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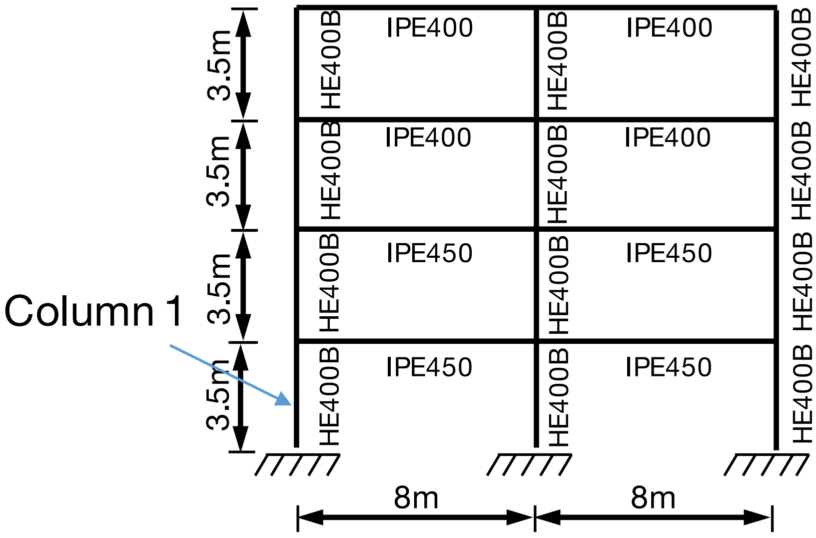
Exercise #8: Seismic Design of Steel MRFs - Steel Columns

The steel moment-resisting frame (MRF) shown in Figure 1 has been designed in a high seismicity zone for gravity and earthquake loading. The cross sections represent the final design of the steel MRF in the North-South loading direction. Steel beams and columns have been designed with S355J2 profile (i.e., $E = 210GPa, f_y = 355MPa$). A behaviour factor of $q = 5$ has been adopted as part of the design process. End plate beam-to-column connections have been utilized for the seismic design. The total floor weight due to gravity loading is $G = 5kN/m^2$ (all included). The stability coefficient is $\theta = 0.13$ for story 1 of the steel MRF. For stories 2, 3, 4 assume that $\theta < 0.10$. The column can be assumed to be fixed at the base (exposed type column-base connection).

The following questions should be addressed:

1. Compute the force demands (N_{Ed}, M_{Ed}, V_{Ed}) for column 1. For these computations use the shear, axial and bending force diagrams shown in Figure 2.
2. Does column 1 satisfy the design requirements for shear demand/shear resistance?
3. What is the section classification of column 1? Can it be used for the seismic design of the steel MRF?
4. Compute the buckling resistance of column 1. Check if it is adequate for the design against compression. Assume that the steel column is restrained laterally with respect to its weak axis every 0.75m.
5. Check the column resistance for the effects of lateral torsional buckling. The steel column is braced laterally only at the first-floor level ($L = 3500mm$).
6. What should the flexural resistance of the column be after considering bending and axial load interaction?
7. Check the column for biaxial bending interaction. Assume that the $M_{z,Ed}$ = -150kN-m (top end) and $M_{z,Ed}$ = 300kN-m (fixed end).
8. Check the stability of the column for interaction of axial load and biaxial bending. Assume that the $M_{z,Ed}$ = -150kN-m (top end) and $M_{z,Ed}$ = 300kN-m (fixed end).
9. Is the strong column/weak beam ratio check satisfied at the first floor exterior joint? Assume that the steel beams are braced laterally to develop their full plastic moment resistance $M_{y,pl,Rd}$. The steel beams are not considered to be fully composite (i.e., the slab does not provide additional strength to the steel beam).

Depending on the assumptions made, the answers can vary.
The axial, shear and bending diagrams are shown in Figures 2, 3, 4, respectively.



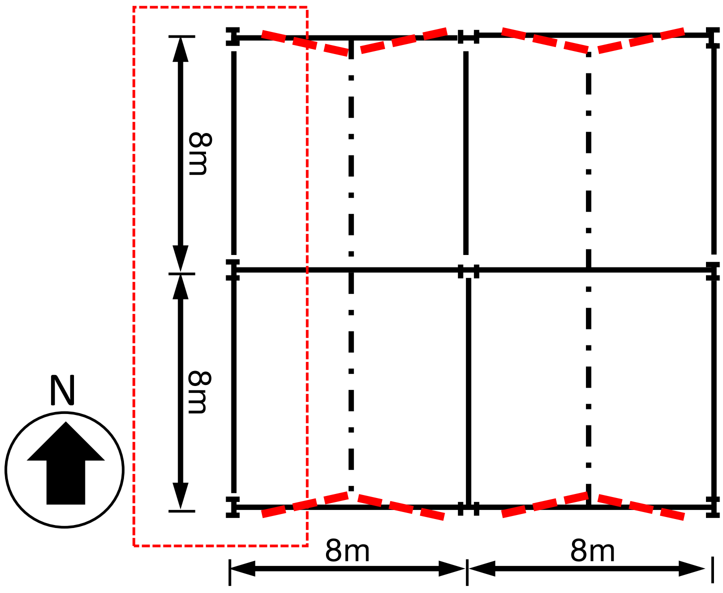
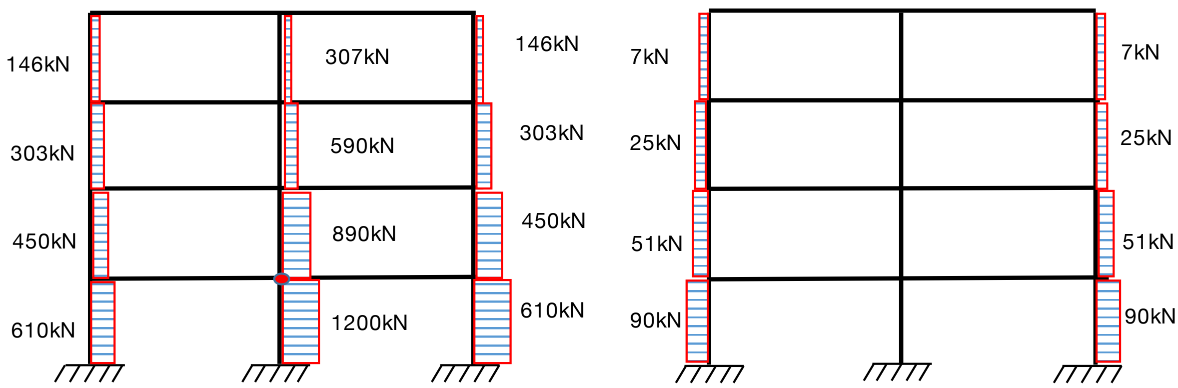
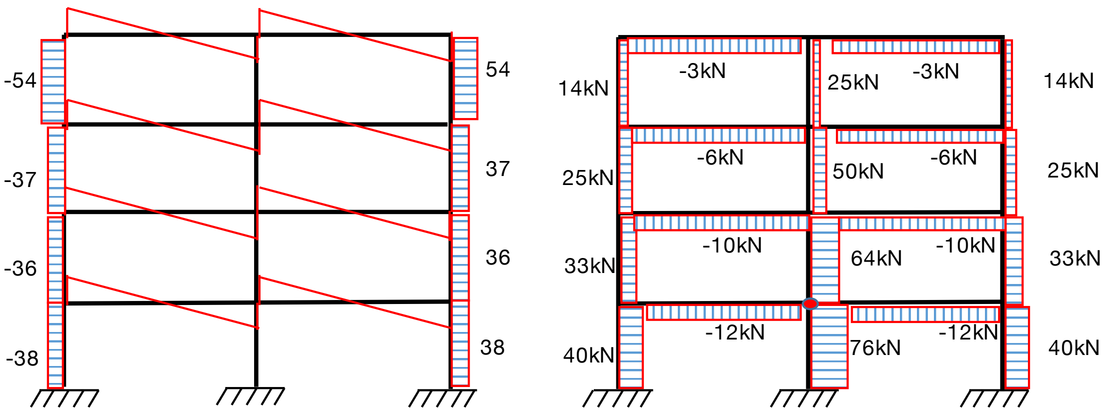


Figure 1. Final design of steel MRF



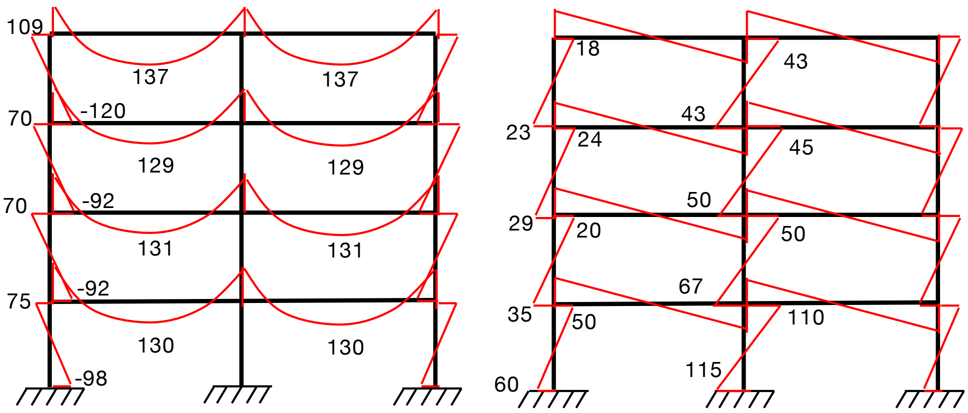
(a) due to gravity (b) due to earthquake

Figure 2. Axial force diagram for gravity and seismic loading



(a) due to gravity (b) due to earthquake

Figure 3. Shear force diagram for gravity and seismic loading

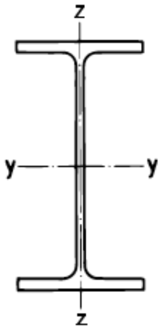


(a) due to gravity (b) due to earthquake

Figure 4. Bending force diagram for gravity and seismic loading

IPE, PEA

IPE- und IPEA-Träger — *Profilés IPE et IPEA*



$A_v = A - 2bt_f + (t_w + 2r)\, t_f$

$A_w = (h - t_f) \cdot t_w$

$S_y = ½\, W_{ply}$

$S_z = ½\, W_{plz}$

$W_{ely} = \dfrac{I_y}{h/2}$

$\overline{W}_y = \dfrac{I_y}{(h - t_f)/2}$

$W_{elz} = \dfrac{I_z}{b/2}$

Maximale Lagerlängen / *Longueurs maximales en stock:*
h ≤ 180 18 m
h ≥ 200 24 m
EURONORM 19 – 57,
DIN 1025/5, ASTM A 6,
Werksnorm/*Norme d'usine*

○ Das Verfahren PP nach SIA 263 ist für dieses Profil aus S355 bei reiner Biegung (n = 0) nicht anwendbar!

* Auch in S355J0 oder S355J2 ab Schweizer Lager erhältlich.

○ *La méthode PP selon SIA 263 n'est pas applicable pour ce profilé en acier S355 en flexion simple (n = 0)!*

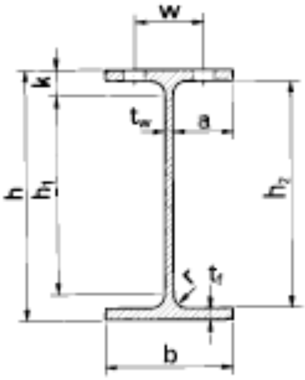
* *Livrable en S355J0 ou S355J2 du stock suisse.*

IPE	m	Statische Werte / *Valeurs statiques*												
	kg/m	A mm²	A_v mm²	A_w mm²	I_y mm⁴	W_{ely} mm³	$\overline{W}_y$ mm³	W_{ply} mm³	i_y mm	I_z mm⁴	W_{elz} mm³	W_{plz} mm³	i_z mm	$K = I_x$ mm⁴
					x 10⁶	x 10³	x 10³	x 10³		x 10⁶	x 10³	x 10³		x 10⁶
80*	6,0	764	358	284	0,801	20,0	21,4	23,2	32,4	0,085	3,69	5,82	10,5	0,0067
100*	8,1	1030	508	387	1,71	34,2	36,3	39,4	40,7	0,159	5,79	9,15	12,4	0,0115
120*	10,4	1320	631	500	3,18	53,0	55,9	60,7	49,0	0,277	8,65	13,6	14,5	0,0169
140*	12,9	1640	764	626	5,41	77,3	81,3	88,3	57,4	0,449	12,3	19,2	16,5	0,0240
160*	15,8	2010	966	763	8,69	109	114	124	65,8	0,683	16,7	26,1	18,4	0,0353
180*	18,8	2390	1125	912	13,2	146	154	166	74,2	1,01	22,2	34,6	20,5	0,0472
200*	22,4	2850	1400	1070	19,4	194	203	221	82,6	1,42	28,5	44,6	22,4	0,0685
220*	26,2	3340	1588	1240	27,7	252	263	285	91,1	2,05	37,3	58,1	24,8	0,0898
240*	30,7	3910	1914	1430	38,9	324	338	367	99,7	2,84	47,3	73,9	26,9	0,127
270*	36,1	4590	2214	1710	57,9	429	446	484	112	4,20	62,2	97,0	30,2	0,157
300*	42,2	5380	2568	2050	83,6	557	578	628	125	6,04	80,5	125	33,5	0,198
330*	49,1	6260	3081	2390	117,7	713	739	804	137	7,88	98,5	154	35,5	0,276
360*	57,1	7270	3514	2780	162,7	904	937	1020	150	10,4	123	191	37,9	0,371
400*	66,3	8450	4269	3320	231,3	1160	1200	1310	165	13,2	146	229	39,5	0,504
450*	77,6	9880	5085	4090	337,4	1500	1550	1700	185	16,8	176	276	41,2	0,661
500*	90,7	11600	5987	4940	482,0	1930	1990	2190	204	21,4	214	336	43,1	0,886
550	106	13400	7234	5910	671,2	2440	2520	2790	223	26,7	254	401	44,5	1,22
600	122	15600	8378	6970	920,8	3070	3170	3510	243	33,9	308	486	46,6	1,65
750 x	137	17500	9290	8460	1599	4250	4340	○ 4860	303	51,7	393	614	54,4	1,36
750 x	147	18700	10540	9720	1661	4410	4510	5110	298	52,9	399	631	53,1	1,57
750 x	173	22100	11640	10700	2058	5400	5560	6220	305	68,7	515	810	55,7	2,71
750 x	196	25100	12730	11600	2403	6240	6450	7170	310	81,8	610	959	57,1	4,06

IPE- und IPEA-Träger — *Profilés IPE et IPEA* — IPE, PEA

Die Profile PER, IPEo und IPEv sind im Walzprogramm einzelner Werke aufgeführt. PEA 80 und PEA 100 sind ebenfalls normiert, aber kaum wirtschaftlich.

Les profilés PER, IPEo et IPEv figurent dans le programme de laminage de quelques aciéries. Les PEA 80 et PEA 100, également normalisés, sont peu économiques.



(grau) Im allgemeinen nur ab Werk lieferbar. Mindestmengen und Termine beachten.

(gris) En général livrable d'usine uniquement. Tenir compte des quantités minimales et des délais.

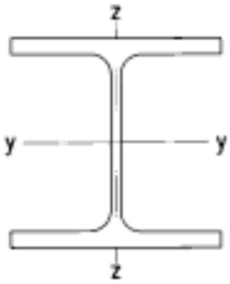
Walztoleranzen siehe Seite 116

Tolérances de laminage voir p. 116

IPE	m	Profilmasse *Dimensions de la section*					Konstruktionsmasse *Dimensions de construction*						Oberfläche *Surface*		IPE
	kg/m	h mm	b mm	t_w mm	t_f mm	r mm	h_1 mm	k mm	a mm	h_2 mm	w mm	$Ø_{max}$	U_m m²/m	U_t m²/t	
80	6,0	80	46	3,8	5,2	5	60	10	21	70			0,328	54,8	80
100	8,1	100	55	4,1	5,7	7	74	13	25	89			0,400	49,5	100
120	10,4	120	64	4,4	6,3	7	92	14	29	107	36	M10	0,475	45,6	120
140	12,9	140	73	4,7	6,9	7	112	14	34	126	38	M10	0,551	42,6	140
160	15,8	160	82	5,0	7,4	9	126	17	38	145	44	M12	0,623	39,4	160
180	18,8	180	91	5,3	8,0	9	146	17	42	164	50	M12	0,698	37,1	180
200	22,4	200	100	5,6	8,5	12	158	21	47	183	56	M12	0,768	34,3	200
220	26,2	220	110	5,9	9,2	12	178	21	52	202	60	M16	0,848	32,4	220
240	30,7	240	120	6,2	9,8	15	190	25	56	220	68	M16	0,922	30,0	240
270	36,1	270	135	6,6	10,2	15	220	25	64	250	72	M20	1,04	28,8	270
300	42,2	300	150	7,1	10,7	15	248	26	71	279	80	M20	1,16	27,5	300
330	49,1	330	160	7,5	11,5	18	270	30	76	307	86	M24	1,25	25,5	330
360	57,1	360	170	8,0	12,7	18	298	31	81	335	90	M24	1,35	23,6	360
400	66,3	400	180	8,6	13,5	21	330	35	85	373	96	M27	1,47	22,2	400
450	77,6	450	190	9,4	14,6	21	378	36	90	421	106	M27	1,61	20,7	450
500	90,7	500	200	10,2	16,0	21	426	37	94	468	110	M27	1,74	19,2	500
550	106	550	210	11,1	17,2	24	468	41	99	516	120	M27	1,88	17,7	550
600	122	600	220	12,0	19,0	24	514	43	104	562	120	M27	2,02	16,6	600
750 x 137		753	263	11,5	17,0	17	685	34	126	719	120	M27	2,51	18,3	750x137
750 x 147		753	265	13,2	17,0	17	685	34	126	719	120	M27	2,51	17,1	750x147
750 x 173		762	267	14,4	21,6	17	685	39	126	719	120	M27	2,53	14,6	750x173
750 x 196		770	268	15,6	25,4	17	685	42	126	719	120	M27	2,55	13,0	750x196

HEB

Breitflanschträger HEB — *Profilés à larges ailes* HEB



$A_v = A - 2bt_f + (t_w + 2r)\,t_f$

$A_w = (h - t_f) \cdot t_w$

$S_y = \frac{1}{2} W_{ply}$

$S_z = \frac{1}{2} W_{plz}$

$W_{ely} = \dfrac{I_y}{h/2}$

$\overline{W}_y = \dfrac{I_y}{(h - t_f)/2}$

$W_{elz} = \dfrac{I_z}{b/2}$

*Auch in S355J0 oder S355J2 ab Schweizer Lager erhältlich

Maximale Lagerlängen / *Longueurs maximales en stock:*

h ≤ 180 18 m

h ≥ 200 24 m

EURONORM 53 – 62, DIN 1025/2

Andere Bezeichnungen / *Autres désignations*: DIN, IPB

**Livrable en S355J0 ou S355J2 du stock suisse*

HEB	m kg/m	A mm²	A_v mm²	A_w mm²	I_y mm⁴ x 10⁶	W_{ely} mm³ x 10³	$\overline{W}_y$ mm³ x 10³	W_{ply} mm³ x 10³	i_y mm	I_z mm⁴ x 10⁶	W_{elz} mm³ x 10³	W_{plz} mm³ x 10³	i_z mm	$K = I_x$ mm⁴ x 10⁶
		Statische Werte / *Valeurs statiques*												
100*	**20,4**	2600	904	540	4,50	89,9	100	104	41,6	1,67	33,5	51,4	25,3	0,0931
120*	**26,7**	3400	1096	708	8,64	144	158	165	50,4	3,18	52,9	81,0	30,6	0,139
140*	**33,7**	4300	1308	896	15,1	216	236	245	59,3	5,50	78,5	120	35,8	0,202
160*	**42,6**	5430	1759	1180	24,9	311	339	354	67,8	8,89	111	170	40,5	0,312
180*	**51,2**	6530	2024	1410	38,3	426	461	481	76,6	13,6	151	231	45,7	0,422
200*	**61,3**	7810	2483	1660	57,0	570	616	643	85,4	20,0	200	306	50,7	0,596
220*	**71,5**	9100	2792	1940	80,9	736	793	827	94,3	28,4	258	394	55,9	0,770
240*	**83,2**	10600	3323	2230	112,6	938	1010	1050	103	39,2	327	498	60,8	1,04
260*	**93,0**	11800	3759	2420	149,2	1150	1230	1280	112	51,3	395	602	65,8	1,26
280*	**103**	13100	4109	2750	192,7	1380	1470	1530	121	65,9	471	718	70,9	1,45
300*	**117**	14900	4743	3090	251,7	1680	1790	1870	130	85,6	571	870	75,8	1,87
320*	**127**	16100	5177	3440	308,2	1930	2060	2150	138	92,4	616	939	75,7	2,29
340*	**134**	17100	5609	3820	366,6	2160	2300	2410	146	96,9	646	986	75,3	2,62
360*	**142**	18100	6060	4220	431,9	2400	2560	2680	155	101	676	1030	74,9	2,98
400*	**155**	19800	6998	5080	576,8	2880	3070	3230	171	108	721	1100	74,0	3,61
450*	**171**	21800	7966	5940	798,9	3550	3770	3980	191	117	781	1200	73,3	4,49
500	**187**	23900	8982	6840	1072	4290	4540	4820	212	126	842	1290	72,7	5,50
550	**199**	25400	10010	7820	1367	4970	5250	5590	232	131	872	1340	71,7	6,12
600	**212**	27000	11080	8840	1710	5700	6000	6420	252	135	902	1390	70,8	6,80
650	**225**	28600	12200	9900	2106	6480	6800	7320	271	140	932	1440	69,9	7,52
700	**241**	30600	13710	11400	2569	7340	7690	8330	290	144	963	1490	68,7	8,42
800	**262**	33400	16180	13400	3591	8980	9360	10230	328	149	994	1550	66,8	9,62
900	291	37100	18880	16000	4941	10980	11400	12580	365	158	1050	1660	65,3	11,5
1000	314	40000	21250	18300	6447	12890	13400	14860	401	163	1090	1720	63,8	12,7

Breitflanschträger HEB *Profilés à larges ailes* HEB

HEB

Anstelle des nicht mehr gewalzten Profils HEB 1100 können HL-Profile verwendet werden, siehe Seiten 40/41.

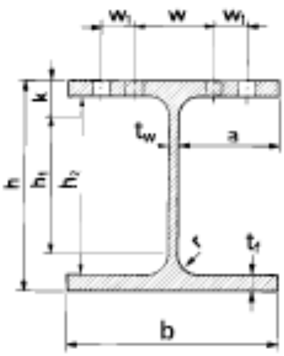
w_1 mit $\varnothing_{max}$ nur für versetzte Schrauben.

Walztoleranzen siehe Seite 116

Au lieu du profilé HEB 1100 qui n'est plus laminé, on utilisera des profilés HL (voir pages 40/41).

w_1 avec $\varnothing_{max}$ seulement pour boulons décalés.

Tolérances de laminage voir page 116



HEB	m	Profilmasse *Dimensions de la section*					Konstruktionsmasse *Dimensions de construction*							Oberfläche *Surface*		HEB
	kg/m	h mm	b mm	t_w mm	t_f mm	r mm	h_1 mm	k mm	a mm	h_2 mm	w mm	w_1 mm	$\varnothing_{max}$	U_m m²/m	U_t m²/t	
100	**20,4**	100	100	6	10	12	56	22	47	80	56		M12	0,567	27,8	**100**
120	**26,7**	120	120	6,5	11	12	74	23	56	98	66		M16	0,686	25,7	**120**
140	**33,7**	140	140	7	12	12	92	24	66	116	76		M20	0,805	23,9	**140**
160	**42,6**	160	160	8	13	15	104	28	76	134	86		M20	0,918	21,5	**160**
180	**51,2**	180	180	8,5	14	15	122	29	85	152	100		M24	1,04	20,3	**180**
200	**61,3**	200	200	9	15	18	134	33	95	170	110		M24	1,15	18,8	**200**
220	**71,5**	220	220	9,5	16	18	152	34	105	188	120		M24	1,27	17,8	**220**
240	**83,2**	240	240	10	17	21	164	38	115	206	96	35	M24	1,38	16,6	**240**
260	**93,0**	260	260	10	17,5	24	176	42	125	225	106	40	M24	1,50	16,1	**260**
280	**103**	280	280	10,5	18	24	196	42	134	244	110	45	M24	1,62	15,7	**280**
300	**117**	300	300	11	19	27	208	46	144	262	120	45	M27	1,73	14,8	**300**
320	**127**	320	300	11,5	20,5	27	224	48	144	279	120	45	M27	1,77	13,9	**320**
340	**134**	340	300	12	21,5	27	242	49	144	297	120	45	M27	1,81	13,5	**340**
360	**142**	360	300	12,5	22,5	27	260	50	143	315	120	45	M27	1,85	13,0	**360**
400	**155**	400	300	13,5	24	27	298	51	143	352	120	45	M27	1,93	12,4	**400**
450	**171**	450	300	14	26	27	344	53	143	398	120	45	M27	2,03	11,9	**450**
500	**187**	500	300	14,5	28	27	390	55	142	444	120	45	M27	2,12	11,3	**500**
550	**199**	550	300	15	29	27	438	56	142	492	120	45	M27	2,22	11,2	**550**
600	**212**	600	300	15,5	30	27	486	57	142	540	120	45	M27	2,32	11,0	**600**
650	**225**	650	300	16	31	27	534	58	142	588	120	45	M27	2,42	10,8	**650**
700	**241**	700	300	17	32	27	582	59	141	636	126	45	M27	2,52	10,5	**700**
800	**262**	800	300	17,5	33	30	674	63	141	734	130	40	M27	2,71	10,4	**800**
900	291	900	300	18,5	35	30	770	65	140	830	130	40	M27	2,91	10,0	900
1000	314	1000	300	19	36	30	868	66	140	928	130	40	M27	3,11	9,9	1000

Solution

1. The following loading combinations should be used for the computation of the force demands on the column:

$$N_{Ed} = N_{Ed,G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot N_{Ed,E}$$
$$V_{Ed} = V_{Ed,G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot V_{Ed,E}$$
$$M_{Ed} = M_{Ed,G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot M_{Ed,E}$$

The overstrength of the members should be computed based on the flexural resistance/demand of the beams at their ends.

Floor 1:
Beam 1 Left: $M_{Ed,E}$ = 35 + 50 = 85kNm, Right: $M_{Ed,E}$ = (110 + 67)/2 = 88.5kNm.
Only the max moment should be considered each time.

The stability coefficient $\theta > 0.10$; P-Delta effects should be considered to compute the first floor beam demands. Therefore,

$M_{Ed,tot} = (75 + 92) \cdot 1/(1-\theta) + 88.5 \cdot 1/(1-\theta) = 294$kNm

NOTE: In $M_{Ed,tot}$, the first term represents the moment due to gravity load and the second one the moment due to the seismic action.

Beam 2 Left: $M_{Ed,E}$ = 29 + 20 = 49kNm, Right: $M_{Ed,E}$ = (50 + 50)/2 = 50kNm

The stability coefficient $\theta < 0.10$; P-Delta effects should not be considered. Therefore,

$M_{Ed,tot}$ = (70+92)+ 50 = 212kNm

NOTE: In $M_{Ed,tot}$, the first term represents the moment due to gravity load and the second one the moment due to seismic action.

Beam 3 Left: $M_{Ed,E}$ = 23 + 24 = 47kNm, Right: $M_{Ed,E}$ = (43 + 45)/2 = 44kNm.

The stability coefficient $\theta < 0.10$; P-Delta effects should not be considered. Therefore,

$M_{Ed,tot}$ = (70+120) + 47 = 237kNm

NOTE: In $M_{Ed,tot}$, the first term represents the moment due to gravity load and the second one the moment due to seismic action.

Beam 4 Left: $M_{Ed,E}$ = 18kNm, Right: $M_{Ed,E}$ = (43)/2 = 21.5kNm.

The stability coefficient $\theta < 0.10$. P-Delta effects should not be considered. Therefore,

$M_{Ed,E}$ = 109 + 21.5 ~ 131kN-m

NOTE: In $M_{Ed,E}$, the first term represents the moment due to gravity load and the second one the moment due to seismic action.

Beams at Floor 1 and 2: IPE 450; $M_{pl} = W_{pl} \cdot f_y / \gamma_{M0} = 1700 \cdot 10^3 \cdot \frac{355}{1000^2} = 603.5kNm$
Beams at Floor 3 and 4: IPE 400; $M_{pl} = W_{pl} \cdot f_y / \gamma_{M0} = 1310 \cdot 10^3 \cdot \frac{355}{1000^2} = 465.05kNm$

$$\Omega = \min\left\{\frac{603.5}{294}, \frac{603.5}{212}, \frac{465.05}{237}, \frac{465.05}{131}\right\} = 1.96$$

1. Force demands for column 1:

Note 1: Earthquake loading is cyclic; therefore, it can act on a structure in both directions; therefore, when the gravity and seismic actions are added, an "absolute" value should be used. In this way, the most critical combination can be taken into consideration for the design.

$$N_{Ed} = (|610| + 1.1 \cdot 1.25 \cdot 1.96 \cdot |90|) = 852.5kN$$
$$V_{Ed} = \frac{1}{1 - 0.13}(|-38| + 1.1 \cdot 1.25 \cdot 1.96 \cdot |40|) = 167.6kN$$

Column 1 top end:
$$M_{Ed} = \frac{1}{1 - 0.13}(|75| + 1.1 \cdot 1.25 \cdot 1.96 \cdot |50|) = 241.1kN - m$$

Column 1 bottom end:
$$M_{Ed} = \frac{1}{1 - 0.13}(|98| + 1.1 \cdot 1.25 \cdot 1.96 \cdot |60|) = 298.5kN - m$$

2. Shear demand/shear resistance check:

A_v = 6998mm^2, f_y = 355MPa

$$V_{pl,Rd} = \frac{A_v \cdot f_y}{\sqrt{3} \cdot \gamma_{M0}} = \frac{6998 \cdot 0.355}{\sqrt{3} \cdot 1.00} = 1434kN$$

Check:
$$\frac{V_{Ed}}{V_{pl,Rd}} = \frac{167.6}{1434} = 0.117 < 0.50$$

Therefore, the shear demand is not critical. The column flexural resistance should not be reduced due to bending-shear interaction.

3. Section classification for HE400B:
Consider the case that you have compression only.

Flange under compression:
$$\frac{c}{t_f} = \frac{0.5b - r - 0.5t_w}{t_f} = \frac{0.5 \cdot 300 - 27 - 0.5 \cdot 13.5}{24} = 4.84 < 7.32 = 9 \cdot \sqrt{\frac{235}{355}}$$
Flange is Class 1

Web subject to compression only. This is more conservative than flexure and compression. If the check fails, then you should check for flexure and compression:

$$\frac{c}{\mathrm{t}} = \frac{h - 2r - 2t_f}{t_w} = \frac{298}{13.5} = 22.07 < 26.85 = 33 \cdot \sqrt{\frac{235}{355}}$$

Therefore, the web is Class 1 and the cross-section is Class 1. Because $q = 5 > 4$, only Class 1 members are permitted to be used in the seismic design process.

If the web was not Class 1 the check should be repeated for flexure and compression, which is the real stress condition of the web.

4. Buckling Resistance:

We are concerned with buckling resistance with respect to the strong axis bending of the column because it is restrained adequately with respect to its weak-axis.

For a hot rolled section (HEB400) with *h/b* = 400/300 = 1.33 > 1.2, t_f = 24mm < 40mm; therefore, for strong-axis y-y buckling, we are allowed to use a buckling curve *a*.

Therefore, the imperfection factor is, *a* =0.21.

- Computation of normalized slenderness ratio, $\bar{\lambda}$

We have a multi-storey MRF that deforms laterally (sway frame). The factors n_{sup} and n_{inf} shall be computed with the Cross method by assuming that we have continuous columns, which is only allowed in seismic applications:

$$n_{sup} = \frac{K_m + K_{m,sup}}{K_m + K_{m,sup} + \sum K_{t,sup}} = \frac{\frac{I_m}{L_m} \cdot 2}{\frac{I_m}{L_m} \cdot 2 + \frac{I_t}{L_t}} = \frac{\frac{576.8x10^6}{3500} \cdot 2}{\frac{576.8x10^6}{3500} \cdot 2 + \frac{337.4x10^6}{8000}}$$
$$= \frac{164800 \cdot 2}{164800 \cdot 2 + 42175} = 0.887$$

Note that IPE450 is not so stiff compared to the columns; therefore, the column top end is not restrained by much.

$n_{inf} = 0$; The first storey column is fixed at the base; therefore, the term $\Sigma K_{t,inf}$ is infinite at the ground floor).

The effective length factor, β, may be computed with the following formula or by interpolating on the nomogram shown in the lecture notes, which approximately gives, $\beta = 1.7$.

$$\beta = \sqrt{\frac{1 - 0.2 \cdot (n_{sup} + n_{inf}) - 0.12 \cdot n_{sup} \cdot n_{inf}}{1 - 0.8 \cdot (n_{sup} + n_{inf}) + 0.6 \cdot n_{sup} \cdot n_{inf}}} = \sqrt{\frac{1 - 0.2 \cdot (0.887) - 0}{1 - 0.8 \cdot (0.887) + 0}} = 1.683$$

Therefore,

$$N_{cr,y} = \frac{\pi^2 E I_y}{(1.683 \cdot L)^2} = \frac{3.14^2 \cdot 210 \cdot 576.8 \cdot 10^6}{(1.683 \cdot 3500)^2} = 34454kN$$

Therefore,

$$\bar{\lambda}_y = \sqrt{\frac{19800 \cdot 0.355}{34454}} = 0.452$$

Computation of buckling factor:

$$\Phi = 0.5 \cdot [1 + 0.21 \cdot (0.452 - 0.2) + 0.452^2] = 0.629$$

$$\chi_y = \frac{1}{\Phi + \sqrt{(\Phi^2 - \bar{\lambda}^2)}} = 0.938$$

Therefore, the buckling resistance of the column in compression should be,

$$N_{b,y,Rd} = \frac{\chi_y \cdot A \cdot f_y}{\gamma_{M1}} = \frac{0.938 \cdot 19800 \cdot 0.355}{1.05} = 6281kN > 852.5kN = N_{Ed}$$

The buckling resistance of the column is very close to its plastic resistance. This is because the normalized column slenderness is very small (i.e., $\bar{\lambda}_y$ =0.452). Columns in steel MRFs designed for seismic action are typically not critical for member buckling.

In the weak axis, the steel column 1 is braced at 0.75m. Therefore, L_b = 750mm. The column is considered to be pinned in the weak axis orientation,

$$N_{z,cr} = \frac{\pi^2 E I_z}{(L_b)^2} = \frac{3.14^2 \cdot 210 \cdot 108 \cdot 10^6}{(750)^2} = 397942kN$$

$$\bar{\lambda}_z = \sqrt{\frac{19800 \cdot 0.355}{397942}} = 0.133$$

Because of the excessive buckling resistance with respect to the weak axis orientation, $\chi_z = 1$ (no reduction for flexural buckling with respect to the weak axis).

Question 5

h/b = 400/300 = 1.33 < 2; according to EN 1993-1-1 the buckling curve to be used is "*a*" and a_{LT} = 0.21

The Plastic flexural resistance of the column with respect to y-y axis

$$M_{pl,y,Rd} = W_{pl,y} \cdot \frac{f_y}{\gamma_{M0}} = 3230 \cdot 10^3 \cdot 0.355/1.00 \cong 1147kNm$$

Plastic flexural resistance with respect to z-z axis

$$M_{pl,z,Rd} = W_{pl,z} \cdot \frac{f_y}{\gamma_{M0}} = 1100 \cdot 10^3 \cdot 0.355/1.00 \cong 390.5kNm$$

Computation of critical moment:

$z_a = 0$ (the cross section is symmetric. We assume for simplicity that the loads are passing through the cross-section shear center).

$M_{Ed,top} = 241.1kNm$
$M_{Ed,bottom} = 298.5kNm$

Because the steel column is in double curvature (opposite moments), $\kappa = \frac{241.1}{298.5} = 0.807$

The coefficient C_1 may be computed as follows,

$$C_1 = 1.75 + 1.05 \cdot \kappa + 0.3 \cdot \kappa^2 = 2.79 > 2.3$$

A value of C_1=2.3 is chosen.

The column is assumed as fixed at the base with exposed type connection, k_v = 0.8 (assumption) and k_φ=0.5. Designers would often select $k_v = 1.0$ and $k_\varphi = 1.0$ in this case because this location is considered to be a potential dissipative zone for a steel MRF.

L = 3500mm (story height because the column is not braced with respect to y-y axis)

Shear modulus: $G = \frac{E}{2 \cdot (1+v)} = 80.8$kN/mm^2

Computation of the torsional constant:

$$I_t = \frac{2 \cdot b \cdot t_f^3 + (h - t_f) \cdot t_w^3}{3} = \frac{2 \cdot 300 \cdot 24^3 + (400 - 24) \cdot 13.5^3}{3} = 3.07x10^6 mm^4$$

Computation of the warping constant,

$$I_w = \frac{t_f \cdot (h - t_f)^2 \cdot b^3}{24} = \frac{24 \cdot (400 - 24)^2 \cdot 300^3}{24} = 3.82x10^{12} mm^6$$

Therefore, the M_{cr} computation is as follows:

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{k_v k_\phi (L)^2} \cdot \left(\frac{I_w}{I_z} \left(\frac{(k_\phi \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_w} + 1 \right) \right)^{0.5}$$
$$= 2.3 \cdot \frac{\pi^2 \cdot 210000 \cdot 108 \cdot 10^6}{0.8 \cdot 0.5 \cdot (3500)^2}$$
$$\cdot \left(\frac{3.82 \cdot 10^{12}}{108 \cdot 10^6} \left(\frac{(0.5 \cdot 3500)^2 \cdot 80769 \cdot 3.07 \cdot 10^6}{\pi^2 \cdot 210000 \cdot 3.82 \cdot 10^{12}} + 1 \right) \right)^{0.5} = 20680kNm$$

Note that if k_v = 1.0 and k_φ=1.0 were assumed, then $M_{cr} = 9296kNm$

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{pl,y,Rd}}{M_{cr}}} = 0.23 < 0.40$$

As the modified slenderness ratio is below 0.4; the column is able to develop its full plastic flexural resistance (i.e., $\chi_{LT} = 1.0$).

The reduction factor χ_{LT} is only calculated below for illustration purposes:

$$\Phi_{LT} = 0.5 \cdot \left[1 + a_{LT} \cdot \left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] = 0.532$$

Therefore,

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.532 + \sqrt{0.532^2 - 0.237^2}} = 0.991 \sim 1.00$$

Note that, the reduction factor is really close to 1.0, as expected.

Therefore,

$$M_{b,Rd} = 0.991 \cdot 1147/1.05 = 1030.6kNm$$

Question 6:

We need to check the axial load ratio first:

$$n = \frac{N_{Ed}}{N_{pl,Rd}} = \frac{852.5}{19800 \cdot 0.355/1.05} = 0.126 < 0.25$$

and

$$N_{Ed} = 852.5kN > \frac{0.5 \cdot h_w \cdot t_w \cdot f_y}{\gamma_{M0}} = \frac{0.5 \cdot (400 - 2(24 + 27)) \cdot 13.5 \cdot 0.355}{1.0} = 714.1kN$$

Therefore, the flexural resistance of the steel column should be reduced for the effects of axial load in both the y-y and z-z axes.

- Strong axis Bending

Position of neutral axis, $a = \frac{A - 2 \cdot b_f \cdot t_f}{A} = \frac{19800 - 2 \cdot 300 \cdot 24}{19800} = 0.273$

Thus,

$$M_{y,N,Rd} = M_{y,pl,Rd} \cdot \frac{1 - n}{1 - 0.5 \cdot a} = 1092 \cdot \frac{1 - 0.126}{1 - 0.5 \cdot 0.273} = 1104.9kNm > M_{y,pl,Rd} = 1092kN$$

- Weak axis Bending

For $n < a$

$$M_{z,N,Rd} = M_{z,pl,Rd} = 390.5/1.05 = 372kNm$$

Question 7:

We shall check for biaxial bending both top and bottom column locations.
For an H-shape cross sections: *a*=2 and $\beta = 5 \cdot n = 5 \cdot 0.146 = 0.73 < 1$ therefore $\beta = 1$.

The biaxial bending check at the column base location (location of highest axial load and bending demands) is as follows,

$M_{y,Ed} = 298.5kNm$ (computed in Question 1)
$M_{z,Ed}$= 300kNm (given by the in-class exercise)

$$\left[\frac{M_{y,Ed}}{M_{y,N,Rd}}\right]^{a} + \left[\frac{M_{z,Ed}}{M_{z,N,Rd}}\right]^{\beta} = \left[\frac{298.5}{1092}\right]^{2} + \left[\frac{300}{372}\right]^{1} = 0.074 + 0.806 = 0.881 < 1$$

The check is verified.

Question 8:

Stability of the column under interaction of axial load and biaxial bending.

$$\frac{N_{ed}}{N_{k,Rd}} + \frac{\omega_y}{1 - \frac{N_{Ed}}{N_{cr,y}}} \cdot \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{\omega_z}{1 - \frac{N_{Ed}}{N_{cr,z}}} \cdot \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1$$

$$N_{k,Rd} = \min\left\{\chi_y \cdot \frac{\mathrm{N_{pl,Rd}}}{\gamma_{M1}};\ \chi_z \cdot \frac{\mathrm{N_{pl,Rd}}}{\gamma_{M1}}\right\} = 6281kN$$

$M_{y,Ed} = 298.5kNm$ (computed in Question 1)
$M_{z,Ed}$= 300 kNm (given by the in-class exercise)

$M_{y,Rd}$ = $0.991 \cdot 1092/1.05$ =1030.6 kNm
$M_{z,Rd}$= 390.5/1.05 = 372 kNm
$N_{Ed} = 979.9kN$
$N_{cr,y} = 34454kN$
$N_{cr,z} = 397942kN$

$$\omega_y = 0.6 + 0.4\left(-\frac{241}{298.5}\right) = 0.277 < 0.4 \rightarrow \omega_y = 0.4$$
$$\omega_z = 0.6 + 0.4\left(-\frac{150}{300}\right) = 0.40 \rightarrow \omega_z = 0.4$$

Therefore,

$$\frac{852.5}{6281} + \frac{0.4}{1 - \frac{852.5}{34454}} \cdot \frac{298.5}{1030.6} + \frac{0.4}{1 - \frac{852.5}{397942}} \cdot \frac{300}{372} = 0.136 + 0.119 + 0.330 = 0.584 < 1$$

Check is verified

Question 9:

We should calculate the strong column/weak beam ratio at the first story exterior joint and this ratio must be greater than 1.3 (capacity design).

The beams are braced such that they can develop their full plastic flexural resistance $M_{pl,Rd}$. End-plate beam-to-column connections are used. We should compute s_h to transfer the shear force of the steel beam to the column centerline.

$$s_h = \min\left\{\frac{h_b}{2}; 3b_f\right\} = \min\left\{\frac{450}{2} = 225; 3 \cdot 190 = 570\right\} = 225\ [mm]$$

The shear demand in the beam is calculated by assuming simultaneous plastic hinges forming at the beam ends. In this case, the length from plastic hinge-to-plastic hinge shall be used,

$$V_{Ed} = 1.1 \cdot \gamma_{ov} \cdot \frac{2M_{pl,Rd}}{L - \frac{h_c}{2} - \frac{h_c}{2} - 2 \cdot s_h} + \frac{G \cdot L \cdot \frac{L}{2}}{2} = 1.1 \cdot 1.25 \cdot \frac{2 \cdot 1700 \cdot \frac{0.355}{1.05}}{8 - 0.4 - 2 \cdot 0.225} + \frac{5 \cdot 8 \cdot \frac{8}{2}}{2} = 301.1kN$$

There is one beam connected to the exterior joint, therefore the moment at the connection is,

$$M_{Rd,b} = 1.1 \cdot \gamma_{ov} \cdot M_{pl,y,Rd} + V_{Ed} \cdot \left(s_h + \frac{h_c}{2}\right) = 1.1 \cdot 1.25 \cdot 1700 \cdot \frac{0.355}{1.00} + 301.1 \cdot \left(0.225 + \frac{0.400}{2}\right) = 957.8kNm$$

The design values of the moment should be reduced by the effects of compressive axial load at the bottom and the top of the beam-to-column joint.

The calculation was already made in Question 6 for the column below the joint (first story column), we can apply the same method for the column above the joint (second story column).

$$N_{Ed,top} = (|450| + 1.1 \cdot 1.25 \cdot 1.96 \cdot |51|) = 587.4kN$$
$$N_{Ed,bot} = (|610| + 1.1 \cdot 1.25 \cdot 1.96 \cdot |90|) = 852.5kN$$

$$n_{top} = \frac{N_{Ed,top}}{N_{pl,Rd}} = \frac{587.4}{19800 \cdot 0.355/1.05} = 0.0877 < 0.25$$

In addition, the following check shall be done:

$$N_{Ed,top} < \frac{0.5 \cdot h_w \cdot t_w \cdot f_y}{\gamma_{M1}} = \frac{0.5 \cdot 298 \cdot 13.5 \cdot 0.355}{1.05} = 680.1kN$$

Therefore, there is no need to reduce the bending resistance due to axial load,

$$M_{y,N,Rd,top} = M_{y,pl,Rd} = 1092kNm$$

Similarly, for the bottom location,

$$n_{bot} = \frac{N_{Ed,bot}}{N_{pl,Rd}} = \frac{852.5}{19800 \cdot 0.355/1.05} = 0.126 < 0.25$$

In addition, the following check shall be done:

$$N_{Ed,bot} > \frac{0.5 \cdot h_{\mathrm{w}} \cdot t_w \cdot f_y}{\gamma_{M1}} = \frac{0.5 \cdot 298 \cdot 13.5 \cdot 0.355}{1.05} = 680.1kN$$

Therefore, in this case we need to reduce the flexural resistance due to axial load in this case,

$$M_{y,N,Rd,bot} = M_{y,pl,Rd} \cdot \frac{1-n}{1-0.5 \cdot a} = 1092 \cdot \frac{1-0.126}{1-0.5 \cdot 0.273} = 1104.9kNm$$
$$> 1092kNm \ (assume\ 1092kNm)$$

The SCWB ratio is computed as follows,

$$\Sigma M_c = M_{N,pl,Rd,top} + M_{N,pl,Rd,bot} = 1092 + 1092 = 2184kNm$$

Note that we neglect the shear transfer from the columns to the joint in order to be conservative when we conduct the SCWB ratio check.

The SCWB ratio is computed,

$$\frac{\Sigma M_c}{\Sigma M_b} = \frac{2184}{957.8} = 2.28 > 1.3$$

Therefore, the check is verified.