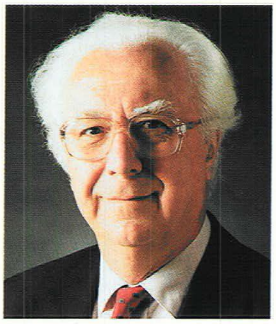
A Re-definition of the Stiffness of Reinforced Concrete Elements and its Implications in Seismic Design



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Summary

It is postulated that for the purposes of seismic design the ductile behaviour of lateral force-resisting structural components, elements and indeed the entire building system, can be satisfactorily simulated by simple bi-linear force-displacement relationships. This enables the displacement relationships between the system and its lateral force-resisting elements at a particular limit state to be readily evaluated. To this end some widely used fallacies, relevant to the transition from elastic to inelastic behaviour, are exposed. A re-definition of yield displacements and consequently stiffness, allows much more realistic predictions of the most important feature of seismic response, element displacements, to be made. The concepts introduced are rational yet very simple. Their applications are closely interwoven with the designer's intensions. The strategy provides the designers with unexpected freedom in the assignment of strengths to lateral force resisting elements, such as frames or structural walls. Contrary to current design practice, whereby a specific global displacement ductility capacity is prescribed for a particular structural class, the designer can determine the acceptable displacement demand to be imposed on the system. This should protect critical elements against excessive displacement demands.

Introduction

A study of the assessment of the structural performance of existing buildings with earthquake risk triggered inquiries adressing the likely response of buildings as constructed, rather than their compliance with a particular code. A major perceived need was the estimation of torsion-induced displacements of elements of ductile systems [1, 2, 3]. In the process several issues emerged with apparent conflict with ingredients of our current design pratice. The description of progressively emerging fallacies, firmly entrenched in our routine seismic design techniques [4], is the subject of this presentation.

The motivation for the introduction of some unfamiliar, but not necessarily new, principles, relevant to ductile structural response, was the need to emphasize the importance of earthquake-induced displacements, rather than a particular method of assigning strengths to elements of a system. Identification of structural behaviour, rationale and transparency of a viable design strategy, combined with simplicity of application, were central issues of this motivation.

Design Criteria

The primary purpose of this study was to address means by which performance criteria, conforming with the appropriate design limit state, may be rationally executed. These criteria are:

- Earthquake-induced deformations should limit the expected displacement ductility demand on any element to its ductility capacity, $\mu_{\Delta imax}$, stipulated in codes.
- Maximum magnitudes of interstorey displacements, to be expected at locations remote from the centre of mass, should not exceed those considered acceptable for buildings, typically 2–2.5% of the storey height.
- Performance criteria may require displacements associated with a specific limit state, to be less than those allowed by the limits listed above.

Terminology used

- In the study of earthquake-induced displacements of buildings, reference will be made to the *structural system*.

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- A structural system comprises lateral force-resisting elements, generally arranged in two orthogonal directions. Due to torsional effects, elements of the system may be subject to different displacements. Typical elements are bents of ductile frames or interconnected walls in the same plane.
- A lateral force-resisting element may comprise several components. Components will be subjected to identical displacements. Typical components are beams or columns or walls.

These terms are illustrated in *Fig. 1*.

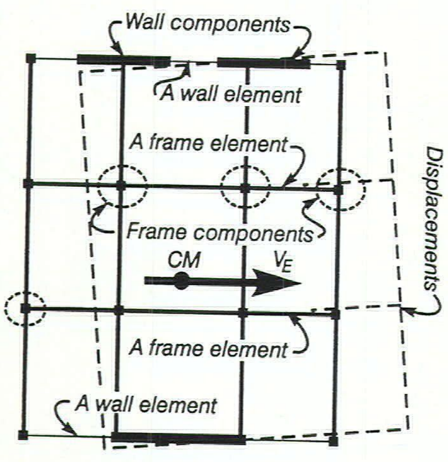


Fig. 1: Nomenclature

Traditional Concepts of the Theory of Elasticity

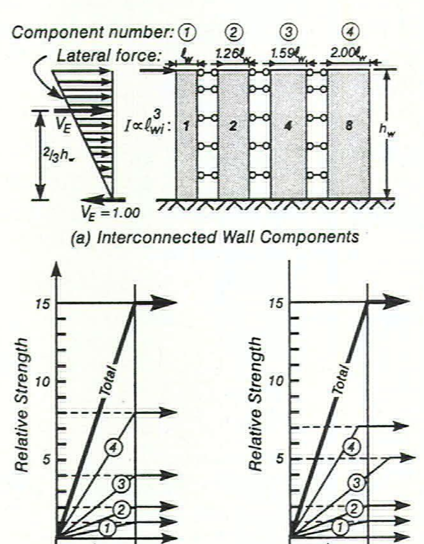
The requirements for static equilibrium and deformation compatibility in statically indeterminate structure, is well established. These principles are still widely used when strength to lateral force-resisting ductile elements of a system are assigned.

With the introduction of equivalent lateral static seismic design forces and the acceptance of ductile response, the same technique continued to be widely used. It implied the notion that strength assigned proportionally to element stiffness will result in the simultaneous onset of yielding in all elements.

Subsequently it was realized that, because, of expected significant inelastic earthquake-imposed deformations, some deviation from strength distribution according to elastic behaviour, is quite acceptable. Thus the practice of strength re-distribution was adapted. It was assumed that for components a reduction of flexural strength will result in an earlier onset of yielding, whereas elements with excess strength will yield a little later. These changes in the onset of yielding were not expected to change component ductility demands significantly enough to be a cause of concern.

Instead of abstract derivations of relevant relationships, some simple examples, commonly encountered in design practice, will be used to show the applicability of certain principles. *Fig. 2(a)* shows four rectangular reinforced concrete cantilever walls of identical heigths and widths. The interconnection of these wall components is such that at any stage of seismic response, identical displacements will be imposed on all four elements. The lengths of the wall components, l_{wi}, is such that the relative second moments of area of the sections, being proportional to l^3_{wi}, are 1, 2, 4 and 8, respectively. The total relative stiffness of the element, comprising four component cantilever walls, is thus 15. The relative component strengths, based on traditionally defined component stiffness, are shown in *Fig. 2(b)*. It also shows that, according to traditional assumptions, all elements will commence yielding at a relative lateral displacement of Δ.

Fig. 2(c) shows the implications of 1/15 of the total strength being redistributed from component (4) to component (3) without reducing the total strength of the four-component element. The traditional definition of component stiffness, based on the second moment of area of the section, suggests that the yield displacements of these two components will change. A departure from the strength assigned in proportion to component stiffness implies thus, as shown in *Fig. 2(c)*, a corresponding decrease or increase, respectively, of the component yield displacement. The fallacies resulting from this traditional usage of component stiffness are examined subsequently.



(b) Strength-displacement relationships

(c) Strength-displacement relationships with some strength redistribution

Fig. 2: Interconnected cantilever wall components

A Definition of Yield Deformations

To allow the very convenient use of bilinear modelling of ductile structural response, it is essential to define the transition point from linear elastic to linear post-yield behaviour. Two types of interrelated deformations, such as yield curvature and yield displacement, need to be considered.

Yield Curvature

To illustrate an acceptable simulation of the non-linear moment-curvature relationship, an example of a rectangular reinforced concrete wall section, shown in *Fig. 3*, will be used. The technique associated with the analyses is well established [5]. Approximations [6], most useful for seismic design purposes, are less well known. The strain pattern associated with the onset of yielding of the reinforcement at the extreme tension fibre enables the corresponding curvature to be expressed as:

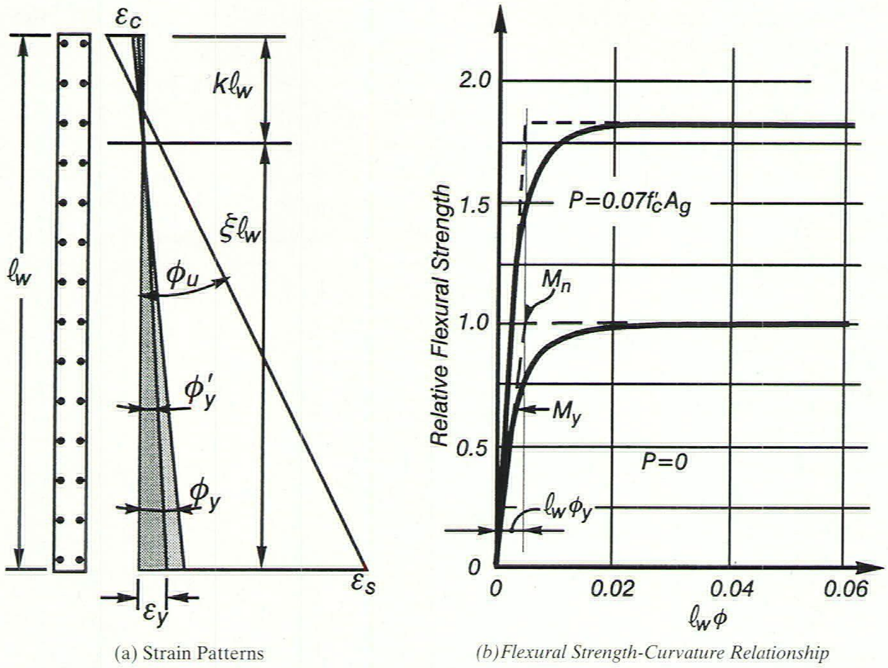
$$\phi'_y \approx \epsilon_y/(l-k)\, l_w = \epsilon_y/(\xi l_w) \tag{1}$$

where ϵ_y, is the yield strain of the steel used and ξl_w is the length of the flexural tension zone of the elastic section.

For the purpose of bi-linear modelling of the flexural strength-curvature relationship at the critical section of the wall component, it is convenient to introduce the term «nominal yield curvature», subsequently referred to simply as yield curvature, by linear extrapolation to the nominal flexural strength of the section, M_n, thus

$$\phi_y = (M_n/M_y)\phi'_y = [(M_n/M_y)/\xi l_w]\epsilon_y \propto (\epsilon_y/l_w) \tag{2}$$

where M_y is the moment associated with the first yield, i.e., ϕ'_y, given by eq. (1). From extensive parametric analyses [6] it has been established that for a given cross section, the value of $(M_n/M_y)/\xi$ remains essentially constant



(a) Strain Patterns

(b) Flexural Strength-Curvature Relationship

Fig. 3: Flexural strength-curvature relationships for a wall section

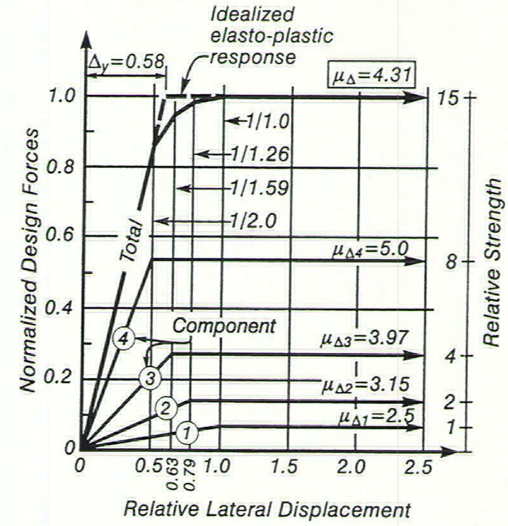


Fig. 4: Bi-linear idealization of component and element force-deformation relationships

irrespective of the ratio and arrangement of reinforcement. As an example for design purposes, it may be assumed [6] that the nominal yield curvature of a rectangular wall section is:

$$\phi_y \approx 2\epsilon_y/l_w \qquad (3)$$

A higher degree of precision in seismic design is not warranted.

The important message of eq. (3) is, that the yield curvature of a reinforced concrete section is proportional to the yield strain of the steel used and inversely proportional to the length or depth of the section.

The bi-linear modelling of flexural strength-curvature relationship, without post yield stiffness, is presented in *Fig. 3(b)*. It is also found that moderate axial compression load on a wall, commonly encountered in multistorey buildings, $P \approx 0.07f'_cA_g$, does not change the yield curvature to any significance. However, the strength of the sections may be significantly increased. The important relationships are recapitulated in *Fig. 3*, where f'_c denotes the compression strength of the concrete and A_g is the gross sectional area of the wall.

Yield Displacement

Once the reference yield curvature of a component, such as a cantilever wall, is established, the corresponding displacement for a given set of lateral forces can be readily established at any desired level of the structure. For example, the reference yield displacement at the top of a wall component i, shown in *Fig. 2(a)*, when subjected to lateral static forces, when based on eq. (3), can be approximated by

$$\Delta_{yi} = C\phi_{yi}h_{wi}^2 \approx (2Ch_{wi}^2\epsilon_y)/l_{wi} \propto 1/l_{wi} \qquad (4)$$

where C is a coefficient which quantifies the distribution pattern of lateral design forces, and h_{wi} is the height of the wall component. Because, as shown in *Fig. 2(a)*, the heights of the walls, h_{wi}, and the grade of the flexural reinforcement used will be the same for all wall components, the bracketed term in eq.(4) will be a constant. Therefore, yield displacements will be inversely proportional to wall lengths. This simple relationship can be conveniently used in design whenever relative properties of components are sufficient to establish, for example, ductility relationships. Examples will subsequently show the relevance of this very important relation.

Implications of the Redefined Yield Displacements

The idealized bi-linear ductile behaviour of a four-component lateral force-resisting wall element, shown in *Fig. 2(a)*, is presented in *Fig. 4*. The assignment of component strengths in accordance with traditional practice, as shown in *Fig. 2(b)*, was used. A study of the relationships demonstrate that:

(1) The relative yield displacement of each wall component is inversely proportional to its length.

(2) As eq. (4) states, the yield displacement is independent of the strength assigned to a component.

(3) When the lateral force-resisting element, shown in *Fig. 2(a)*, is subjected to increasing displacements, the components will commence yielding in a predetermined order. In the example shown, component (4) will be the first and component (1) the last one to yield. At this stage the full strength of the four-component element is developed.

(4) As a corollary, and contrary to traditional assumptions, depicted in *Fig. 2(b)*, the simultaneous yielding of components with different lengths is not possible.

(5) The displacement capacity of a lateral force resisting element is limited by that of the component with the smallest yield displacement, i.e., largest length. In this example it has been assumed that the displacement ductility capacity of the wall components is $\mu_{\Delta imax} = 5$. Therefore, the displacement of this element at the ultimate limit state must be limited to the capacity of component (4) with $\Delta_{u4} = 5 \times 0.5 = 2.5$ displacement units. In *Fig. 4* the yield displacement of component (1) was chosen as a displacement unit. It follows then that tahe ductility demand on components (1), (2) and (3) will be non-critical.

(6) Because the sequence of yielding is set and is independent of the

strength of the components, strength to components may be assigned in any arbitrary manner. This provides the designer with considerable freedom of choice. It can be exploited so as to improve overall structural performance, as well as to arrive at more economical solutions. Some examples are provided subsequently.

An unrecognised significant advantage of this approach is that it enables the designer to control the major sources of the detrimental effects of the torsional response of buildings. These are eccentricities with respect to the centre of the mass of the system. If desired, the strength to elements of a system may be assigned so that the centres of strength and mass will coincide [7].

A Re-definition of Stiffness

In the context of this study, stiffness relates the total lateral force to a corresponding horizontal displacement.

Component Stiffness

Idealised bi-linear strength-displacement relationships are presented in *Fig. 4*. In these possible post-yield stiffness have been ignored. It is evident that a realistic approximation of component stiffness is:

$$k_i = V_{ni}/\Delta_{yi} \quad (5)$$

that is, nominal strength/yield displacement.

The important implications of this definition are that:

- The nominal strength of the component is the choice of the designer. It must be known before stiffness can be quantified.
- Stiffness is proportional to strength!
- The nominal yield displacement of a component, as previously defined, depends on the geometry of the component and the relevant yield strain of the material used. For design purposes nominal yield displacements, Δ_{yi}, are independent of strength!

Element Stiffness

The stiffness of a lateral force-resisting element, comprising a number of components subjected to identical displacements, such as shown in *Fig. 2(a)*, may be defined by the superposition of the bi-linear response of the components. The stiffness of each component in *Fig. 4* was clearly defined by eq. (5). The superposition of the response of the four components leads to a non-linear transition from the elastic to the post-yield behaviour of the element. For design purposes this can again be replaced by bi-linear simulation, shown by dashed lines in *Fig. 4* as total response. Accordingly the stiffness of the element is:

$$k_e = \Sigma k_i \quad (6)$$

This in turn allows the nominal yield displacement of the element to be defined as

$$\Delta_{ye} = \Sigma V_{ni}/\Sigma k_i \quad (7)$$

The value of the element yield displacements so derived is shown distinctly in *Fig. 4*. Because the simultaneous yielding of components with different yield displacements, Δ_{yi}, is not possible, some components will yield at displacements less than that of the element, Δ_{ye}. Larger displacement will be required to develop the nominal strength of those components for wich $\Delta_{yi} > \Delta_{ye}$. The non-linear transition from elastic to plastic behaviour, shown by the full line marked «Total» in *Fig. 4*, illustrates this feature.

The above examples show the fallacy of the common assumption, that the element and system or global displacement ductility factor is the same as that specified in codes for appropriately detailed components. A displacement ductility capacity of 5 of the element depicted in *Fig. 4*, implies a ductility demand on component (4) of $\mu_{\Delta 4} = 5 \times 0.58/0.5 = 5.8$.

System Stiffness

As *Fig. 1* shows, a building system will comprise a set of parallel lateral force-resisting elements. The strength, yield displacement and stiffness of each element are determined, as described. The procedure applicable to elements may then be used to estimate the stiffness and yield displacement of the entire system, comprising of a number of elements.

The sole purpose of the system nominal yield displacement, this being a reference value, is to quantify the system displacement ductility. Therefore, any convenient value that is compatible with overall behaviour under the action of lateral forces, may be used. The nominal system yield displacement, Δ_{ys}, used here, is associated with the uniform translation of all its elements and the centre of mass. The definition is thus identical to that represented by eq. (7). In this definition, displacements due to torsional phenomena are not included. In seismic design, the influence of torsion on the displacement of the centre of mass may be considered to represent unwarranted analytical refinements.

Ductility Relationships

Once the nominal strengths and yield displacements, and hence the stiffness, of components or elements have been set, the relationships between different displacements and displacement ductilities, to be considered in the design, are readily established.

Components of Lateral Force-resisting Elements

Bi-linear responses of 4-component elements were presented in *Fig. 4*. It was stated that the displacement capacity of this element is controlled by that of its critical component (4) with the smallest yield displacement. The sum of component relative stiffness was found to be $\Sigma k_i = 1.72$. Equation (7) allows the yield displacement of the element in *Fig. 4* to be determined as $\Delta_{ye} = 1/1.72 = 0.58$ displacement units. Therefore, with the ultimate displacement limited to $5 \times 0.5 = 2.5$ units, the element displacement ductility demand must be limited to $\mu_\Delta \leq 2.5/0.58 = 4.31$.

It was emphasised that an entirely different assignment of strength to the four components of the element is also acceptable. Such a choice will affect the stiffness and hence yield displacement of the element. These may require greater restriction on the acceptable displacement ductility demand on the element. This is controlled by the strength-independent displacement capacity of component (4).

Lateral Force-resisting Elements of Building Systems

Similar limitations apply when the system displacement ductility is to be limited to ensure that the displacement ductility capacity of the critical element is not exceeded. However, in the consideration of the displacements of elements of a system, torsional effect need also to be accounted for.

Allocation of Strength

It was postulated that, irrespective of the way strengths were assigned to components, the sequence of yielding will depend only on the yield displacements of the components. As previously stated, for given material properties (ϵ_y), these are controlled by the geometry rather than the strength of the components. It was, therefore, concluded that, within rational limits, strength to components may be assigned in any arbitrary manner. The criterion to be satisfied is that the total strength of the element, i.e., the sum of the strengths of ist components, must be maintained. Two examples are presented here to show how arbitrary, yet astute, choices of strength allocation to components may lead to appealing structural solutions.

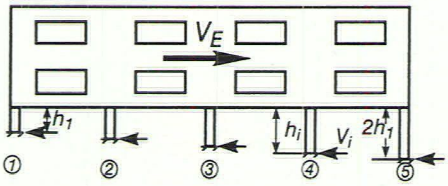


Fig. 5: A structure supported on columns with variable heights

A Structure Supported on Columns with Variable Heights

As *Fig. 5* shows, a two storey rigid block structure is supported on five columns of variable heights. The relative heigths of the columns (1) to (5), h_i, are 1.00, 1.25, 1.50, 1.75 and 2.00, respectively. The inertia force, V_E, needs to be transmitted by the 5 columns with identical cross sections. Because of identical sectional dimensions, the yield curvatures at both ends of all columns, ϕ_y, will be practically identical. Small differences in the values of yield displacements result from different axial compression loads on the columns. Consequently, the yield displacements will be approximately $\Delta_{yi} \approx \phi_y h_i^2/6$, i.e., proportional to the square of the column heights. Therefore, irrespective of flexural strength, the yield displacement of column (5) is approximately 4 times that of column (1).

Component strengths, yield displacements and hence stiffness, allow the non-linear shear force – lateral displacement relationships for two different elements, considered above, to be plotted. These are presented in *Fig. 6*. In the first case the column shear forces and hence relevant strenghts, are assigned in accordance with conventional $1/h_i^3$ proportionality. In the other case shear forces are made inversely proportional to column heights. For this case the bi-linear simulation of the five columns is also shown in *Fig. 6*. With the use of eq. (7), the bi-linear simulation allows the element yield displacements, Δ_{ye}, for each case, i.e., 1.364 and 1.667 displacement units, respectively, to be estimated. These are inversely proportional to element stiffness.

The critical component is column (1) with the smallest relative (unit) yield displacement. It is assumed that the displacement ductility demand on appropriately detailed columns should be limited to 6. Therefore, the displacement demand on the element at the ultimate limit state must be limited to the displacement capacity of column (1), i.e., $\Delta_u \leq 6 \times 1.0 = 6.0$. This implies then that the element displacement ductility demands, using the traditional and arbitrary assignment of column strengths, should be limited to $\mu_{\Delta c} \leq 6.0/1.364 = 4.40$ and $\mu_{\Delta a} \leq 6.0/1.667 = 3.60$, respectively.

The major points highlighted by this example are:

- The arbitrary distribution of the base shear force, V_E, among columns, relying on equal flexural strengths, enables the same detailing of the vertical reinforcement in all five columns.
- The extremely disproportionate shear demand, associated with the conventional allocation of component strengths, is eliminated.
- The reduction of the element stiffness by a factor of 1.364/1.667 = 0.82 is not likely to affect adversely dynamic response.
- The advantages of arbitrary strength allocation to components outweigh the disadvantage associated with the somewhat reduced energy dissipation capacity of the element. Note that post-yield stiffness, inevitably present, has not been considered in this comparison.
- The examples show that an allowance for a global displacement ductility capacity of 6, which may have been adopted for ductile frames, would grossly underestimate the ductility demand on the critical component, i.e. column (1). (For example $\mu_{\Delta 1 max} = 6 \times 1.667 = 10 >> 6$).
- The example demonstrates how the element displacement ductility demand should be limited to approximately 4, if the critical component is to be protected against excessive displacement demands. As a corollary, the study of the example structure reveals that it is not possible to utilize the ductility capacity of components with larger yield displacements. For example $\mu_{\Delta 5} = 6/4 = 1.5 << 6$. Such columns may thus be designed and detailed according to recommendations for limited ductility demands.
- Existing design procedures, based on traditional definitions of component stiffness, do not allow displacement ductility demands on constituent components with different geometric properties to be related to that imposed on the element.

A Structure Supported by Columns with Unequal Depth

This example element, presented in *Fig. 7*, is similar to that seen in *Fig. 5*. However, in this symmetrical variant the columns are of equal heights but have different depth, h_{ci}. The conventional stiffness proportional allocation

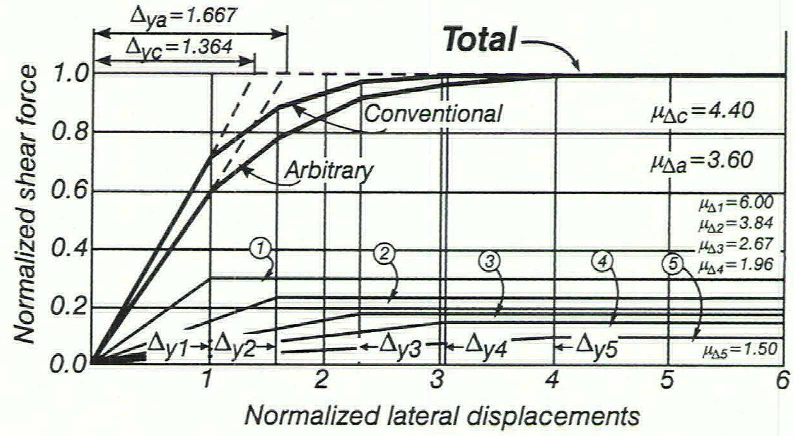


Fig. 6: Force-displacement modelling of the element shown in Fig. 5

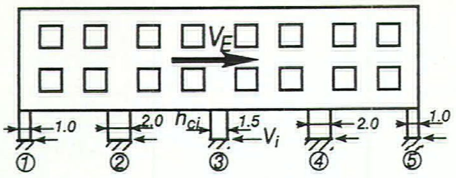


Fig. 7: A structure supported on columns with different depths

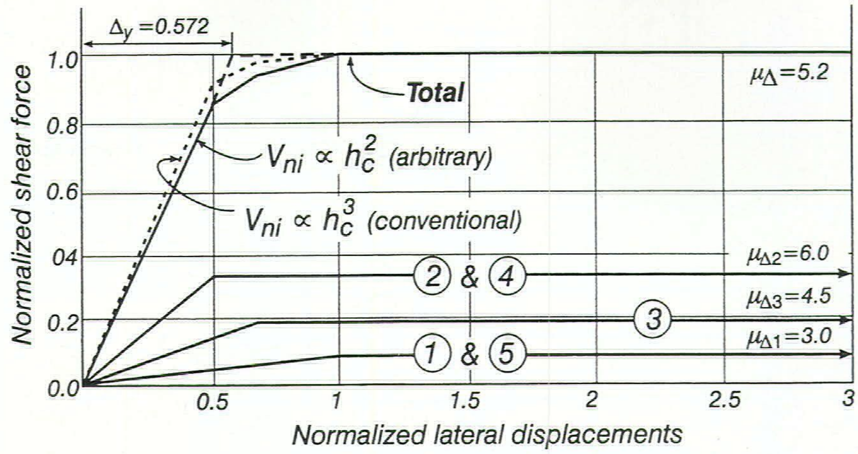


Fig. 8: Force-displacement modelling of the structure shown in Fig. 7

of shear strength would follow the principles described in connection with the model element shown in *Fig. 2*(a). Details of this are not given here.

The relative depths of columns (1), (2) and (3) are 1.0, 2.0 and 1.5, respectively. Correspondingly the relative yield displacements, being inversely proportional to the depth of the sections (eq. (4)), are $\Delta_{y1} = 1.000$, $\Delta_{y2} = 0.500$ and $\Delta_{y3} = 0.667$, respectively. The conventional assignment of (shear) strengths, in proportion to h_i^3, would result in these 3 columns having to resist 4.7%, 37.4% and 15.8% of the total base shear, V_E, respectively. The lateral force-displacement response of the 5-component element so designed, is shown by the dashed line in *Fig 8*.

Making use of the designer's freedom in choosing any arbitrary assignment of component strengths, it is decided to make column base shear strengths proportional to the square of the section depths. This technique leads to a well balanced design, whereby the ratio of the vertical reinforcement is approximately the same in all columns.

As the full lines in *Fig. 8* show, the strengths of the three types of columns would then correspond to 8.2%, 32.6% and 18.4% respectively, of the total base shear, V_E. The bi-linear idealization for each column component so designed is also shown in *Fig. 8*. It is seen that the difference between the overall responses, based on the conventional and arbitrary assignment of component strengths, is negligible. It should be noted that the yield displacements of the columns are the same in both cases. Assuming again that the displacement capacity of the columns is 6, the element displacement capacity is controlled by that of columns (2) and (4), i.e., $\mu_{\Delta i}\,\Delta_{yi} = 6 \times 0.5 = 3.0$ displacement units. Therefore, the displacement ductility demand on the five-column element should be limited to $\mu_{\Delta e} \leq 6.0/0.572 = 5.2$. The full displacement ductility capacity of the columns having yield displacements larger that 0.500 displacement units, cannot be utilized.

Conclusions

1. The recognition of the criteria of performance-based seismic design necessitates more attention to be given to realistic estimates of lateral force-induced structural deformations.
2. Improved techniques for the estimation of yield displacements are postulated. The latter depend only on material properties, such as limiting strains, and the geometry of components of elements of the structure. For design purposes generally yield displacements may be considered to be independent of the strength assigned to components or elements.
3. Because the sequence of the onset of yielding of components of a plastic mechanism is independent of their strength, within rational limits, strengths may be assigned to components in any way that suits the designer's intentions.
4. Re-defined stiffness, relating freely chosen strengths to strength-independent yield displacements, enables a more realistic assessment of the stiffness of elements or of a system to be made.
5. Clearly defined yield displacements of components enable displacement and displacement ductility demands on the system to related to the displacement ductility capacity of the critical components.

References

[1] PAULAY, T. *Seismic Design for Torsional Response of Ductile Buildings*, Bulletin of the NZ Society for Earthquake Engineering, Vol. 29, No. 3, September, pp. 178–198, 1996.

[2] PAULAY, T. *A Review of Code Provisions for Torsional Seismic Effects in Buildings*, Bulletin of the NZ Society for Earthquake Engineering, Vol. 30, No. 3, September, pp. 252–263, 1997.

[3] PAULAY, T. *Mechanism of Ductile Building Systems as affected by Torsion*, Proceedings NZ Society for Earthquake Engineering, Technical Conference, Wairakei, pp. 11–118, 1998.

[4] PRIESTLEY, M. J. N. *Myths and Fallacies in Earthquake Engineering – Conflicts Between Design and Reality*, American Concrete Institute (SP-157), Recent Development in Lateral Force Transfer in Buildings, pp. 231–25, 1995.

[5] PARK, R. and PAULAY, T. *Reinforced Concrete Structures*, John Wiley and Sons, New York, pp. 769, 1975.

[6] PRIESTLEY, M. J. N., and KOWALSKY, M. J. *Aspects of Drift and Ductility Capacity of Rectangular Structural Walls*, Bulletin of the NZ Society for Earthquake Engineering, Vol. 31, No. 2, pp. 73–85, 1997.

[7] PAULAY, T. *A Simple Displacement Comptability-based Design Strategy for Reinforced Concrete Buildings*, Proceedings of the 12th World Conference on Earthquake Engineering, Paper No. 0062, 2000.