FAT3 EXERCISE: STRESS INTENSITY FACTOR CALCULATIONS – CORRECTION

1. Internal imperfection. The hanger is made of S690 steel, so f_y = 690 MPa.

Its thoughness can be deduced from the Charpy impact energy value (also called CVN) using BS 7910, see the course FAT3 "Introduction to Fracture Mechanics" slide, reproduced below.

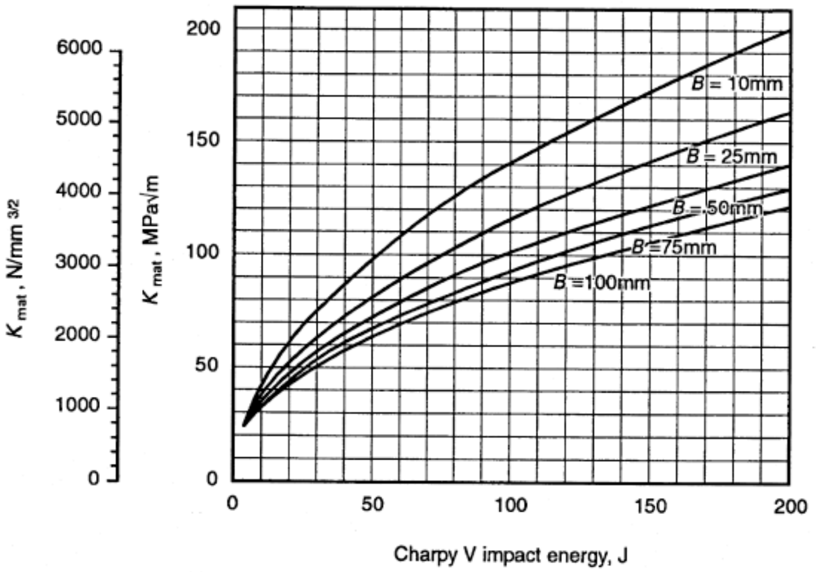


Figure J.2 — K_{mat} plotted against Charpy V impact energy for lower shelf and transitional behaviour

The graph gives, for B = 100 mm (taken here as the diameter of the bar), 100 MPa·m$^{1/2}$ or in S.I. units: K_{Ic} = 3160 MPa·mm$^{1/2}$

Ultimate stress, design value, of tensile loading: σ_{Ed} = 0.8·690 = 552 MPa.

The imperfection, admitted as a crack, is circular and internal, the correction factor is (see TGC10 § 13.6 and example 13.1):

Y = 2/π

Note: it is implicitly assumed that the size of the crack is relatively small compared to the dimensions of the part, i.e. Y_f = 1.0. The error due to this assumption is discussed in the complement.

The maximum size is given by:

$$a \leq \frac{1}{\pi}\left(\frac{K_{Ic}}{Y \cdot \sigma_{Ed}}\right)^2 = \frac{1}{\pi}\left(\frac{3160}{2/\pi \cdot 552}\right)^2 = 25.7\ \text{mm}$$

That's huge!

In fact, the plasticization failure (net cross-section) of the bar probably occurs before, i.e. as soon as:

$$\pi \cdot (R^2 - a^2) \cdot 690 > \pi \cdot R^2 \cdot 552$$

with R = 50 mm : a > 22.4 mm

Indeded, it is therefore the criterion of fracture in the net section that controls in this case (i.e. the steel has sufficient toughness and ductility).

And in our case one can add that the size is such that it is a **defect**, because unlike an imperfection, it would not be tolerated following NDE quality controls of the weld (NDE = non-dest. examination, or NDT = non-destructive testing).

2. Surface imperfection of 30 mm :

The surface stress (is thus allowed up to the maximum depth of the imperfection):

$\sigma_{Ed} = 552 \cdot 1.1 = 607.2$ MPa.
$a/c = 0.5 < 1$, so the point in depth corresponds to the maximum K.
Surface length, 2c = 30 mm
$a = 0.5 \cdot 30/2 = 7.5$ mm

$$Y_e = \left[1 + 1.464\left(\frac{a}{c}\right)^{1.65}\right]^{-1/2} = 0.826$$

$Y_f = 1.0$ (finite dimensions large enough to neglect)

$$Y_s = Y_s = 1 + 0.12\left(1 - 0.75\frac{a}{c}\right) = 1 + 0.12(1 - 0.75 \cdot 0.5) = 1.075$$

Checking of the validity of formula Y_s : a/2t = 7.5/100 = 0.075 < 0.1 OK

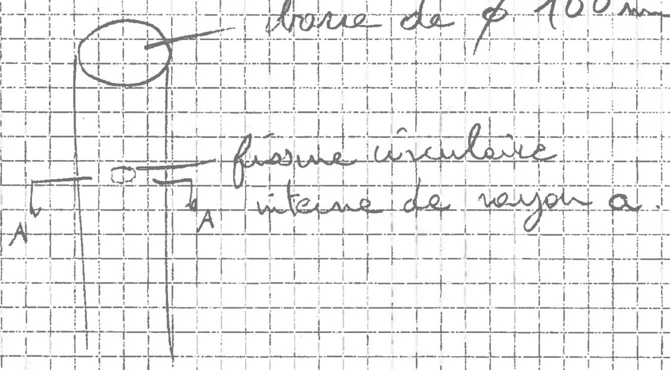
$Y = 0.826 \cdot 1.075 = 0.888$

$$K_{Ed} = Y \cdot \sigma_{Ed}\sqrt{\pi \cdot a} = 0.888 \cdot 607.2\sqrt{\pi \cdot 0.5 \cdot 15} = 2617.3 \text{ MPa} \cdot \text{mm}^{1/2} < K_{Ic} = 3160 \text{ MPa} \cdot \text{mm}^{1/2}$$ **OK**

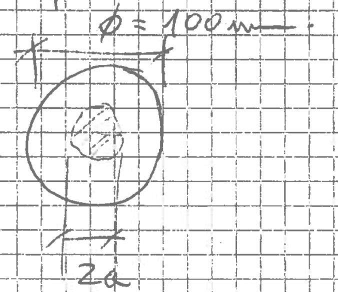
The imperfection is not critical, it can be tolerated as is (but it may grow with fatigue, that's another problem to study).

Complement to question 1 : take into account the finite dimensions:

Calcul additionnel de Y_f



Coupe A-A :



Première estimation (sans Y_f) de la fissure critique donne :

$a = 25{,}7$ mm

Donc Y_f, avec $2t = \phi = 100$ mm

$$Y_f = \sqrt{\frac{\phi}{\pi a}\tan\frac{\pi a}{\phi}} = \sqrt{\frac{100}{\pi \cdot 25{,}7}\tan\frac{\pi \cdot 25{,}7}{100}} = 1{,}137$$

Recalcul de la taille critique :

$$a \leq \frac{1}{\pi}\left(\frac{K_{Ic}}{2/\pi \cdot Y_f \cdot 552}\right)^2 = 19{,}9 \text{ mm}.$$

on fait une erreur de 9% en négligeant Y_f

2ème itération : $Y_f = 1{,}074$

$a = 22{,}29$ mm.

3ème itération : $Y_f = 1{,}097$ la valeur finale doit plutôt être $Y_f \cong 1{,}09$

$$\Rightarrow a \leq \frac{1}{\pi}\left(\frac{3160}{2/\pi \cdot 1{,}09 \cdot 552}\right)^2 \cong 21{,}7 \text{ mm}.$$

⇒ La rupture par fissuration et par plastification sont simultanées.

An error of 9% is made by neglecting Y_f, the corresponding increase in $K_{applied}$ leads to a decrease in the allowable defect size, which ends up to be very similar in size to the net section defect size found.