FACULTE ENVIRONNEMENT NATUREL, ARCHITECTURAL ET CONSTRUIT ENAC
INSTITUT D'INGÉNIERIE CIVILE IIC
Laboratoire des Structures Métalliques Résilientes RESSLab

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : http://resslab.epfl.ch
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

Exercise #1 – Statics, deflections, periods of vibration and drift verifications

Problem #1: Review on statics and deflection calculations

Compute the deflection at the top of the two 2-storey frames shown in Figure 1. Assume that the point of inflection in the second story columns is at mid-height, even though this assumption is not always valid.

The following properties should be considered:

- All columns: $I_c = 1.66x10^9 mm^4$,
- All beams: $I_b = 1.25x10^9 mm^4$,
- $E = 30{,}000 MPa$.

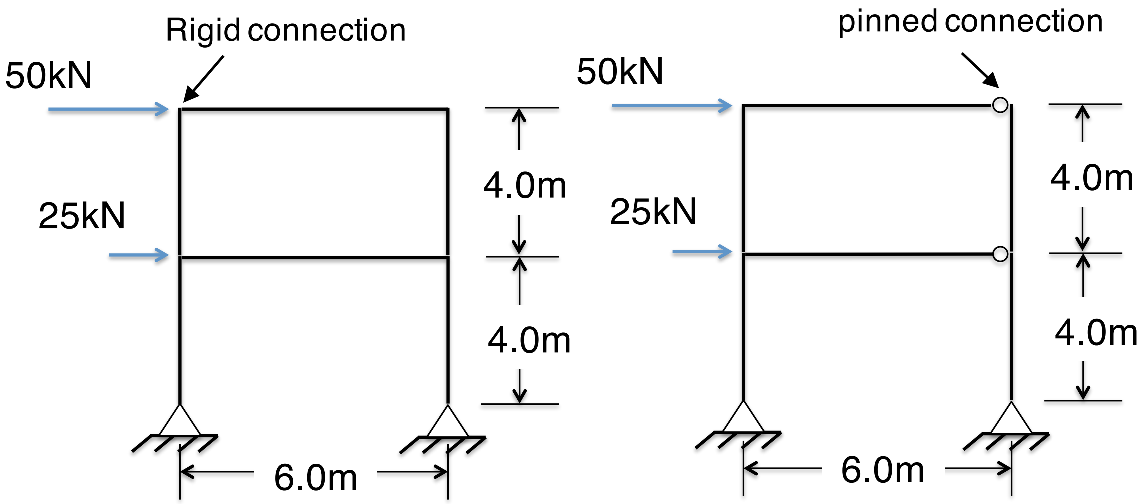


Figure 1. Two-storey frames

Problem #2: Lateral stiffness, natural period and seismic action computations

The one-storey steel industrial building is located at a high seismicity zone that is characterized by the pseudo-acceleration response spectrum shown in Figure 2-1 and its simplification, which is shown with the red solid line. The building, which is shown in Figure 2-2, can be idealized as a single-degree-of-freedom (SDF) system in each of its principal orthogonal directions. The columns are all I-shaped steel cross sections ($E = 200GPa$) that are fixed at their base and connected to a roof truss system at the top. The moments of inertia of the I-shaped cross sections are as follows, $I_y = 34.4x10^{-6}m^4$ and $I_y = 7.64x10^{-6}m^4$. The roof truss has a flexural stiffness that is significantly greater than that of the columns in the direction where the columns bend about their strong axis (assumed as fixed at the base) but has negligible flexural stiffness in the direction where the columns are bending about their weak axis (assumed as pinned at the base). In addition, in the two perimeter frames in the east-west (EW) direction, slender X-braces, made of 25-mm-diameter circular steel rods, are installed in three bays (a total of six braces per braced perimeter frame; only the ones in tension contribute to the lateral stiffness). The total dead load acting on the roof of the structure is equal to $1.06kPa$ while the total dead load acting on the perimeter walls is equal to $0.48kPa$.

1. Compute the natural period of vibration of the building in each one of its principal loading directions (N-S and E-W). Clearly list your assumptions (because depending on those, the expected period may vary). In any case, $T_1 \leq 0.4sec$.

2. Compute the expected base shear for seismic loading in both horizontal directions for the pseudo-acceleration response spectrum given in Figure 2-1.

3. Does the building meet a drift limit of 1% in both loading directions?

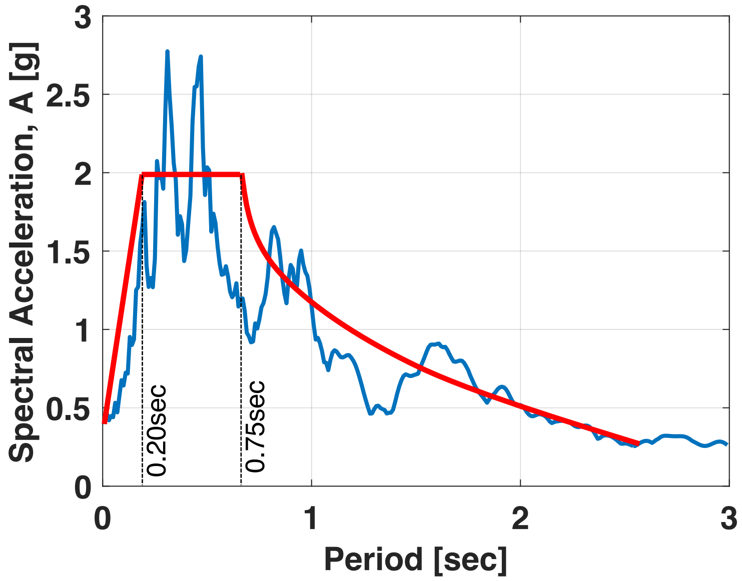


Figure 2-1. Pseudo-acceleration response spectrum

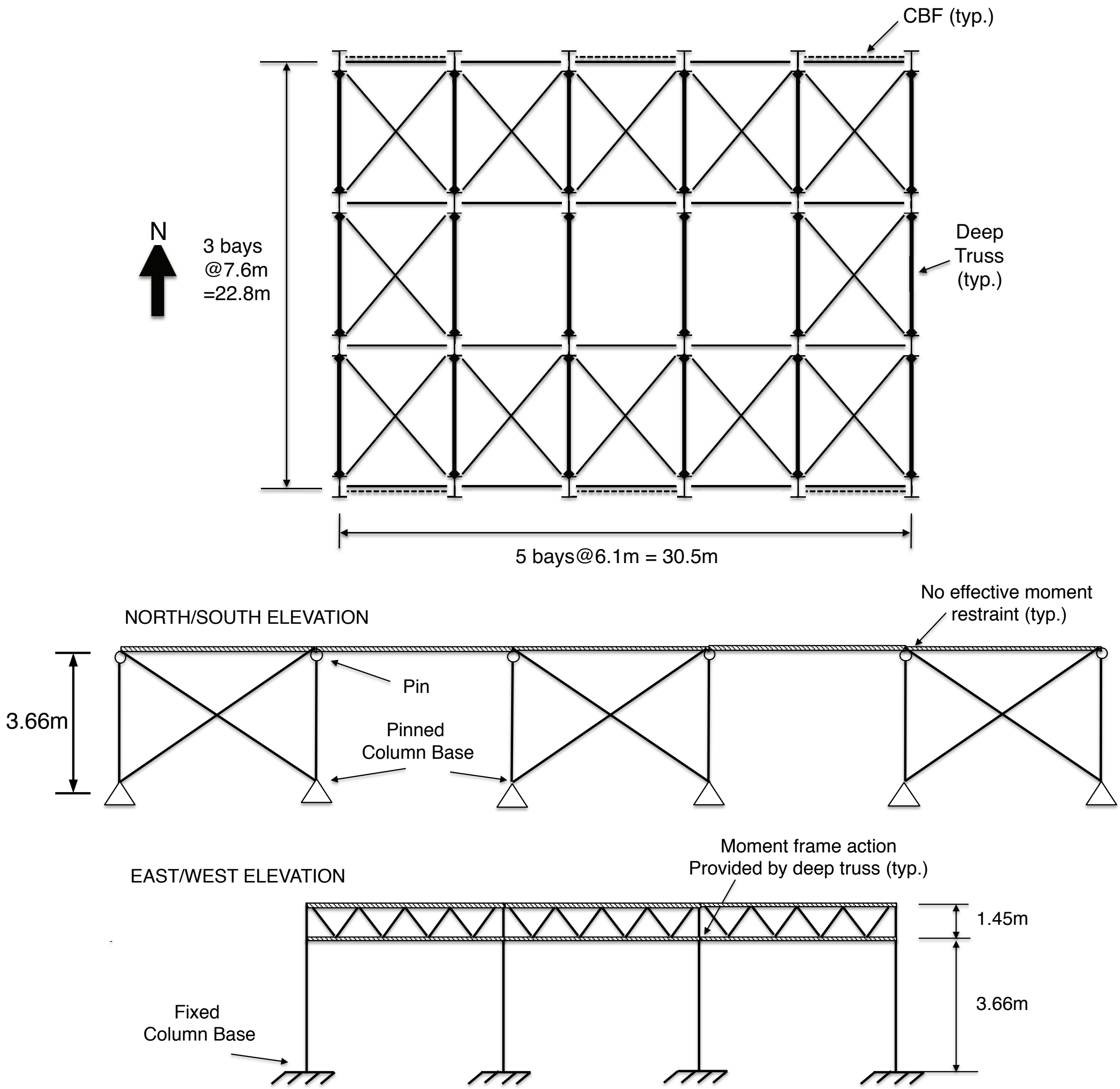
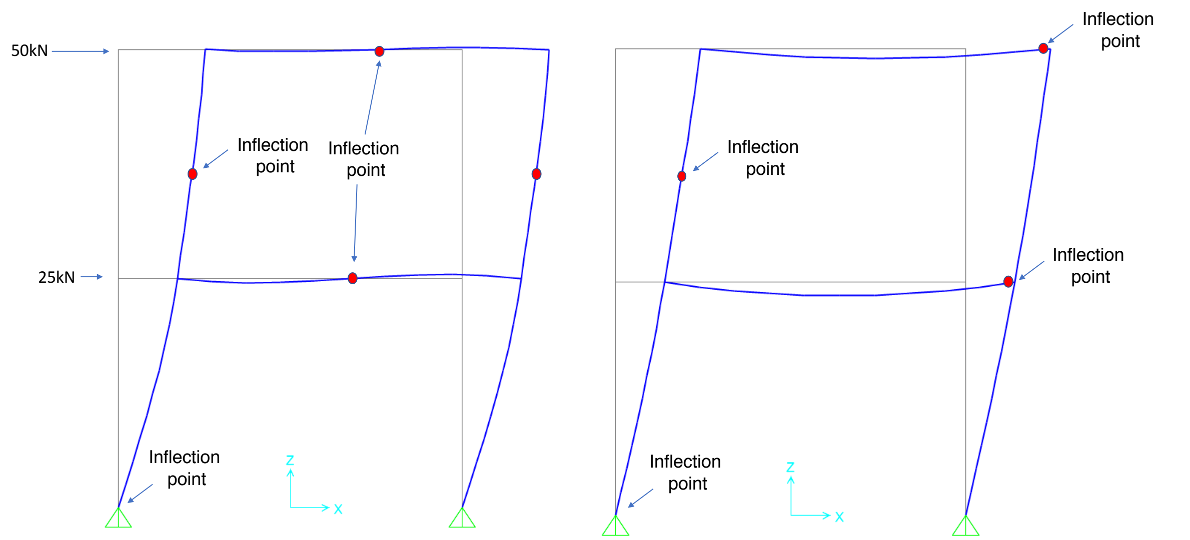


Figure 2-2. Single-storey industrial building; plan view and elevations

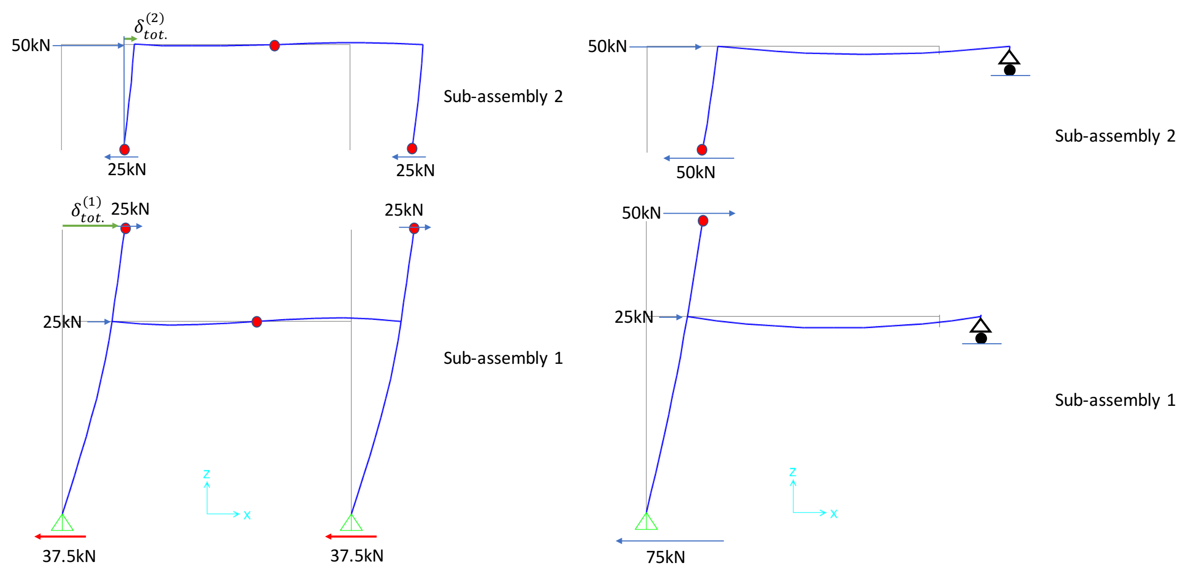
Problem 1 - Solution

Frame 1 (Left)



Left frame Right frame

We will split the frame into two sub-assemblies to compute the total deflection based on the inflection point locations. This is done as follows:



Left frame Right frame

The total deflection at the roof of the building is as follows,

$$\delta_{tot.} = \delta_{tot.}^{(1)} + \delta_{tot.}^{(2)}$$

Sub-assembly 1:

Deflection due to bending in the columns (assume infinitely rigid beam):

Bottom story column deflection:

$$\delta_{c,bot.}^{(1)} = \frac{F_{bot.}}{2 \cdot \left(\frac{3EI_c}{h^3}\right)} = \frac{75}{2 \cdot \left(\frac{3 \cdot 30 \cdot 1.66e9}{4000^3}\right)} = 16.06mm$$

Upper story half column deflection:

$$\delta_{c,top}^{(1)} = \frac{F_{top}}{2 \cdot \left(\frac{3EI_c}{(h/2)^3}\right)} = \frac{50}{2 \cdot \left(\frac{3 \cdot 30 \cdot 1.66e9}{(4000/2)^3}\right)} = 1.34mm$$

Deflection due to bending in the beam (assume rigid columns):

$$\delta_b^{(1)} = \theta \cdot \left(h + \frac{h}{2}\right) = \frac{M_b}{K_b} \cdot \frac{3h}{2} = \frac{M_b}{\frac{3 \cdot E \cdot I_b}{L_b/2}} \cdot \frac{3h}{2} = \frac{37.5 \cdot 4000 + 25 \cdot 2000}{\frac{3 \cdot 30 \cdot 1.25e9}{3000}} \cdot \frac{3 \cdot 4000}{2}$$
$$= 32mm$$

$$\delta_{tot.}^{(1)} = 16.06 + 1.34 + 32 = 49.4mm$$

Sub-assembly 2:

Deflection due to bending in the columns (assume infinitely rigid beam):

Story column deflection:

$$\delta_c^{(2)} = \frac{F}{2 \cdot \left(\frac{3EI_c}{(h/2)^3}\right)} = \frac{50}{2 \cdot \left(\frac{3 \cdot 30 \cdot 1.66e9}{(4000/2)^3}\right)} = 1.34mm$$

Deflection due to bending in the beam (assume infinitely rigid column):

$$\delta_b^{(2)} = \theta \cdot \frac{h}{2} = \frac{M_b}{K_b} \cdot \frac{h}{2} = \frac{M_b}{\frac{3 \cdot E \cdot I_b}{L_b/2}} \cdot \frac{h}{2} = \frac{25 \cdot 2000}{\frac{3 \cdot 30 \cdot 1.25e9}{3000}} \cdot \frac{4000}{2} = 2.67mm$$

$$\delta_{tot.}^{(2)} = 1.34 + 2.67 = 4.01mm$$

Therefore, the total deflection at the top of the moment-frame is as follows,

$$\delta_{tot.} = \delta_{tot.}^{(1)} + \delta_{tot.}^{(2)} = 49.4 + 4.01 = 53.4mm$$

Frame 2 (right):

The total deflection at the roof of the right frame is as follows,

$$\delta_{tot.} = \delta_{tot.}^{(1)} + \delta_{tot.}^{(2)}$$

Sub-assembly 1:

Deflection due to bending in the columns (assume infinitely rigid beam):

Bottom story column deflection:

In this case, the right column does not take any shear. Therefore,

$$\delta_{c,bot.}^{(1)} = \frac{F_{bot.}}{\left(\frac{3EI_c}{h^3}\right)} = \frac{75}{\left(\frac{3 \cdot 30 \cdot 1.66e9}{4000^3}\right)} = 32.13mm$$

Upper story half column deflection:

In this case, the right column does not take any shear. Therefore,

$$\delta_{c,top}^{(1)} = \frac{F_{top}}{\left(\frac{3EI_c}{(h/2)^3}\right)} = \frac{50}{\left(\frac{3 \cdot 30 \cdot 1.66e9}{(4000/2)^3}\right)} = 2.68mm$$

Deflection due to bending in the beam (assume rigid columns):

$$\delta_b^{(1)} = \theta \cdot \left(h + \frac{h}{2}\right) = \frac{M_b}{K_b} \cdot \frac{3h}{2} = \frac{M_b}{\frac{3 \cdot E \cdot I_b}{L_b}} \cdot \frac{3h}{2} = \frac{75 \cdot 4000 + 50 \cdot 2000}{\frac{3 \cdot 30 \cdot 1.25e9}{6000}} \cdot \frac{3 \cdot 4000}{2}$$
$$= 128mm$$

$$\delta_{tot.}^{(1)} = 32.13 + 2.68 + 128 = 162.81mm$$

Sub-assembly 2:

Deflection due to bending in the columns (assume infinitely rigid beam):

Story column deflection:

$$\delta_c^{(2)} = \frac{F}{\left(\frac{3EI_c}{(h/2)^3}\right)} = \frac{50}{\left(\frac{3 \cdot 30 \cdot 1.66e9}{(4000/2)^3}\right)} = 2.68mm$$

Deflection due to bending in the beam (assume infinitely rigid column):

$$\delta_b^{(2)} = \theta \cdot \frac{h}{2} = \frac{M_b}{K_b} \cdot \frac{h}{2} = \frac{M_b}{\frac{3 \cdot E \cdot I_b}{L_b}} \cdot \frac{h}{2} = \frac{50 \cdot 2000}{\frac{3 \cdot 30 \cdot 1.25e9}{6000}} \cdot \frac{4000}{2} = 10.67mm$$

$$\delta_{tot.}^{(2)} = 10.67 + 2.68 = 13.34mm$$

Therefore, the total deflection at the top of the moment-frame is as follows,

$$\delta_{tot.} = \delta_{tot.}^{(1)} + \delta_{tot.}^{(2)} = 162.81 + 13.34 = 176.15mm$$

Problem 2 – Solution

1. To compute the natural period in each of the principal directions, the seismic mass associated with these SDF systems shall be determined. It is assumed that the total dead load acting on the roof as well as the dead load corresponding to the top half of the perimeter walls will contribute to the total mass that is lumped at the top of the SDF system. First the weight is computed as follows:

W = 1.06kPa x 30.5m x 22.8m + 0.48kPa x 2 x (30.5m + 22.8m) x 1.83m = $830kN$.

The above weight corresponds to a mass, $m = W/g = 85kNs^2/m$.

Stiffness in North-South Direction (NS)

The lateral stiffness of the building in the NS direction is provided by six moment-resisting frames. Since the columns are fixed at their base and the flexural stiffness of the roof truss is much greater than the stiffness of the columns bending about their strong axis, we can determine that the lateral stiffness provided by each column in the moment frame is:

$$k_i = \frac{12EI_x}{h^3} = \frac{12\left(200\times10^6 kN/m^2\right)\left(34.4\times10^{-6} m^4\right)}{\left(3.66m\right)^3} = 1684kN/m$$

Therefore, the total stiffness of the building in the NS direction is:

$$k_{NS} = \sum_{i=1}^{24} k_i = 24\left(1684kN/m\right) = 40416kN/m$$

Stiffness in the East-West Direction (EW)

The lateral stiffness of the building in the EW direction is provided by 12 diagonal braces (six in each perimeter frame). However, because of the high slenderness of the bracing members, it is expected that the braces will buckle in compression at a very low lateral load and will not therefore contribute to the lateral stiffness of the building in this direction (i.e., can be assumed as tension-only braces). As such, only the stiffness provided by a total of six braces (three in each perimeter frame) acting in tension for any given direction of loading is considered in this calculation.

Assume that the length of the brace is L_b. Each diagonal brace provides the building with a lateral stiffness in this direction of:

$$k_i = \frac{EA}{L_b}\cos^2\theta$$

in which,

$$A = \frac{\pi d^2}{4} = 490\times10^{-6} m^2, L_b = \sqrt{3.66^2 + 6.10^2} = 7.11m$$

$$\theta = \tan^{-1}\left(\frac{3.66}{6.10}\right) = 0.858$$

Therefore,

$$k_i = \frac{\left(490\times10^{-6} m^2\right)\left(200\times10^{6} kN/m^2\right)}{7.11}\cos^2\left(31^o\right) = 10147 kN/m$$

The total stiffness provided by the braces acting in tension in the EW direction is therefore:

$$k_{EW-braces} = 6\times10147 kN/m = 60881 kN/m$$

Finally, the natural periods per loading direction can be calculated as follows:

$$T_{NS} = 2\pi\sqrt{\frac{m}{k_{NS}}} = 2\pi\sqrt{\frac{85kNs^2/m}{40416kN/m}} = 0.288\,sec$$

$$T_{EW} = 2\pi\sqrt{\frac{m}{k_{EW-braces}}} = 2\pi\sqrt{\frac{85kNs^2/m}{60881kN/m}} = 0.235\,sec$$

2. The base shear will be obtained using the spectrum for both horizontal directions. In both cases, T_{NS} and T_{EW} are within the range of 0.20sec to 0.75sec. Therefore, the anticipated elastic spectral acceleration ordinate is as follows,

$$A = 2\cdot g$$

The base shear can be obtained with the following relation:

$$V_b = A\cdot m_{Tot}$$

$$V_{b,NS} = V_{b,EW} = 2\cdot 9.81\cdot 85 = 1667.7kN$$

3. The corresponding deflection for the anticipated spectral acceleration is as follows:

$$u_{NS} = \frac{A}{\omega_{NS}^2} = A\left(\frac{T_{\mathrm{NS}}}{2\pi}\right)^2 = 2\cdot 9810\cdot\left(\frac{0.288}{6.28}\right)^2 = 41.3mm$$

$$u_{EW} = \frac{A}{\omega_{EW}^2} = A\left(\frac{T_{\mathrm{EW}}}{2\pi}\right)^2 = 2\cdot 9810\cdot\left(\frac{0.235}{6.28}\right)^2 = 27.5mm$$

Therefore, the corresponding drift limits are as follows:

$$\frac{u_{NS}}{h} = \frac{41.3}{3660} = 0.011 = 1.1\% > 1.0\%$$

$$\frac{u_{EW}}{h} = \frac{27.5}{3660} = 0.0075 = 0.75\% < 1.0\%$$

To satisfy the drift requirement in the NS direction, the second moment of area of the columns shall be increased accordingly.