a composite twin-girder with a slab on which two slow lanes need to be considered (one in each direction). It is reasonable to assume that each slow lane is carrying the same traffic, this means that $Q_{\text{m2}} = Q_{\text{m1}};\ N_2 = N_1$. For this composite twin-girder section, the transverse distribution line is given in the data, i.e. η_1=0.675 and η_2=0.325.
Finally, in the fatigue curve, if the usual assumption that our loads correspond to the zone where the slope is m $= m_2 = 5$ (because we are studying welded steel details), then it follows:

$$\lambda_4 = \left[1 + \frac{N_2}{N_1}\left(\frac{\eta_2 \cdot Q_{m2}}{\eta_1 \cdot Q_{\text{m1}}}\right)^5\right]^{1/5} \cong 1.00$$

There is also the condition that the product of all λ_i factors must remain less than λ_{max}. The formulas for λ_{max} are found in EN1993-2 and values depend on the « span length » as for λ_1. These calculations lead to the values given in Table 2 below, in the case of details where the stress (and stress range) comes from bending moments.

Table 2 – Value of the λ_{max}. factor

Section Position	L (m)	λ_{max}
Side span	60	2.0
Close to the supports	(80+60)/2 = 70	1.8 + 0.9 (L-30)/50 = 2.52
Centre span	80	2.0

Finally, the resulting values of the factor λ can be determined using:
$\lambda = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4$ but $\lambda \leq \lambda_{max}$
The results are summarized in Table 3.
In this example, we can see that **λ_{max} controls for all sections!**
i.e. the checks are all made with respect to the fatigue limit, because of the very high bidirectional traffic on the bridge.

Table 3 – Summary of damage equivalency factor values λ.

Section position (between 0 m and 540 m)	λ
Banks spans: - between 0 m and 0.85*L1 = 51 m - between 149 m and 200 m	2.71 ($\leq \lambda_{max}$ 2.0) so 2.0
Support areas: - between 51 m and L1+0.15*L2=92 m - between 128 m and 149 m	2.77 ($\leq \lambda_{max}$ 2.52) so 2.52
Central span: - between 92 m and 128 m	2.44 ($\leq \lambda_{max}$ 2.0) so 2.0

1b. And what is the value of the resulting λ for a longitudinal weld located in the beam web and in the side span ?

This detail will mainly be subjected to the shear forces/stresses. For a continuous beam, one must therefore take for the «span length»:

- in span regions, $0.4 \cdot Li$

This gives: $L_{rep} = 0.4 \cdot 60 = 24$ m and therefore:

$$\lambda_1 = 2.55 - 0.7\frac{L_{rep} - 10}{70} = 2.41$$
$$\lambda_2 = 1.32$$
$$\lambda_3 = 1.00$$
$$\lambda_4 = 1.00$$
$$\lambda_{max} = 2.5 - 0.5\frac{L_{rep} - 10}{15} = 2.03$$

$$\lambda = 2.41 \cdot 1.32 \cdot 1 \cdot 1 = 3.18\ (\leq 2.03)\ \ thus\ \boldsymbol{\lambda = 2.03}$$

2. Determine the stress differences in the sections to be checked.

The load of the fatigue model Q_{fat} is 480 kN.
The elastic section modulus (lower flange) are $W_{b,0,inf,span} = 1.37 \cdot 10^8$ mm^3 and, on support, $W_{b,0,inf,support} = 4.52 \cdot 10^8$ mm^3.
For the calculation of the maximum and minimum moments due to the load model, the transverse distribution line must also be taken into account because the load model FLM3 is to be applied on lane 1. One must therefore multiply the value obtained with the factor η_1=0.675.

<u>Section 1 – Side span x=30 m</u>
Using the line of influence given in the statement, it is found:
M_{max} = 0.675*480 [kN]* 12.38[kNm/kN] = 4011 kNm
M_{min} =0.675*480 [kN]* -3.59 [kNm/kN] = -1163 kNm

Moment difference: ΔM=5174 kNm
$W_{b,0,inf,span} = 1.37 \cdot 10^8$ mm^3
Stress difference: $\Delta\sigma(Q_{fat}) = \Delta M / W_{b,0,inf,span} = 5174 \cdot 106 / 1.37 \cdot 10^8 = 37.8$ MPa

<u>Section 2 – On Supports x=60 m</u>
Using the line of influence given in the statement, it is found:
M_{max} = 0.675*480 [kN]* 1.54[kNm/kN] = 499 kNm
M_{min} =0.675*480 [kN]* -7.19 [kNm/kN] = -2330 kNm

Moment difference: ΔM=2829 kNm
$W_{b,0,inf,support} = 4.52 \cdot 10^8$ mm^3
Stress difference: $\Delta\sigma(Q_{fat}) = \Delta M/W_{b,0,inf,span} = 2829 \cdot 106/4.52 \cdot 10^8 = 6.3$ MPa

Section 3 – Centre span x=100 m
Using the line of influence given in the statement, it is found:
M_{max} = 0.675*480 [kN]* 13.33[kNm/kN] = 4319 kNm
M_{min} =0.675*480 [kN]* -1.92 [kNm/kN] = -622 kNm

Moment difference: ΔM=4941 kNm
$W_{b,0,inf,span} = 1.37 \cdot 10^8$ mm^3
Stress difference: $\Delta\sigma(Q_{fat}) = \Delta M/W_{b,0,inf,span} = 4941 \cdot 106/1.37 \cdot 10^8 = 36.1$ MPa

3. Perform the fatigue check of the 3 joint details forming the lower part of the transverse frame.

The fatigue check formula is as follows:

$$\gamma_{Ff} \cdot \Delta\sigma_{E2} = \gamma_{Ff} \cdot \lambda \cdot \Delta\sigma_{\square}(Q_{fat}) \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

The values of the partial safety factors, γ_{Ff} and γ_{Mf}, must be introduced. By default in the current code the load factor γ_{Ff} is considered to be equal to 1.0. As for the partial coefficient of resistance γ_{Mf}, it depends on the consequences of the ruin, as well as on the method of evaluation.

After "failure" of a detail of the twin-girder (the failure being here defined as a through crack in the flange), the fatigue crack will continue to grow in the flange and then in the web and certainly lead at some point to the final failure of the section. These are therefore details of first importance and the consequences of failure can be taken as high. However, the consequences should be put into perspective as:

- on the one hand the bridge is a continuous beam, i.e. a hyperstatic system;
- on the other hand, this type of section can be easily inspected, by walking and observing for any through crack from the maintenance passerelle, thus rather short inspection intervals can be set (e.g. every 2 or 3 years).

Therefore, the value for the factor can be taken as $\gamma_{Mf} = 1.15$.

The last data missing for the checks are the fatigue detail categories, they are defined below for each of the section.

Section 1 – Side span x=30 m
The detail to check is the weld of the vertical stiffener on the lower flange (note: same as for the centre span). Referring to EN 1993-1-9 (Table 8.4 – detail 7), one finds that the detail category is FAT80 assuming that a distance weld toe to weld toe ≤ 50 mm. The checking of the detail reads:

$$\gamma_{Ff} \cdot \lambda \cdot \Delta\sigma_{\square}(Q_{fat}) \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$$75.6\ MPa = 1.0 \cdot 2.0 \cdot 37.8 \leq \frac{80}{1.15} = 69.6\ MPa \qquad \textbf{KO}$$

Section 2 – On support x=60 m
The detail to check is the weld of the vertical T-stiffener on the lower flange (with a longitudinal attachment part). Referring to EN 1993-1-9 (Table 8.4 – detail 1), in this case the strength of the detail must be reduced due to the length of the attachment, which is in this case certainly L > 100 mm, the detail category is FAT56.

The checking of the detail reads:

$$\gamma_{Ff} \cdot \lambda \cdot \Delta\sigma(Q_{fat}) \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$$15.9\ MPa = 1.0 \cdot 2.52 \cdot 6.3 \leq \frac{56}{1.15} = 48.7\ MPa$$ **OK**

<u>Section 3 – Centre span x=100 m</u>

The detail to check is similar to the one in section 1 (vertical stiffener on the lower flange), thus Table 8.4 – detail 7. Distance weld toe to weld toe ≤ 50 mm, thus it is again detail category FAT80.

The checking of the detail reads:

$$\gamma_{Ff} \cdot \lambda \cdot \Delta\sigma(Q_{fat}) \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$$72.2\ MPa = 1.0 \cdot 2.0 \cdot 36.1 \leq \frac{80}{1.15} = 69.6\ MPa$$ **KO**

The details that are located in the span do not comply with the checks but only for a few MPa. The following solutions are possible:

- Design the sections in order to increase (in this case slighly) the resisting moment, that is by increasing the thickness of the flange or the static height, or with a coverplate (but this adds other fatigue details and the ends of the coverplate has a low fatigue resistance, will have to be carefully placed and checked).
- Fabricate the detail in a more favorable way (do not weld on the flange, if possible interrupt it above it).
- Use a post-weld treatment to improve the detail category, especially if you assess an existing structure or it is the only detail that does not comply (Note : there will be a new annex on this subject in the revised version of Eurocode EN 1993-1-9, which should be released in 2026 or 2027).

ANNEXE, facteurs partiels de dommage équivalent λ_i

A partir de l'équation de la courbe de fatigue, on a les proportionnalités suivantes : $\lambda \propto \Delta\sigma; N^{1/m}; V^{1/m}$

La vérification à satisfaire est :

$$\gamma_{Ff} \cdot \lambda \cdot \Delta\sigma(Q_{fat}) \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

Avec le facteur de dommage équivalent composé des facteurs partiels λ_i :

$$\lambda = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 < \lambda_{max}$$

<u>Pour le poids et volume de trafic :</u>

- λ_1 pour le trafic en poids et volume de référence (voir les graphiques donnés dans les normes, diffèrent selon le type de structure et le modèle de charge). De même pour λ_{max}

- λ_2 pour le trafic routier : $\lambda_2 = \frac{Q_{m1}}{Q_0} \cdot \left(\frac{N_{obs}}{N_0}\right)^{\frac{1}{m}}$

  $m = 5$

  Q_{m1} = poids moyen des poids lourds

  Q_0 = 480 kN (poids de référence)

  $N_0 = 0.5 \cdot 10^6$ cycles de charge sur la voie lente

  $$Q_m = \left(\frac{\sum n_i Q_i^5}{\sum n_i}\right)^{1/5}$$

- λ_2 pour le trafic ferroviaire : $\lambda_2 = \left(\frac{V_{obs}}{V_0}\right)^m$

- λ_2 pour des calculs plus précis, le trafic ferroviaire à voie étroite : $\lambda_2 = \frac{Q_{obs}}{Q_0} \cdot \left(\frac{N_{obs}}{N_0}\right)^{\frac{1}{m}}$

  Q_{obs} Poids moyen par axe du trafic réel sur l'ouvrage [kN]
  Q_0 Poids moyen par axe du modèle, égal à 120 [kN]
  N_{obs} Nombre total d'axes du trafic réel sur l'ouvrage [n/année]
  N_0 Nombre total d'axes du modèle, égal à 1 Mio [n/année]

<u>Influence de la durée de vie :</u>

$$\lambda_3 = \left(\frac{T_{Ld}}{100}\right)^{1/m}$$

T_{Ld} = durée de service prévue (en années)

<u>Influence de plusieurs voies :</u>

- λ_4 pour le trafic routier :

$$\lambda_4 = \left[1 + \frac{N_2}{N_1}\left(\frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}}\right)^5 + \frac{N_3}{N_1}\left(\frac{\eta_3 Q_{m3}}{\eta_1 Q_{m1}}\right)^5 + \ldots + \frac{N_k}{N_1}\left(\frac{\eta_k Q_{mk}}{\eta_1 Q_{m1}}\right)^5\right]^{1/5}$$

k nb de voies supportant un trafic lourd

N_j nb de poids lourds par an sur la voie *j*

Q_{mj} poids moyen des poids lourds sur la voie *j*

η_j valeur de la ligne de répartition transversale au centre de la voie *j* qui produit l'étendue de contrainte, avec un signe positif

- λ_4 pour le trafic ferriviaire :

$$\lambda_4 = \left(n + [1-n]\left[a^5 + (1-a)^5\right]\right)^{1/5}$$

$$a = \Delta\sigma_1 / \Delta\sigma_{1+2}$$

n portion du trafic se croisant sur le pont (par défaut 12%)