FACULTE ENVIRONNEMENT NATUREL, ARCHITECTURAL ET CONSTRUIT ENAC
INSTITUT D'INGÉNIERIE CIVILE IIC
Laboratoire des Structures Métalliques Résilientes RESSLAB

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : http://resslab.epfl.ch
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

Exercise #9: Seismic design of bracing connections and other members

The steel frame building shown in Figure 1-2 has been designed with concentrically braced frames (CBFs) in the North-South (y-y) loading direction and steel moment resisting frames (MRFs) in the East-West (x-x) loading direction. Both frames have been designed for gravity and earthquake loading. The CBF design comprises an X-bracing configuration with welded bracing connections. The cross sections shown in the figure represent the final design of the steel CBF. The steel components (beams, columns, and braces) have been designed with S355J2 profile (i.e., $E = 210GPa$, $f_y = 355MPa$). The stability coefficient θ is less than 0.10 in all stories. The weight of each floor due to gravity (for all three floors) equals to $G = 7kN/m^2$.

The following questions should be answered:

1. Check if the maximum overstrength Ω from all the brace members does not differ from the minimum one by more than 25%.
2. Check if the diagonal bracing of the first story meets the requirements of normalized slenderness. For simplicity, assume that l_k is half the centerline length of the brace (i.e., $l_k = (3.0^2 + 2.0^2)^{0.5} = 3.61m$).
3. Check the stability of the first story column for interaction of axial load and biaxial bending. The steel column has pin ends in the y-y loading direction. The column is fixed at the base in the x-x loading direction. Assume that $M_{x,Ed} = 61kNm$ (bottom fixed end) and that $M_{x,Ed} = -78kNm$ (top end). You may assume that the buckling length of the column in the (sway permitted) MRF direction is $1.5L$ (i.e., L is the column length). Moreover, assume that the axial force due to earthquake loading in the x-x direction equals to zero (i.e., $N_{Ed,Ex} = 0$).
4. Calculate the action forces on the first-floor steel beam due to gravity and earthquake loading. Compute the bending, shear and axial force diagrams of the steel beam. Assume $k_v = k_\varphi = 1.0$ for your calculations.
5. Check the stability of the first-floor steel beam due to axial force and bending interaction. The steel beam is braced laterally every $l_b/4$ (1500mm) and it does not experience weak axis bending. Assume $k_v = k_\varphi = 1.0$.
6. Design and verify the welded bracing connections of the CBF including their gusset plate. The following considerations should be included:
    a. Calculate the force demand of each connection based on the axial resistance of each bracing member.
    b. Determine the weld length including the weld resistance.
    c. Design the gusset plate for both tensile and compressive loading.
    d. Develop a preliminary drawing for a typical bracing connection.

The axial force diagrams for the seismic loading of the steel CBF are shown in Figure 2-2.

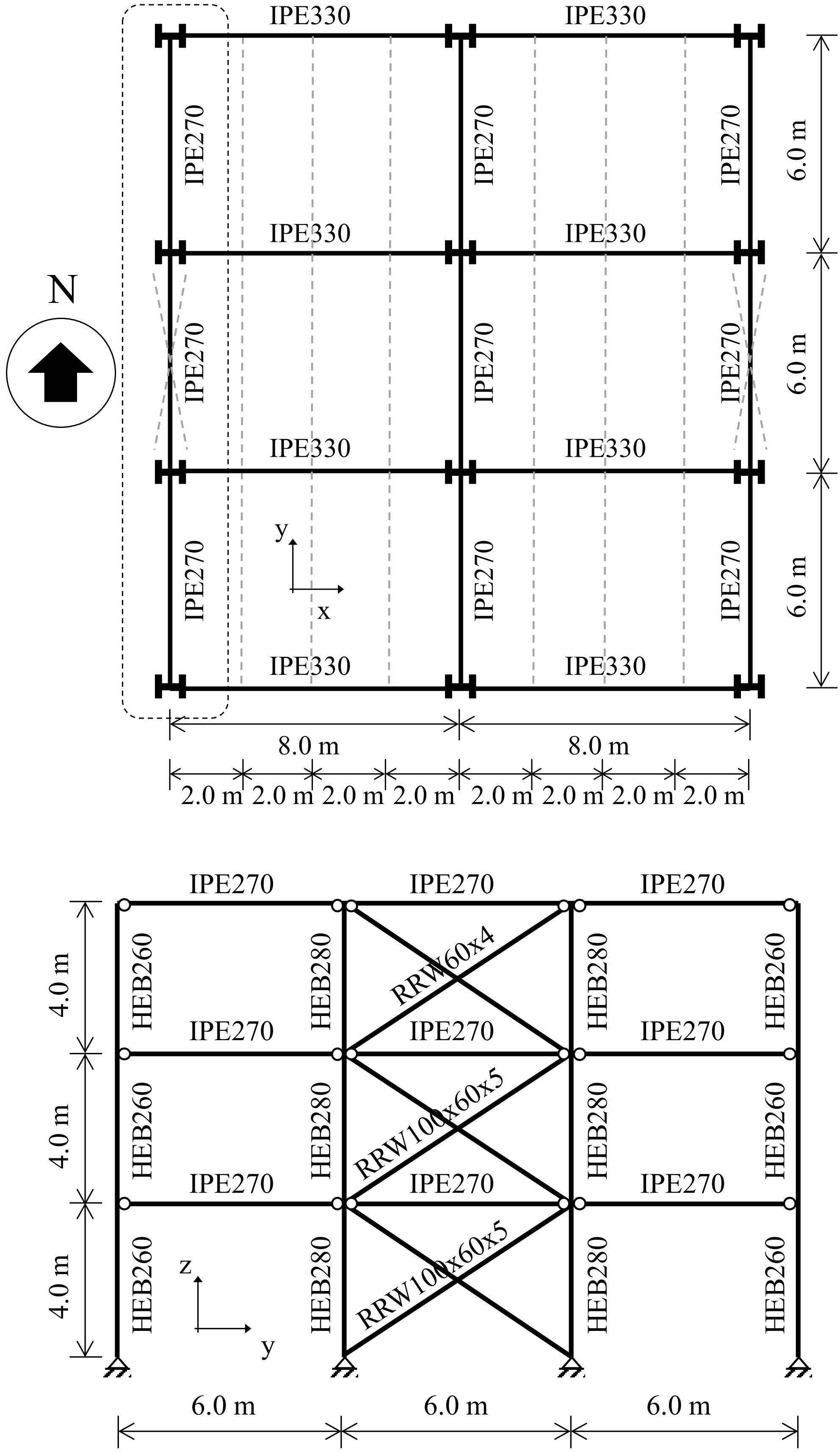


Figure 1-2. Final design of steel CBF – Plan view and elevation

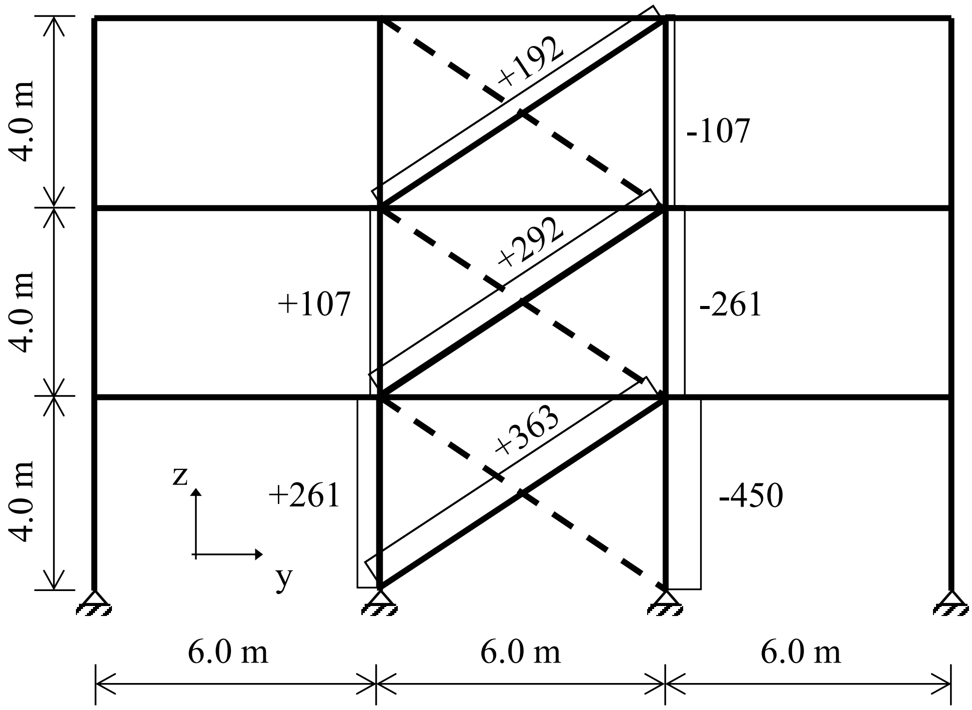
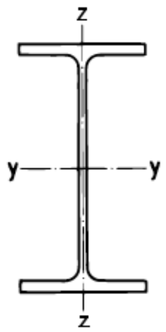


Figure 2-2. Axial force diagram due to seismic loading

IPE, PEA — IPE- und IPEA-Träger — *Profilés IPE et IPEA*



$A_v = A - 2bt_f + (t_w + 2r)\, t_f$

$A_w = (h - t_f) \cdot t_w$

$S_y = ½\, W_{ply}$

$S_z = ½\, W_{plz}$

$W_{ely} = \dfrac{I_y}{h/2}$

$\overline{W}_y = \dfrac{I_y}{(h-t_f)/2}$

$W_{elz} = \dfrac{I_z}{b/2}$

Maximale Lagerlängen / *Longueurs maximales en stock:*
h ≤ 180   18 m
h ≥ 200   24 m
EURONORM 19 – 57,
DIN 1025/5, ASTM A 6,
Werksnorm/*Norme d'usine*

○ Das Verfahren PP nach SIA 263 ist für dieses Profil aus S355 bei reiner Biegung (n = 0) nicht anwendbar!

* Auch in S355J0 oder S355J2 ab Schweizer Lager erhältlich.

○ *La méthode PP selon SIA 263 n'est pas applicable pour ce profilé en acier S355 en flexion simple (n = 0)!*

* *Livrable en S355J0 ou S355J2 du stock suisse.*

IPE	m	Statische Werte / *Valeurs statiques*												
	kg/m	A mm²	A_v mm²	A_w mm²	I_y mm⁴	W_{ely} mm³	$\overline{W}_y$ mm³	W_{ply} mm³	i_y mm	I_z mm⁴	W_{elz} mm³	W_{plz} mm³	i_z mm	$K = I_x$ mm⁴
					x 10⁶	x 10³	x 10³	x 10³		x 10⁶	x 10³	x 10³		x 10⁶
80*	6,0	764	358	284	0,801	20,0	21,4	23,2	32,4	0,085	3,69	5,82	10,5	0,0067
100*	8,1	1030	508	387	1,71	34,2	36,3	39,4	40,7	0,159	5,79	9,15	12,4	0,0115
120*	10,4	1320	631	500	3,18	53,0	55,9	60,7	49,0	0,277	8,65	13,6	14,5	0,0169
140*	12,9	1640	764	626	5,41	77,3	81,3	88,3	57,4	0,449	12,3	19,2	16,5	0,0240
160*	15,8	2010	966	763	8,69	109	114	124	65,8	0,683	16,7	26,1	18,4	0,0353
180*	18,8	2390	1125	912	13,2	146	154	166	74,2	1,01	22,2	34,6	20,5	0,0472
200*	22,4	2850	1400	1070	19,4	194	203	221	82,6	1,42	28,5	44,6	22,4	0,0685
220*	26,2	3340	1588	1240	27,7	252	263	285	91,1	2,05	37,3	58,1	24,8	0,0898
240*	30,7	3910	1914	1430	38,9	324	338	367	99,7	2,84	47,3	73,9	26,9	0,127
270*	36,1	4590	2214	1710	57,9	429	446	484	112	4,20	62,2	97,0	30,2	0,157
300*	42,2	5380	2568	2050	83,6	557	578	628	125	6,04	80,5	125	33,5	0,198
330*	49,1	6260	3081	2390	117,7	713	739	804	137	7,88	98,5	154	35,5	0,276
360*	57,1	7270	3514	2780	162,7	904	937	1020	150	10,4	123	191	37,9	0,371
400*	66,3	8450	4269	3320	231,3	1160	1200	1310	165	13,2	146	229	39,5	0,504
450*	77,6	9880	5085	4090	337,4	1500	1550	1700	185	16,8	176	276	41,2	0,661
500*	90,7	11600	5987	4940	482,0	1930	1990	2190	204	21,4	214	336	43,1	0,886
550	106	13400	7234	5910	671,2	2440	2520	2790	223	26,7	254	401	44,5	1,22
600	122	15600	8378	6970	920,8	3070	3170	3510	243	33,9	308	486	46,6	1,65
750 x	137	17500	9290	8460	1599	4250	4340	○ 4860	303	51,7	393	614	54,4	1,36
750 x	147	18700	10540	9720	1661	4410	4510	5110	298	52,9	399	631	53,1	1,57
750 x	173	22100	11640	10700	2058	5400	5560	6220	305	68,7	515	810	55,7	2,71
750 x	196	25100	12730	11600	2403	6240	6450	7170	310	81,8	610	959	57,1	4,06

IPE- und IPEA-Träger *Profilés IPE et IPEA* IPE, PEA

Die Profile PER, IPEo und IPEv sind im Walzprogramm einzelner Werke aufgeführt.
PEA 80 und PEA 100 sind ebenfalls normiert, aber kaum wirtschaftlich.

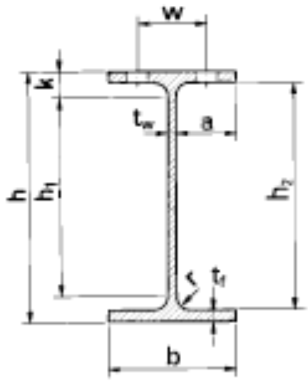
Les profilés PER, IPEo et IPEv figurent dans le programme de laminage de quelques aciéries.
Les PEA 80 et PEA 100, également normalisés, sont peu économiques.

Im allgemeinen nur ab Werk lieferbar. Mindestmengen und Termine beachten.

En général livrable d'usine uniquement. Tenir compte des quantités minimales et des délais.

Walztoleranzen siehe Seite 116

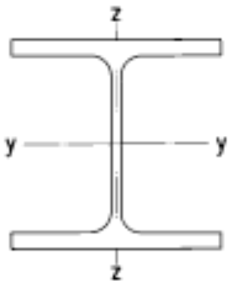
Tolérances de laminage voir p. 116



IPE	m	Profilmasse *Dimensions de la section*					Konstruktionsmasse *Dimensions de construction*						Oberfläche *Surface*		IPE
	kg/m	h mm	b mm	t_w mm	t_f mm	r mm	h_1 mm	k mm	a mm	h_2 mm	w mm	$Ø_{max}$	U_m m²/m	U_t m²/t	
80	**6,0**	80	46	3,8	5,2	5	60	10	21	70			0,328	54,8	**80**
100	**8,1**	100	55	4,1	5,7	7	74	13	25	89			0,400	49,5	**100**
120	**10,4**	120	64	4,4	6,3	7	92	14	29	107	36	M10	0,475	45,6	**120**
140	**12,9**	140	73	4,7	6,9	7	112	14	34	126	38	M10	0,551	42,6	**140**
160	**15,8**	160	82	5,0	7,4	9	126	17	38	145	44	M12	0,623	39,4	**160**
180	**18,8**	180	91	5,3	8,0	9	146	17	42	164	50	M12	0,698	37,1	**180**
200	**22,4**	200	100	5,6	8,5	12	158	21	47	183	56	M12	0,768	34,3	**200**
220	**26,2**	220	110	5,9	9,2	12	178	21	52	202	60	M16	0,848	32,4	**220**
240	**30,7**	240	120	6,2	9,8	15	190	25	56	220	68	M16	0,922	30,0	**240**
270	**36,1**	270	135	6,6	10,2	15	220	25	64	250	72	M20	1,04	28,8	**270**
300	**42,2**	300	150	7,1	10,7	15	248	26	71	279	80	M20	1,16	27,5	**300**
330	**49,1**	330	160	7,5	11,5	18	270	30	76	307	86	M24	1,25	25,5	**330**
360	**57,1**	360	170	8,0	12,7	18	298	31	81	335	90	M24	1,35	23,6	**360**
400	**66,3**	400	180	8,6	13,5	21	330	35	85	373	96	M27	1,47	22,2	**400**
450	**77,6**	450	190	9,4	14,6	21	378	36	90	421	106	M27	1,61	20,7	**450**
500	**90,7**	500	200	10,2	16,0	21	426	37	94	468	110	M27	1,74	19,2	**500**
550	**106**	550	210	11,1	17,2	24	468	41	99	516	120	M27	1,88	17,7	**550**
600	**122**	600	220	12,0	19,0	24	514	43	104	562	120	M27	2,02	16,6	**600**
750 x 137		753	263	11,5	17,0	17	685	34	126	719	120	M27	2,51	18,3	**750x137**
750 x 147		753	265	13,2	17,0	17	685	34	126	719	120	M27	2,51	17,1	**750x147**
750 x 173		762	267	14,4	21,6	17	685	39	126	719	120	M27	2,53	14,6	**750x173**
750 x 196		770	268	15,6	25,4	17	685	42	126	719	120	M27	2,55	13,0	**750x196**

HEB

Breitflanschträger HEB — *Profilés à larges ailes* HEB



$A_v = A - 2bt_f + (t_w + 2r)\, t_f$

$A_w = (h - t_f) \cdot t_w$

$S_y = ½\, W_{ply}$

$S_z = ½\, W_{plz}$

$$W_{ely} = \frac{I_y}{h/2}$$

$$\overline{W}_y = \frac{I_y}{(h - t_f)/2}$$

$$W_{elz} = \frac{I_z}{b/2}$$

*Auch in S355J0 oder S355J2 ab Schweizer Lager erhältlich

Maximale Lagerlängen / *Longueurs maximales en stock:*
h ≤ 180 18 m
h ≥ 200 24 m

EURONORM 53 – 62, DIN 1025/2

Andere Bezeichnungen / *Autres désignations*: DIN, IPB

**Livrable en S355J0 ou S355J2 du stock suisse*

HEB	m	Statische Werte / *Valeurs statiques*												
	kg/m	A mm²	A_v mm²	A_w mm²	I_y mm⁴	W_{ely} mm³	$\overline{W}_y$ mm³	W_{ply} mm³	i_y mm	I_z mm⁴	W_{elz} mm³	W_{plz} mm³	i_z mm	$K - I_x$ mm⁴
					x 10⁶	x 10³	x 10³	x 10³		x 10⁶	x 10³	x 10³		x 10⁶
100*	**20,4**	2600	904	540	4,50	89,9	100	104	41,6	1,67	33,5	51,4	25,3	0,0931
120*	**26,7**	3400	1096	708	8,64	144	158	165	50,4	3,18	52,9	81,0	30,6	0,139
140*	**33,7**	4300	1308	896	15,1	216	236	245	59,3	5,50	78,5	120	35,8	0,202
160*	**42,6**	5430	1759	1180	24,9	311	339	354	67,8	8,89	111	170	40,5	0,312
180*	**51,2**	6530	2024	1410	38,3	426	461	481	76,6	13,6	151	231	45,7	0,422
200*	**61,3**	7810	2483	1660	57,0	570	616	643	85,4	20,0	200	306	50,7	0,596
220*	**71,5**	9100	2792	1940	80,9	736	793	827	94,3	28,4	258	394	55,9	0,770
240*	**83,2**	10600	3323	2230	112,6	938	1010	1050	103	39,2	327	498	60,8	1,04
260*	**93,0**	11800	3759	2420	149,2	1150	1230	1280	112	51,3	395	602	65,8	1,26
280*	**103**	13100	4109	2750	192,7	1380	1470	1530	121	65,9	471	718	70,9	1,45
300*	**117**	14900	4743	3090	251,7	1680	1790	1870	130	85,6	571	870	75,8	1,87
320*	**127**	16100	5177	3440	308,2	1930	2060	2150	138	92,4	616	939	75,7	2,29
340*	**134**	17100	5609	3820	366,6	2160	2300	2410	146	96,9	646	986	75,3	2,62
360*	**142**	18100	6060	4220	431,9	2400	2560	2680	155	101	676	1030	74,9	2,98
400*	**155**	19800	6998	5080	576,8	2880	3070	3230	171	108	721	1100	74,0	3,61
450*	**171**	21800	7966	5940	798,9	3550	3770	3980	191	117	781	1200	73,3	4,49
500	**187**	23900	8982	6840	1072	4290	4540	4820	212	126	842	1290	72,7	5,50
550	**199**	25400	10010	7820	1367	4970	5250	5590	232	131	872	1340	71,7	6,12
600	**212**	27000	11080	8840	1710	5700	6000	6420	252	135	902	1390	70,8	6,80
650	**225**	28600	12200	9900	2106	6480	6800	7320	271	140	932	1440	69,9	7,52
700	**241**	30600	13710	11400	2569	7340	7690	8330	290	144	963	1490	68,7	8,42
800	**262**	33400	16180	13400	3591	8980	9360	10230	328	149	994	1550	66,8	9,62
900	291	37100	18880	16000	4941	10980	11400	12580	365	158	1050	1660	65,3	11,5
1000	314	40000	21250	18300	6447	12890	13400	14860	401	163	1090	1720	63,8	12,7

Breitflanschträger HEB — *Profilés à larges ailes HEB* — HEB

Anstelle des nicht mehr gewalzten Profils HEB 1100 können HL-Profile verwendet werden, siehe Seiten 40/41.

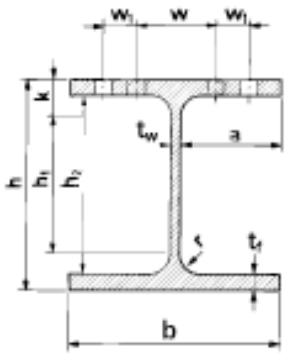
Au lieu du profilé HEB 1100 qui n'est plus laminé, on utilisera des profilés HL (voir pages 40/41).

w_1 mit $Ø_{max}$ nur für versetzte Schrauben.

w_1 avec $Ø_{max}$ seulement pour boulons décalés.

Walztoleranzen siehe Seite 116

Tolérances de laminage voir page 116



HEB	m	Profilmasse *Dimensions de la section*					Konstruktionsmasse *Dimensions de construction*							Oberfläche *Surface*		HEB
	kg/m	h mm	b mm	t_w mm	t_f mm	r mm	h_1 mm	k mm	a mm	h_2 mm	w mm	w_1 mm	$Ø_{max}$	U_m m^2/m	U_t m^2/t	
100	**20,4**	100	100	6	10	12	56	22	47	80	56		M12	0,567	27,8	**100**
120	**26,7**	120	120	6,5	11	12	74	23	56	98	66		M16	0,686	25,7	**120**
140	**33,7**	140	140	7	12	12	92	24	66	116	76		M20	0,805	23,9	**140**
160	**42,6**	160	160	8	13	15	104	28	76	134	86		M20	0,918	21,5	**160**
180	**51,2**	180	180	8,5	14	15	122	29	85	152	100		M24	1,04	20,3	**180**
200	**61,3**	200	200	9	15	18	134	33	95	170	110		M24	1,15	18,8	**200**
220	**71,5**	220	220	9,5	16	18	152	34	105	188	120		M24	1,27	17,8	**220**
240	**83,2**	240	240	10	17	21	164	38	115	206	96	35	M24	1,38	16,6	**240**
260	**93,0**	260	260	10	17,5	24	176	42	125	225	106	40	M24	1,50	16,1	**260**
280	**103**	280	280	10,5	18	24	196	42	134	244	110	45	M24	1,62	15,7	**280**
300	**117**	300	300	11	19	27	208	46	144	262	120	45	M27	1,73	14,8	**300**
320	**127**	320	300	11,5	20,5	27	224	48	144	279	120	45	M27	1,77	13,9	**320**
340	**134**	340	300	12	21,5	27	242	49	144	297	120	45	M27	1,81	13,5	**340**
360	**142**	360	300	12,5	22,5	27	260	50	143	315	120	45	M27	1,85	13,0	**360**
400	**155**	400	300	13,5	24	27	298	51	143	352	120	45	M27	1,93	12,4	**400**
450	**171**	450	300	14	26	27	344	53	143	398	120	45	M27	2,03	11,9	**450**
500	**187**	500	300	14,5	28	27	390	55	142	444	120	45	M27	2,12	11,3	**500**
550	**199**	550	300	15	29	27	438	56	142	492	120	45	M27	2,22	11,2	**550**
600	**212**	600	300	15,5	30	27	486	57	142	540	120	45	M27	2,32	11,0	**600**
650	**225**	650	300	16	31	27	534	58	142	588	120	45	M27	2,42	10,8	**650**
700	**241**	700	300	17	32	27	582	59	141	636	126	45	M27	2,52	10,5	**700**
800	**262**	800	300	17,5	33	30	674	63	141	734	130	40	M27	2,71	10,4	**800**
900	291	900	300	18,5	35	30	770	65	140	830	130	40	M27	2,91	10,0	900
1000	314	1000	300	19	36	30	868	66	140	928	130	40	M27	3,11	9,9	1000

Suggested Solution

Question 1

The overstrength in CBFs is calculated from the brace tensile capacity-to-demand ratios because the braces act as seismic fuses. Since they carry just axial load, the overstrength factor is calculated as follows:

$\Omega_i = N_{pl,Rd,i}/N_{Ed,i}$

Where: $N_{Ed,i} = N_{Ed,G,i} + N_{Ed,E,i}$

$N_{pl,Rd,i} = \frac{f_y \cdot A_i}{\gamma_{M0}}$

However, for the given x-diagonal steel braces we do not consider the influence of axial load due to gravity because this load is picked by the frame (Beams and columns without braces). Only the seismic load is considered for the brace design, according to Figure 2; Therefore, $N_{Ed,i} = N_{Ed,E,i}$ in this case.

The computation of the overstrength is resumed in the table below:

Story	Section	A [mm^2]	$N_{pl,Rd}$ [kN]	N_{Ed} [kN]	Ω [-]	$(\Omega$-$\Omega_{min})/\Omega_{min}$
3	RRW60x4	879	312.05	192	1.63	13%
2	RRW100x60x5	1473	522.92	292	1.79	24%
1	RRW100x60x5	1473	522.92	363	1.44	0%

The overstrength difference does not exceed 25% for the 2nd and 3rd floor. This means that plasticity in the braces is expected to happen in a uniform manner.

For the calculations in the remaining questions the minimum overstrength factor should be used, $\Omega = \Omega_{min} = 1.44$ (similar concept with steel MRFs)

Question 2

Requirements for maximum slenderness:

The braces in compression have been conservatively neglected from the analysis by assuming a tension-only system. For the x-bracing system to behave in a desirable way, $1.3 \leq \bar{\lambda} \leq 2.0$ should be met.

The braces are connected in their mid-length. Their buckling length can be approximated as 50% of their total length, resulting in $l_k = \sqrt{3.0^2 + 2.0^2} = 3.61m$ for both in and out of plane buckling.

The steel brace is an RRW100x60x5 (first story) therefore,

$$N_{cr} = \pi^2 \cdot E \cdot \frac{I_z}{l_k^2} = 3.14^2 \cdot 210 \cdot \frac{0.836 \cdot 10^6}{3610^2} = 132.96kN$$

Weak axis is critical, therefore: the check should be performed only in this axis for both upper and lower normalized slenderness limit.

Moreover, $A \cdot f_y = 1473 \cdot 0.355 = 522.92kN$

Therefore,

$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{cr}}} = \sqrt{\frac{522.92}{132.96}} = 1.98$

Consequently, $1.3 \leq \bar{\lambda} \leq 2.0$ and the steel brace meets the requirements for normalized slenderness.

Question 3

We need to compute the column axial load demand (consider using absolute values of loads since the seismic action is a cyclic load):

$$N_{Ed,G} = 3 \cdot (6 \cdot 4) \cdot 7 = 504kN$$

$$N_{Ed} = N_{Ed,G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot N_{Ed,E} = 504 + 1.1 \cdot 1.25 \cdot 1.44 \cdot 450 = 1395kN$$

Note here that, since we have the axial force acting in the columns, we can use this formula, according to the code. For the next question that the axial forces in the beams are not given, we will proceed to an assumption.

Buckling resistance of the steel column:

Strong axis (MRF direction)

$$l_k = 1.5L = 1.5 \cdot 4000 = 6000mm$$

$\frac{h}{b} = 1.0 < 1.2$ and $tf < 100mm$ therefore the buckling coefficient is $\alpha = 0.34$ (y-y axis)

$$N_{cr,y} = \frac{\pi^2 \cdot E \cdot I_y}{l_k^2} = 3.14^2 \cdot 210 \cdot \frac{192.7 \cdot 10^6}{6000^2} = 11083kN$$

Weak axis (CBF direction)

$l_k = L = 4000mm$ (column is pinned at the bottom and the beams intersecting at the top are pinned to the column in the weak-axis)

$\frac{h}{b} = 1.0 < 1.2$ and $tf < 100mm$ therefore the buckling coefficient is $\alpha = 0.49$ (z-z axis)

$$N_{cr,z} = \frac{\pi^2 \cdot E \cdot I_z}{l_k^2} = 3.14^2 \cdot 210 \cdot \frac{65.9 \cdot 10^6}{4000^2} = 8528kN$$

$$\bar{\lambda}_y = \sqrt{\frac{A \cdot f_y}{N_{cr,y}}} = \sqrt{\frac{13100 \cdot 0.355}{11083}} = 0.65$$

$$\bar{\lambda}_z = \sqrt{\frac{A \cdot f_y}{N_{cr,z}}} = \sqrt{\frac{13100 \cdot 0.355}{8528}} = 0.74$$

$$\Phi_y = 0.5 \cdot \left(1 + \alpha \cdot (\bar{\lambda}_y - 0.2) + \bar{\lambda}_y^{\ 2}\right) = 0.5 \cdot (1 + 0.34 \cdot (0.65 - 0.2) + 0.65^2) = 0.79$$

$$\Phi_z = 0.5 \cdot \left(1 + \alpha \cdot \left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z^{\ 2}\right) = 0.5 \cdot (1 + 0.49 \cdot (0.74 - 0.2) + 0.74^2) = 0.90$$

$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \bar{\lambda}_y^{\ 2}}} = \frac{1}{0.79 + \sqrt{0.79^2 - 0.65^2}} = 0.81$$

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^{\ 2}}} = \frac{1}{0.90 + \sqrt{0.90^2 - 0.74^2}} = 0.70$$

Therefore,

$N_{b,y,Rd} = \chi_y \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 0.81 \cdot 13100 \cdot \frac{0.355}{1.05} = 3588kN > 1395kN$ (check is ok)
$N_{b,z,Rd} = \chi_z \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 0.70 \cdot 13100 \cdot \frac{0.355}{1.05} = 3100kN > 1395kN$ (check is ok)

Axial load –flexure interaction:

h/b = 280/280 = 1.0 < 2; therefore, the buckling curve to be used is "*a*" (i.e., $a_{LT} = 0.21$), according to EN 1993-1-1.

Plastic bending resistance with respect to y-y axis

$$M_{pl,y,Rd} = W_{pl,y} \cdot \frac{f_y}{\gamma_{M0}} = 1530 \cdot 10^3 \cdot 0.355/1.00 \cong 543.2kNm$$

Plastic bending resistance with respect to z-z axis

$$M_{pl,z,Rd} = W_{pl,z} \cdot \frac{f_y}{\gamma_{M0}} = 718 \cdot 10^3 \cdot 0.355/1.00 \cong 254.9kNm$$

Computation of critical moment:

$z_g = 0$ (cross section is symmetric and loads are passing through the cross-section shear center).

$M_{y,Ed,top} = -78kN - m$
$M_{y,Ed,bottom} = 61kN - m$

therefore, $k = \frac{61}{78} = 0.78$

The steel column is fixed at the base in the y-y direction; however, conservatively, we assume that the warping constant is $k_v = 1.0$

From, $k_v = 1.0$, $k_\varphi = 1.0$ (conservative assumption), $k = 0.5$, $C_1 = 2.75 > 2.3, C_1 = 2.3$

L = 4000mm

Shear modulus: $G = \frac{E}{2 \cdot (1+v)} = 80.8\text{kN/mm}^2$
Computation of torsional and warping constants:

$$K = \frac{2 \cdot b \cdot t_f^3 + (h - t_f) \cdot t_w^3}{3} = \frac{2 \cdot 280 \cdot 18^3 + (280 - 18) \cdot 10.5^3}{3} = 1.19x10^6 mm^4$$

$$I_\omega = \frac{t_f \cdot (h - t_f)^2 \cdot b^3}{24} = \frac{18 \cdot (280 - 18)^2 \cdot 280^3}{24} = 1.13x10^{12} mm^6$$

Therefore, the computation of M_{cr} is as follows:

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{k_v k_\varphi (L_D)^2} \cdot \left(\frac{I_w}{I_z} \cdot \left(\frac{(k_\varphi \cdot L_D)^2 \cdot G \cdot K}{\pi^2 \cdot E \cdot I_\omega} + 1\right)\right)^{0.5}$$
$$= 2.3 \cdot \frac{\pi^2 \cdot 210 \cdot 65.9 \cdot 10^6}{1.0 \cdot 1.0 \cdot (4000)^2} \cdot \left(\frac{1.13 \cdot 10^{12}}{65.9 \cdot 10^6} \cdot \left(\frac{(1.0 \cdot 4000)^2 \cdot 80.8 \cdot 1.19 \cdot 10^6}{\pi^2 \cdot 210 \cdot 1.13 \cdot 10^{12}} + 1\right)\right)^{0.5} \sim 3309.2 kNm$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} \cdot f_y}{M_{cr}}} = \sqrt{\frac{543.2}{3309.2}} = 0.41 > 0.40$$

Therefore, the column bending resistance should be reduced due to lateral torsional buckling (i.e., $\chi_{LT} \neq 1.0$). Note here that if k_v, k_φ would not have been assumed as 1, this reduction would have been avoided.

$$\Phi_{LT} = 0.5 \cdot \left(1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2\right) = 0.5 \cdot (1 + 0.21 \cdot (0.41 - 0.2) + 0.41^2)$$
$$= 0.60$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.60 + \sqrt{0.60^2 - 0.41^2}} = 0.95$$

Strong Axis Interaction: (Note that $M_{z,Ed} = 0$ because the column is pinned in the CBF direction):

$$\frac{N_{Ed}}{\chi_z \cdot A \cdot \frac{f_y}{\gamma_{M1}}} + \frac{\omega_y}{1 - \frac{N_{Ed}}{N_{y,cr}}} \cdot \frac{M_{y,Ed}}{\chi_{LT} \frac{M_{pl,y,Rd}}{\gamma_{M1}}} \leq 1$$

To compute ω_y you should consider the moment sign in this case such that the moment gradient can reduce the interaction due to bending if the member is in double curvature. Therefore,

$$\omega_y = 0.6 + 0.4 \cdot \left(-\frac{61}{78}\right) = 0.29 < 0.40; therefore, \omega_y = 0.40$$

$$\frac{N_{Ed}}{\chi_z \cdot A \cdot \frac{f_y}{\gamma_{M1}}} + \frac{\omega_y}{1 - \frac{N_{Ed}}{N_{y,cr}}} \cdot \frac{M_{y,Ed}}{\chi_{LT} \frac{M_{pl,y,Rd}}{\gamma_{M1}}} = \frac{1395}{3100} + \frac{0.40}{1 - \frac{1395}{11083}} \cdot \frac{78}{0.95 \cdot 517.3} = 0.45 + 0.07 = 0.52$$

Therefore, the column satisfies all the checks for interaction of axial load and bending.

Question 4

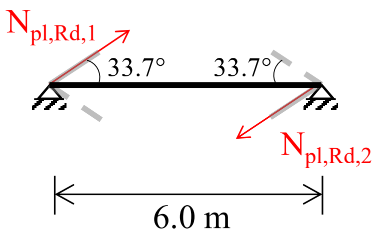
We need to estimate the axial load demand due to the seismic action in the steel beam. We will do this approximately without the use of a structural analysis program. However, because of this reason, we will safely estimate the axial load by using the plastic resistance of the steel brace. This involves a number of steps.

Step 1: Treat the beam as simple supported because its connections at both ends do not carry moments.

Angle, $\alpha = atan(4.0/6.0) = 0.59 rad\ (33.7°)$

Step 2: Seismic action

For the seismic action, the axial forces that should be considered in the bracing system that is intersecting to the beam of the first floor are as follows:

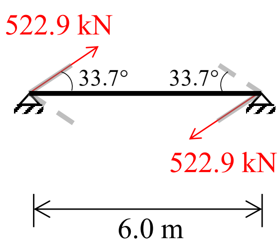


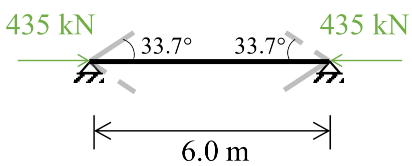
Note here that we assume that the braces in compression do not contribute to the frame resistance. This is a conservative assumption, since they reduce by 30% the axial force in the beams (assuming that their resistance is $0.3N_{pl,Rd}$). Whereas, if a similar approach is followed for the columns, then the contribution of the braces in compression should not be neglected, since they increase the axial compressive force demand in the columns.

As such,

$N_{pl,Rd,1} = N_{pl,Rd,2} = N_{pl,Rd} = 522.9\ kN$ (brace is the same size in stories 1 and 2)

Step 3: Reactions:





It should be noted that the vertical components of the bracing system axial forces are directly taken by the columns; thus, there are no shear or moment demands in the beam due to the seismic action.

Step 4: Gravity loading

The simple supported beams are loaded uniformly by

$g = G \cdot l_{x,eff} = 7 \cdot 1 = 7kN/m$

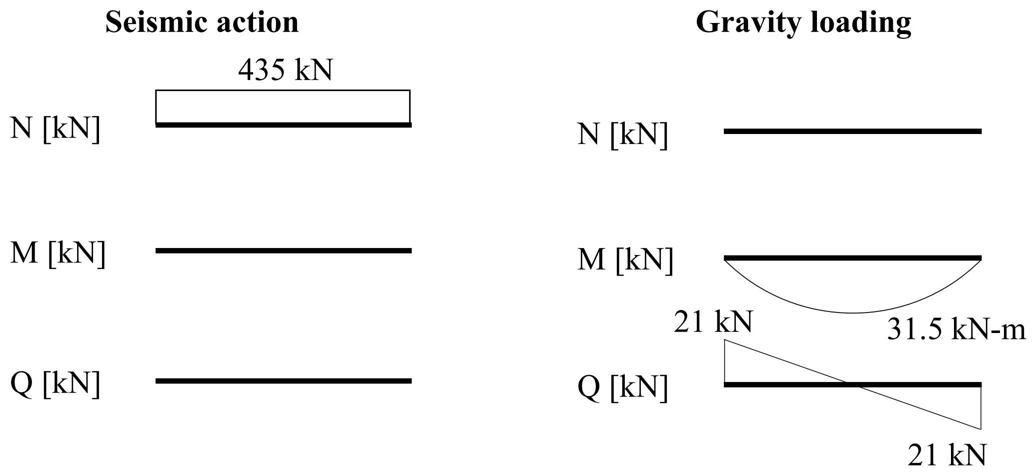
Consequently, the moment in the centre of the beam equals to

$M_{max} = g \cdot \frac{{l_b}^2}{8} = 7 \cdot \frac{6^2}{8} = 31.5kN - m$

Moreover, the maximum shear force in the beam ends equals to

$V_{max} = g \cdot \frac{l_b}{2} = 7 \cdot \frac{6}{2} = 21kN$

Step 5: Internal force diagrams for the given actions



Question 5

The beam size is an IPE270:

$$V_{pl,Rd} = \frac{A_v \cdot f_y}{\sqrt{3} \cdot \gamma_{M0}} = \frac{2214 \cdot 0.355}{\sqrt{3} \cdot 1.00} = 453.8kN$$

Shear demand:

Ω = 1.44 (we use the smallest Ω)

$V_{Ed} = V_{Ed,G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot V_{Ed,E} = 21 + 0 = 21\ kN < 453.8kN = V_{pl,Rd}$

Bucking resistance of the steel beam (y-axis buckling):

$l_k = L = 6000mm$

$\frac{h}{b} = \frac{270}{135} = 2 > 1.2$ and $tf = 10.2mm < 100mm$ therefore the imperfection curve is a and the imperfection factor is $\alpha = 0.21$ (y-y axis)

$$N_{cr,y} = \frac{\pi^2 \cdot E \cdot I_y}{l_k^2} = 3.14^2 \cdot 210 \cdot \frac{57.9 \cdot 10^6}{6000^2} = 3330.1kN$$

$$\bar{\lambda}_y = \sqrt{\frac{A \cdot f_y}{N_{cr,y}}} = \sqrt{\frac{4590 \cdot 0.355}{3330.1}} = 0.7$$

$$\Phi_y = 0.5 \cdot \left(1 + \alpha \cdot \left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y{}^2\right) = 0.5 \cdot (1 + 0.21 \cdot (0.7 - 0.2) + 0.7^2) = 0.8$$

$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \bar{\lambda}_y{}^2}} = \frac{1}{0.8 + \sqrt{0.8^2 - 0.7^2}} = 0.84$$

Bucking resistance of the steel beam (z-axis buckling):

$l_k = L = 1500mm$ (the steel beam is braced laterally every $l_b/4$)

$\frac{h}{b} = \frac{270}{135} = 2 > 1.2$ and $tf = 10.2mm < 100mm$ therefore the imperfection curve is b and the imperfection factor is $\alpha = 0.34$ (z-z axis)

$$N_{cr,z} = \frac{\pi^2 \cdot E \cdot I_z}{l_k^2} = 3.14^2 \cdot 210 \cdot \frac{4.2 \cdot 10^6}{1500^2} = 3865.0kN$$

$$\bar{\lambda}_z = \sqrt{\frac{A \cdot f_y}{N_{cr,z}}} = \sqrt{\frac{4590 \cdot 0.355}{3865.0}} = 0.65$$

$$\Phi_z = 0.5 \cdot \left(1 + \alpha \cdot \left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z{}^2\right) = 0.5 \cdot (1 + 0.34 \cdot (0.65 - 0.2) + 0.65^2) = 0.79$$

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z{}^2}} = \frac{1}{0.79 + \sqrt{0.79^2 - 0.65^2}} = 0.81$$

Therefore, z-axis buckling controls and
$N_{Ed} = N_{Ed,G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot N_{Ed,E} = 0 + 1.1 \cdot 1.25 \cdot 1.44 \cdot 435 = 861.3\ kN$

Note: If you do explicit structural analysis with a software without the simplifications we did in Question 4, then you can directly use as $N_{Ed,E}$ what the structural analysis program provides for the steel beam. Herein, we conservatively use as an axial demand due to the seismic action for the steel beam what we estimated from $N_{pl,Rd}$ of the steel brace (see Question 4).

$N_{b,z,Rd} = \chi_z \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 0.81 \cdot 4590 \cdot \frac{0.355}{1.05} = 1257.0kN > 861.3kN$ (check is ok)

Axial load –flexure interaction:

The beam in its weak axis does not experience any bending; in the strong axis it experiences $M_{Ed} = M_{Ed,G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot M_{Ed,E} = 31.5 + 0 = 31.5\ kN$ at its center.
Therefore, we should check for axial load – strong axis bending interaction.

h/b = 270/135 = 2; therefore the buckling curve to be used is "*a*" (i.e., $a_{LT} = 0.21$) , according to EC3.

Plastic bending resistance with respect to y-y axis

$$M_{pl,y,Rd} = W_{pl,y} \cdot \frac{f_y}{\gamma_{M0}} = 484 \cdot 10^3 \cdot \frac{0.355}{1.00} = 171.8kNm$$

Computation of critical moment:

$z_g = 0$ (cross section is symmetric – assume that loads are passing through the cross-section shear center).

By checking the most critical part of the beam and assuming uniform loading in this part, we have $C_1 = 1.0$ (conservative assumption)
The steel beam is connected to the column with a shear connection; therefore, $k_v = k_\varphi = 1.0$.

L = 1500mm

Shear modulus: $G = \frac{E}{2\cdot(1+v)} = 80.8\text{kN/mm}^2$

Computation of torsional constant:

$$K = \frac{2 \cdot b \cdot t_f^3 + (h - t_f) \cdot t_w^3}{3} = \frac{2 \cdot 135 \cdot 10.2^3 + (270 - 10.2) \cdot 6.6^3}{3} = 0.12x10^6 mm^4$$

$$I_w = \frac{t_f \cdot (h - t_f)^2 \cdot b^3}{24} = \frac{10.2 \cdot (270 - 10.2)^2 \cdot 135^3}{24} = 7.06x10^{10} mm^6$$

Therefore, the computation of M_{cr} is as follows:

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{k_v k_\varphi (L_D)^2} \cdot \left(\frac{I_w}{I_z} \cdot \left(\frac{(k_\varphi \cdot L_D)^2 \cdot G \cdot K}{\pi^2 \cdot E \cdot I_\omega} + 1 \right) \right)^{0.5}$$
$$= 1.00 \cdot \frac{\pi^2 \cdot 210 \cdot 4.2 \cdot 10^6}{1.0 \cdot 1.0 \cdot (1500)^2} \cdot \left(\frac{7.06 \cdot 10^{10}}{4.2 \cdot 10^6} \cdot \left(\frac{(1.0 \cdot 1500)^2 \cdot 80.8 \cdot 0.12 \cdot 10^6}{\pi^2 \cdot 210 \cdot 7.06 \cdot 10^{10}} + 1 \right) \right)^{0.5} = 537.7kNm$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} \cdot f_y}{M_{cr}}} = \sqrt{\frac{171.8}{537.73}} = 0.57 > 0.40$$

$$\Phi_{LT} = 0.5 \cdot \left(1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^{\ 2}\right) = 0.5 \cdot (1 + 0.21 \cdot (0.57 - 0.2) + 0.57^2) = 0.70$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^{\ 2}}} = \frac{1}{0.70 + \sqrt{0.70^2 - 0.57^2}} = 0.90$$

Reduction of bending due to shear-bending interaction:
$V_{Ed} = 21\ kN < 0.5 \cdot 453.8 kN$
Therefore, no reduction due to shear is required.

Strong Axis Interaction: (Note that $M_{z,Ed} = 0$)

$$\frac{N_{Ed}}{\chi_z \cdot A \cdot \frac{f_y}{\gamma_{M1}}} + \frac{\omega_y}{1 - \frac{N_{Ed}}{N_{y,cr}}} \cdot \frac{M_{y,Ed}}{\chi_{LT} \frac{M_{pl,y,Rd}}{\gamma_{M1}}} \leq 1$$

$\omega_y = 1$, since uniform moment diagram is assumed in the mid spans on the beam

$$\frac{N_{Ed}}{\chi_z \cdot A \cdot \frac{f_y}{\gamma_{M1}}} + \frac{\omega_y}{1 - \frac{N_{Ed}}{N_{y,cr}}} \cdot \frac{M_{y,Ed}}{\chi_{LT} \frac{M_{pl,y,Rd}}{\gamma_{M1}}} = \frac{861}{1257} + \frac{1.0}{1 - \frac{861}{3330.1}} \cdot \frac{31.5}{0.9 \cdot 163.6} = 0.68 + 0.29 = 0.97 < 1$$

Therefore, the beam satisfies all the checks for interaction of axial load and bending.

Question 6

Step 1: Compute, $R_d = 1.1\, \gamma_{ov}\, N_{pl,brace}$; where $N_{pl,brace}$ is calculated in the first question.

For the first and second storey: $R_d = 1.1 \cdot 1.25 \cdot 522.9 = 719\ kN$
For the third storey: $R_d = 1.1 \cdot 1.25 \cdot 312.1 = 429$ kN

Step 2: Weld length and weld resistance

To size the length of the weld we check the block shear rupture.

$N_{eff,Rd} = \frac{1}{\gamma_{M2}}\left(0.9\, f_u A_{t,net} + \frac{f_y}{\sqrt{3}} 4Lt\right) \geq R_d$ where we may assume $A_{t,net} \approx 0$ to be on the conservative side.

First and second storey:
$\frac{1}{1.25}\left(0 + \frac{0.355}{\sqrt{3}} . 4 \cdot L \cdot 5\right) \geq 719 \rightarrow L \geq 219\ mm$; therefore, use $L = 220 mm$

Third storey:

$\frac{1}{1.25}\left(0+\frac{0.355}{\sqrt{3}}\cdot\ 4\cdot L\cdot 4\right)\ \geq 429 \rightarrow L\ \geq 164\ mm$, therefore, use $L = 164\ mm$

Step 3: Size the gusset plate in tension and compression

a. Net Section verification at the Whitmore section:

$N_{t,Rd} = \frac{1}{\gamma_{M2}} f_y L_w t_p \geq R_d$ where $L_w = 2L_{weld}\tan(30) + width_{brace}$

First and second storey:
$L_w = 2\cdot 220\cdot\tan(30^o) + 100 = 354\ mm$
$N_{t,Rd} = \frac{1}{1.25}0.355\cdot 354\cdot t_p\ \geq 719kN \rightarrow t_p\ \geq 7.15\ mm$: use $t_p = 7.5\ mm$

Third storey:
$L_w = 2\cdot 164\cdot\tan(30) + 60 = 249.4\ mm$
$N_{t,Rd} = \frac{1}{1.25}0.355\cdot 249.4\cdot t_p\ \geq 429\ kN \rightarrow t_p\ \geq 6.1\ mm$: use $t_p = 6.5\ mm$

b. Compressive verification at the Whitmore section:

For first and second storey: as the buckling length for both weak and strong axis is the same, we calculate the buckling resistance for the weak axis. Here, we consider conservatively assume that the buckling length of a bracing member is equal to $l_k = (3.0^2 + 2.0^2)^{0.5} = 3.61$m.

$$N_{cr,z} = \pi^2\cdot E\cdot\frac{I_z}{l_k^2} = 3.14^2\cdot 210\cdot\frac{0.836\cdot 10^6}{3610^2} = 133kN$$

$$\bar{\lambda}_z = \sqrt{\frac{A\cdot f_y}{N_{cr,z}}} = \sqrt{\frac{1473\cdot 0.355}{133}} = 1.98$$

According to SIA 263/2013 (Fig. 7), for hot-rolled ST355 square hollow structural profiles, the imperfection factor is α =0.21.

$$\Phi_z = 0.5\cdot\left(1+\alpha\cdot\left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z^{\ 2}\right) = 0.5\cdot(1 + 0.21\cdot(1.98 - 0.2) + 1.98^2) = 2.65$$

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^{\ 2}}} = \frac{1}{2.65 + \sqrt{2.65^2 - 1.98^2}} = 0.227$$

$$N_{b,z,Rd} = \chi_z\cdot A\cdot\frac{f_y}{\gamma_{M1}} = 0.227\cdot 1473\cdot\frac{0.355}{1.05} = 113\ kN$$

Similarly, in the third storey, we have a hollow square structural profile; therefore,

$$N_{cr} = \pi^2\cdot E\cdot\frac{I_z}{l_k^2} = 3.14^2\cdot 210\cdot\frac{0.454\cdot 10^6}{3610^2} = 72.2\ kN$$

$$\bar{\lambda}_z = \sqrt{\frac{A\cdot f_y}{N_{cr,z}}} = \sqrt{\frac{874\cdot 0.355}{72.2}} = 2.07$$

$$\Phi_z = 0.5 \cdot \left(1 + \alpha \cdot (\bar{\lambda}_z - 0.2) + \bar{\lambda}_z^{\,2}\right) = 0.5 \cdot (1 + 0.21 \cdot (2.07 - 0.2) + 2.07^2) = 2.84$$

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^{\,2}}} = \frac{1}{2.84 + \sqrt{2.84^2 - 2.07^2}} = 0.209$$

$$N_{b,z,Rd} = \chi_z \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 0.209 \cdot 847 \cdot \frac{0.355}{1.05} = 59.9\ kN$$

Hence,

$$N_{cr,gusset} = \frac{\pi^2 E L_w t_{gusset}^3}{12(kL)^2} \geq R_d$$

Assumptions: $k = 0.65$ and $L = 2t_p + \frac{L_w}{2}\cot(33.7)$ (see Figure 3)

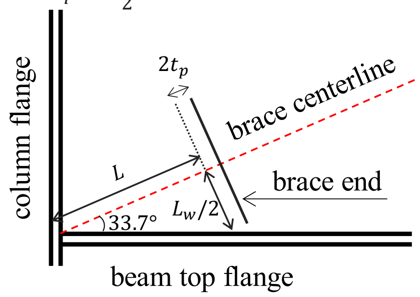


Figure 3. Gusset plate schematic for its buckling resistance calculation

First and second storey:

$$L = 2 \cdot 7.5 + \frac{354}{2}\cot(33.7) = 280.4\ mm$$

$$R_d = 1.1\,\gamma_{ov}\,N_{b,Rd} = 1.1 \cdot 1.25 \cdot 113 = 155.4kN$$

$$N_{cr} = \frac{\pi^2 \cdot 210 \cdot 354 \cdot 7.5^3}{12(0.65 \cdot 280.4)^2} = 776.5kN \ > \ 155.4kN\ ok$$

Third storey:

$$L = 2 \cdot 6.5 + \frac{249}{2}\cot(33.7) = 200\ mm$$

$$R_d = 1.1\,\gamma_{ov}\,N_{b,Rd} = 1.1 \cdot 1.25 \cdot 59.9 = 82.4kN$$

$$N_{cr} = \frac{\pi^2 \cdot 210 \cdot 249.4 \cdot 6.5^3}{12(0.65 \cdot 200)^2} = \ 700\ kN \ > \ 82.4kN\ ok$$

c. Net section verification:

$$N_{net,Rd} = \frac{0.9 f_u A_{t,net}}{\gamma_{M2}} \geq R_d = 1.1\gamma_{ov}N_{pl,Rd} = 1.1\gamma_{ov}f_y A \rightarrow \frac{A_{t,net}}{A} \geq \frac{1.1 \cdot 1.25 \cdot 0.355}{0.9 * 0.470} \cdot 1.25 = 1.44;$$

Note that this check should not control as A_{net} is always smaller than A.

First and second storey:

$$\frac{A_{t,net}}{A} = \frac{1473 - 2 \cdot 5 \cdot 7.5}{1473} = 0.95$$

Third storey:

$$\frac{A_{t,net}}{A} = \frac{879 - 2 \cdot 4 \cdot 6.5}{879} = 0.94$$

The preliminary sketches for the bracing connections are as follows

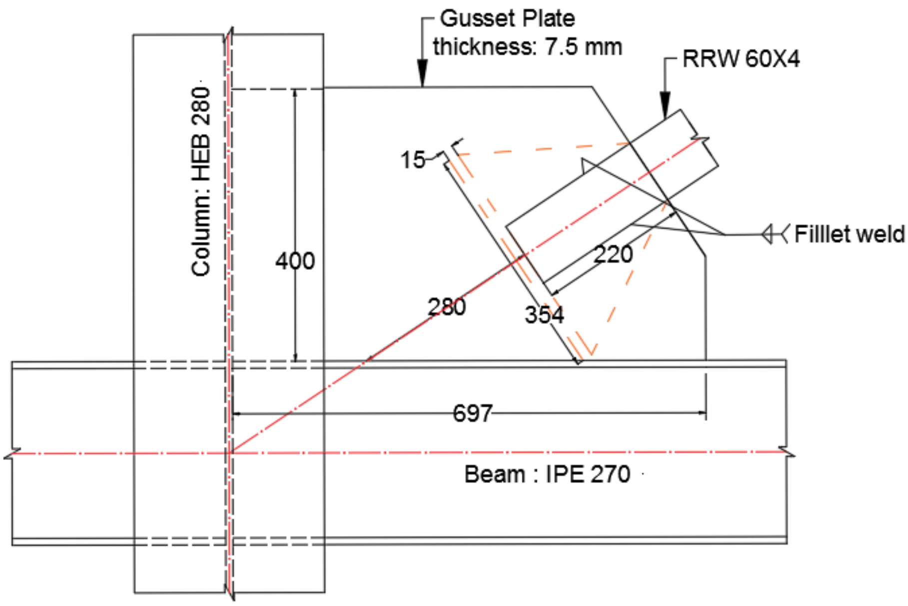


Figure 4. Connection detail for 1st and 2nd storeys (dimensions in mm)

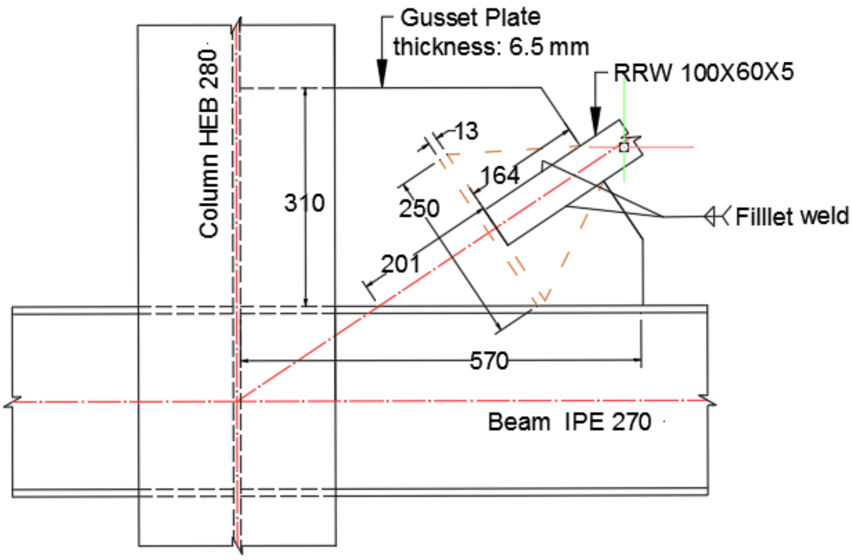


Figure 5. Connection detail for 3rd storey (dimensions in mm)