EPFL

Spring 2025

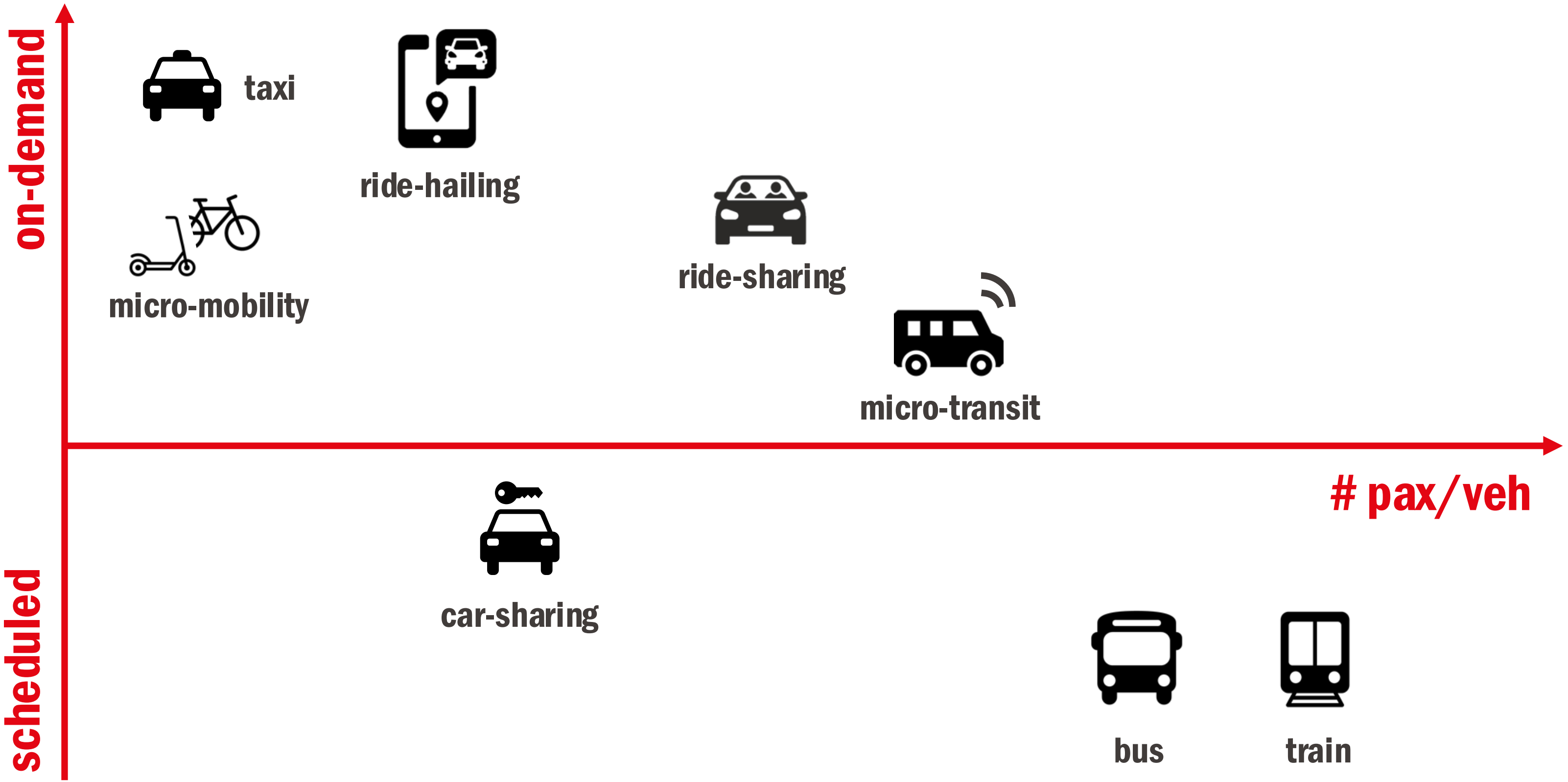
10 Emerging Topic I:
Shared mobility

CIVIL-477 Transportation network modeling & analysis

École
polytechnique
fédérale
de Lausanne

Overview

- A spectrum of shared mobility services

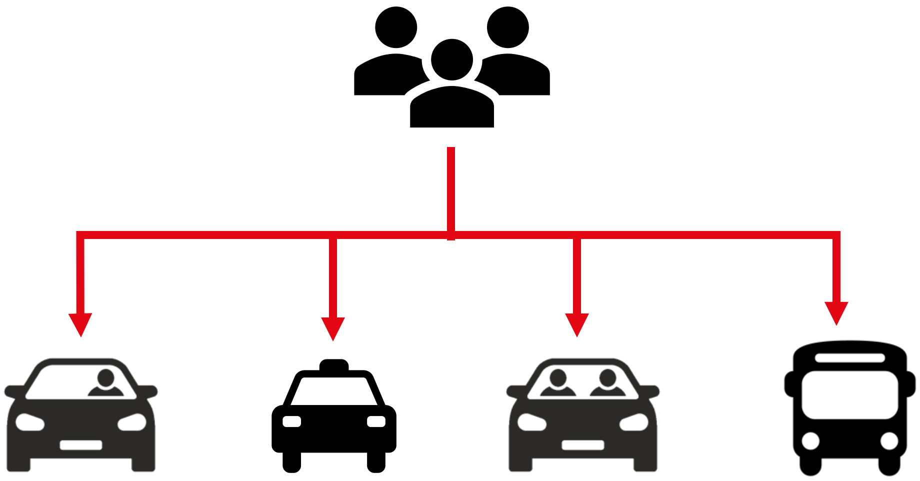


Overview

- Shared mobility services in the transportation system

Demand

Supply



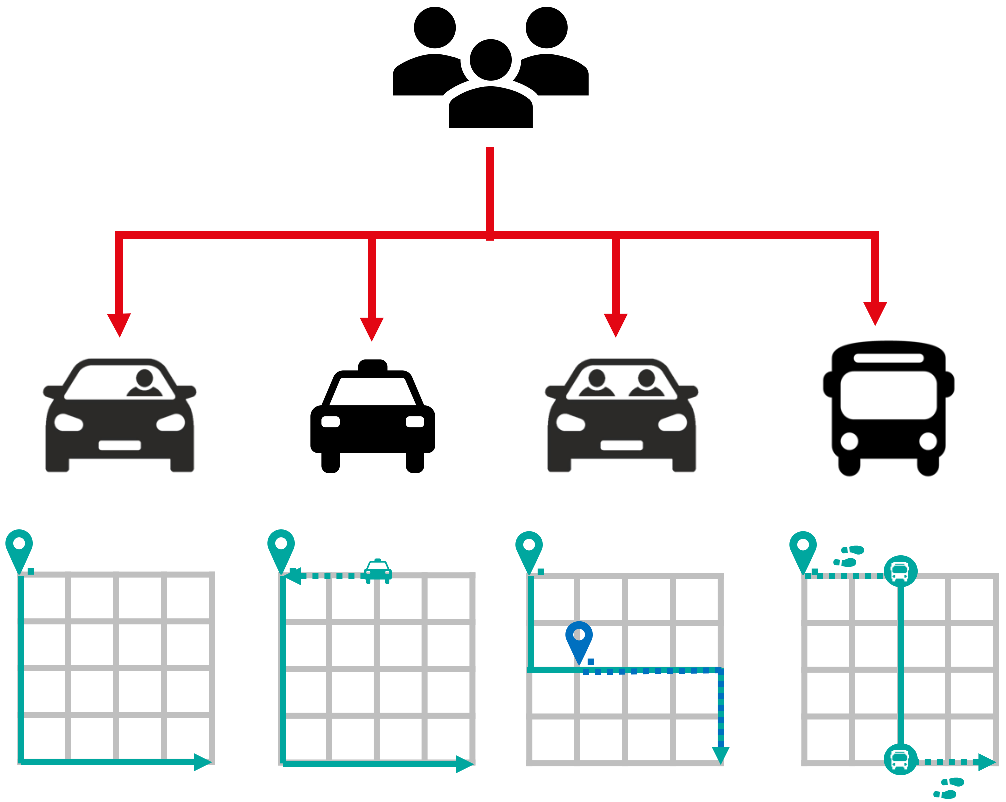
Monetary cost	$$$	$$	$$	$
Access time	+	++	++	+++
Riding time	+	+	++	+++

Overview

- Shared mobility services in the transportation system

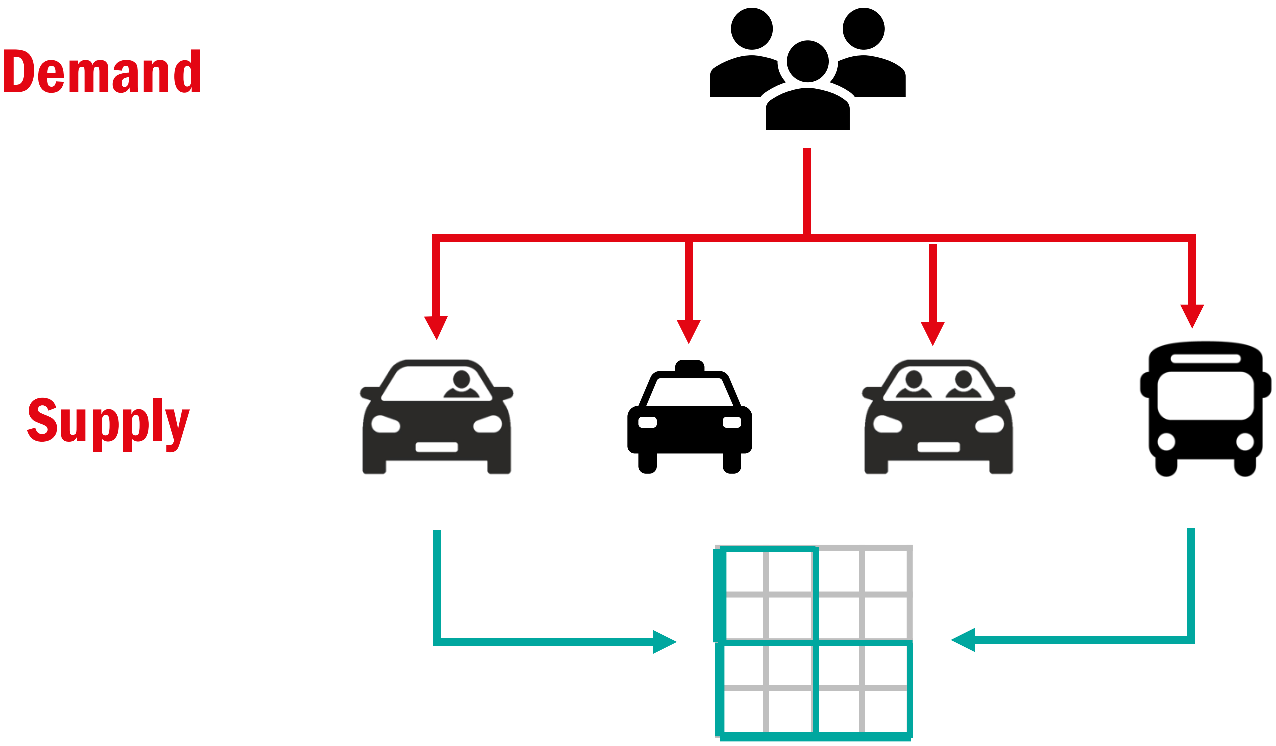
Demand

Supply



Overview

- Shared mobility services in the transportation system

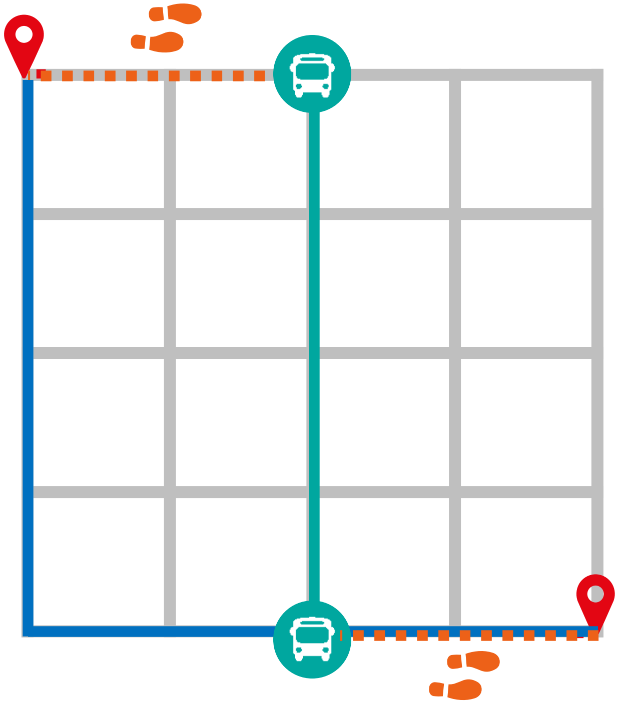


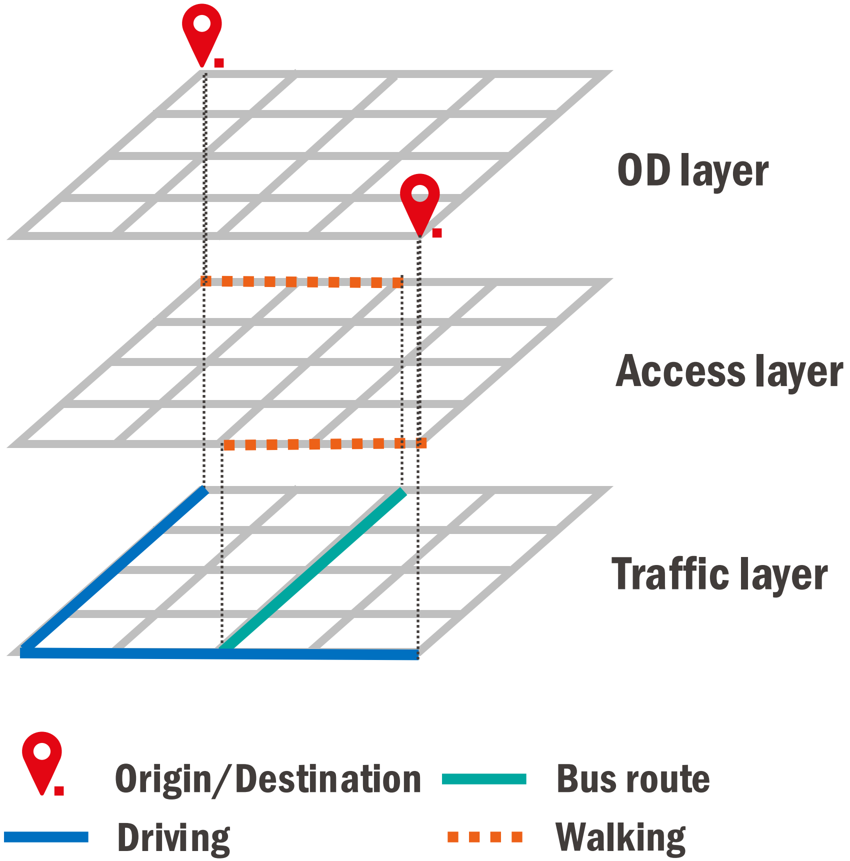
Outline

- Bus
  - Multimodal network
- Ride-sharing
  - Same vs different OD
- Ride-hailing
  - Empty vehicle movements
- First-mile service
  - Integration to mass transit

Mutlimodal network

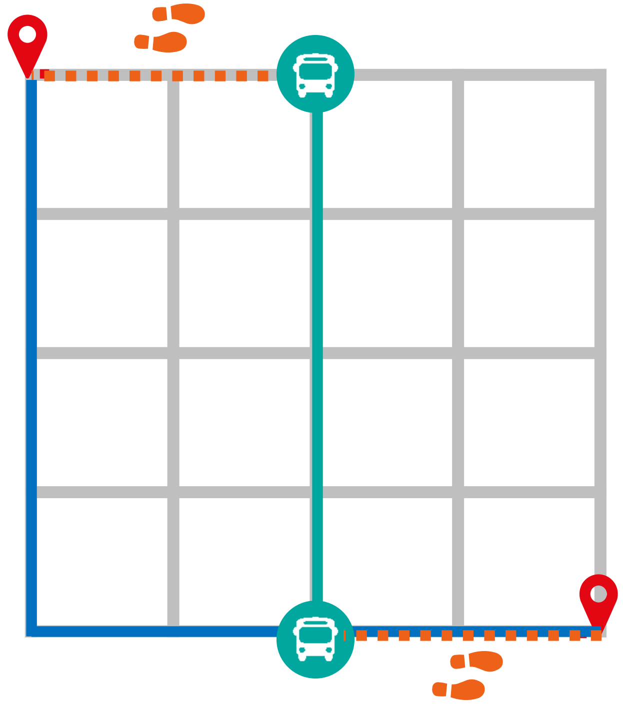
- From physical to super network

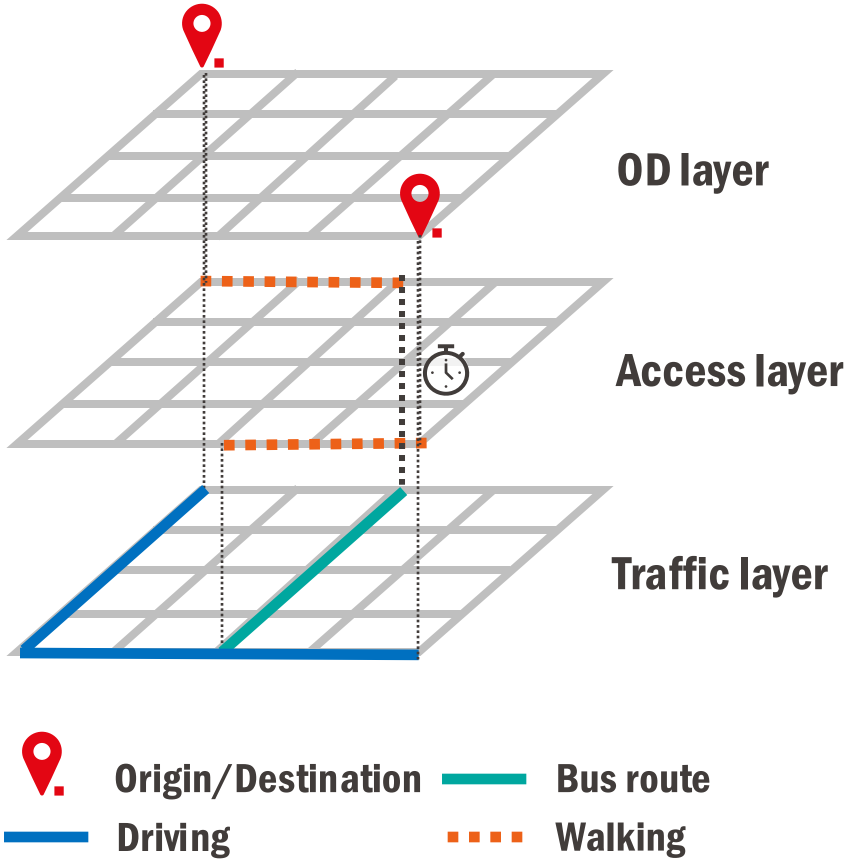




Mutlimodal network

- From physical to super network
  - Waiting time for bus ⏱





Multimodal network

- Static traffic assignment in multimodal network

$$(\mathbf{f}^*)^T(\mathbf{c}^* - \Lambda^T\mu^*) = 0$$ **Complementary condition of path flows**

$$\mathbf{c}^* - \Lambda^T\mu^* \geq \mathbf{0}$$ **Min path cost**

$$\Lambda\mathbf{f}^* = \mathbf{q}$$ **Demand flow conservation**

$$\mathbf{f}^* \geq \mathbf{0}$$ **Path flow feasibility**

- Multimodal path cost
  - Driving path $c_k = \sum_{a\in A_3} \delta_{ak}^3 t_a^3(x_a^{\text{veh}} + x_a^{\text{bus}})$
  - Bus path $c_k = \sum_{a\in A_1} \delta_{ak}^1 t_a^1 + \sum_{a\in A_2} \delta_{ak}^2 t_a^2 + \sum_{a\in A_3} \delta_{ak}^3 t_a^3(x_a^{\text{veh}} + x_a^{\text{bus}})$

access time | **wait/transfer time** | **riding time**

Multimodal network

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- Multimodal path cost
  - Driving path $c_k = \sum_{a\in A_3} \delta_{ak}^3 t_a^3(x_a^{\text{veh}} + x_a^{\text{bus}})$ *** independent of bus travel flow**
  - Bus path $c_k = \sum_{a\in A_1} \delta_{ak}^1 t_a^1 + \sum_{a\in A_2} \delta_{ak}^2 t_a^2 + \sum_{a\in A_3} \delta_{ak}^3 t_a^3(x_a^{\text{veh}} + x_a^{\text{bus}})$

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*** another set of bus links whose travel time depend on traffic link flows**

Multimodal network

- Static traffic assignment in multimodal network

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*** additional stopping added to each bus link**

Multimodal network

- Static traffic assignment in multimodal network

$$(\mathbf{f}^*)^T(\mathbf{c}^* - \Lambda^T\mu^*) = 0$$ **Complementary condition of path flows**

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  - Driving path $c_k = \sum_{a\in A_3} \delta_{ak}^3 t_a^3(x_a^{\text{veh}})$
  - Bus path $c_k = \sum_{a\in A_1} \delta_{ak}^1 t_a^1 + \sum_{a\in A_2} \delta_{ak}^2 t_a^2 + \sum_{a\in A_4} \delta_{ak}^4 t_a^4(x_a^{\text{bus}}, \omega)$

*** link interaction no longer exist when bus lanes are utilized, while link parameters also change**

Multimodal network

- Static traffic assignment in multimodal network

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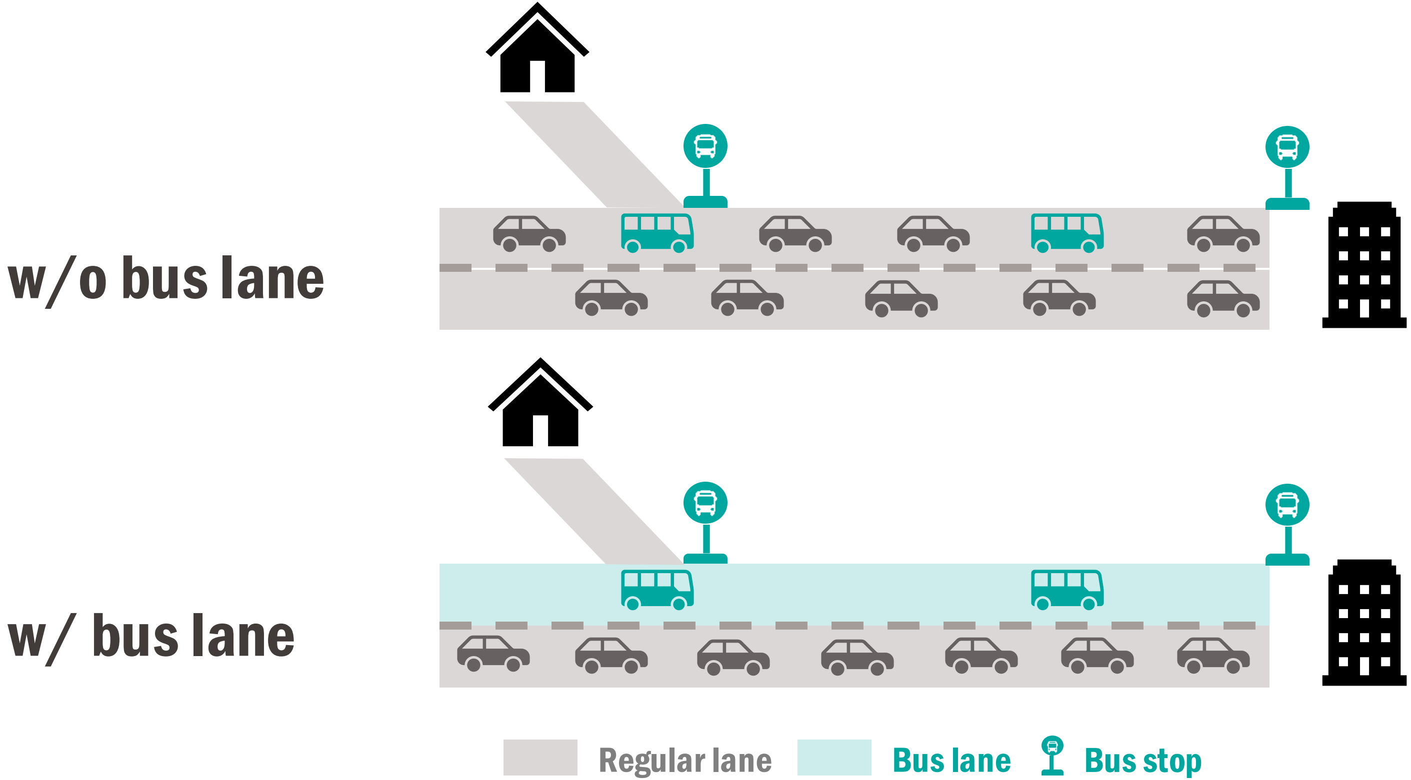
- Multimodal path cost
  - Driving path $c_k = c_0^{\text{drive}} + \sum_{a\in A_3} \delta_{ak}^3 t_a^3(x_a^{\text{veh}})$
  - Bus path $c_k = c_0^{\text{bus}} + \sum_{a\in A_1} \delta_{ak}^1 t_a^1 + \sum_{a\in A_2} \delta_{ak}^2 t_a^2 + \sum_{a\in A_4} \delta_{ak}^4 t_a^4(x_a^{\text{bus}}, \omega)$

*** include monetary cost that makes bus trips more attractive**

Multimodal network

- Static traffic assignment in multimodal network
  - A toy example of bus lane



Questions?

Outline

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  - Same vs different OD
- Ride-hailing
  - Empty vehicle movements
- First-mile service
  - Integration to mass transit

Ride-sharing vs ride-hailing

- Ride-sharing
  - Drivers have own trips
  - Often scheduled in ahead
  - Higher sensitivity on detour and other inconvenience
  - Decentralized matching and pricing

- Ride-hailing
  - Drivers deliver trips to make a revenue
  - Usually on-demand
  - Higher sensitivity on price and waiting time
  - Centralized matching and pricing

Ride-sharing between same OD

- Static traffic assignment + same OD ride-sharing

$$(\mathbf{f}^*)^T(\mathbf{c}^* - \Lambda^T\mu^*) = 0$$ **Complementary condition of path flows**

$$\mathbf{c}^* - \Lambda^T\mu^* \geq \mathbf{0}$$ **Min path cost**

$$\Lambda(\mathbf{f}^*_{\text{solo}} + m\mathbf{f}^*_{\text{share}}) = \mathbf{q}$$ **Demand flow conservation**

$$\mathbf{f}^* \geq \mathbf{0}$$ **Path flow feasibility**

  - $\mathbf{f}_{\text{solo}}, \mathbf{f}_{\text{share}}$: path flow of solo-driving/ride-sharing vehicles
  - m: vehicle occupancy

- Path cost
  - Solo-driving $c_k = \sum_{a\in A}\delta t_a\left(x_a^{\text{solo}} + x_a^{\text{share}}\right), \quad k \in P_w, w \in W$
  - Ride-sharing $c_k = c_0\left(q_w^{\text{share}}\right) + \sum_{a\in A}\delta t_a\left(x_a^{\text{solo}} + x_a^{\text{share}}\right), \quad k \in P_w, w \in W$
    - variable extra cost c_0 depends on the sharing demand $\mathbf{q}_{\text{share}} = m\Lambda\mathbf{f}_{\text{share}}$

- ***Q: What is the issue with this model?***

Ride-sharing between same OD

- Static traffic assignment + same OD ride-sharing

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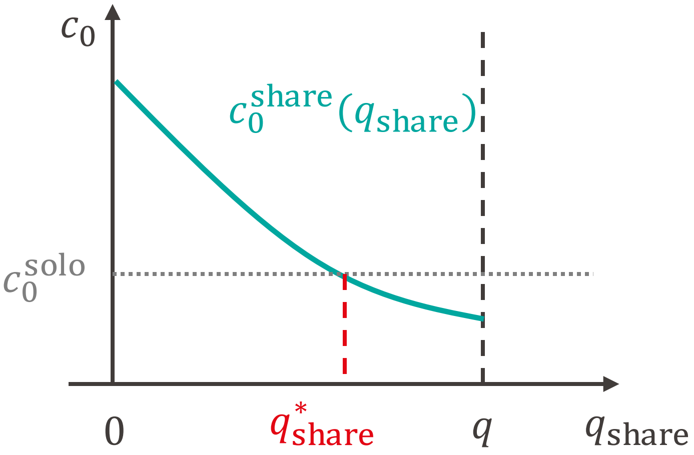
$$\mathbf{f}^* \geq \mathbf{0}$$ **Path flow feasibility**

  - $\mathbf{f}_{\text{solo}}, \mathbf{f}_{\text{share}}$: path flow of solo-driving/ride-sharing vehicles
  - m: vehicle occupancy
- Path cost
  - Solo-driving $c_k = c_0^{\text{solo}} + \sum_{a\in A}\delta t_a\left(x_a^{\text{solo}} + x_a^{\text{share}}\right), \quad k \in P_w, w \in W$
    - fixed extra cost c_0^{solo}
  - Ride-sharing $c_k = c_0^{\text{share}}\left(q_w^{\text{share}}\right) + \sum_{a\in A}\delta t_a\left(x_a^{\text{solo}} + x_a^{\text{share}}\right), \quad k \in P_w, w \in W$
    - variable extra cost c_0^{share} depends on the sharing demand $\mathbf{q}_{\text{share}} = m\Lambda\mathbf{f}_{\text{share}}$
- ***Q: What is the easier way to solve the equilibrium?***

Ride-sharing between same OD

- Static traffic assignment + same OD ride-sharing
  - Step 1: Solve demand split $\mathbf{q}_{\text{solo}}, \mathbf{q}_{\text{share}}$
    - As all vehicles take the shortest path, the cost of solo-driving and ride-sharing only differs in the extra cost

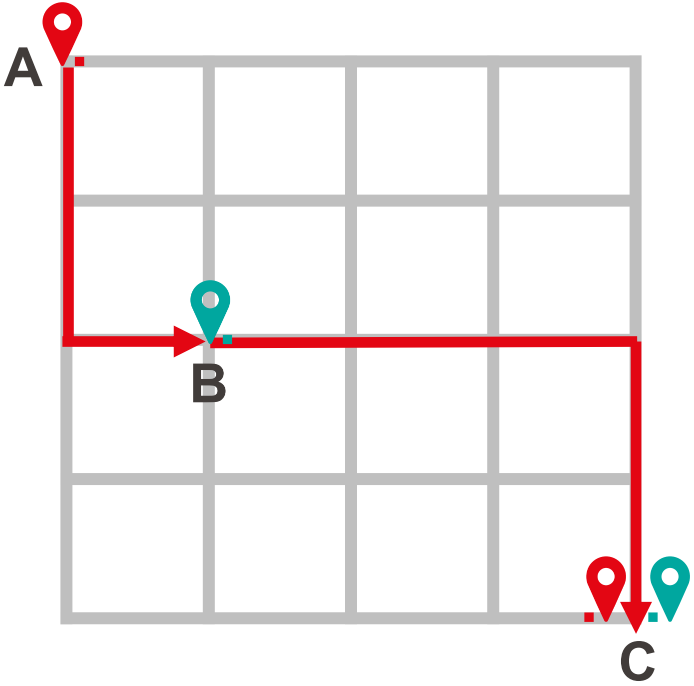


  - Step 2: Solve traffic equilibrium with adjusted demand

$$\tilde{\mathbf{q}} = \mathbf{q}_{\text{solo}} + \frac{\mathbf{q}_{\text{share}}}{m}$$

Ride-sharing between different OD

- Segmentation of trips in ride-sharing



rider demand $q_{BC}^{\text{pax}} = 100$

w/o sharing $q_{AC} = 100$

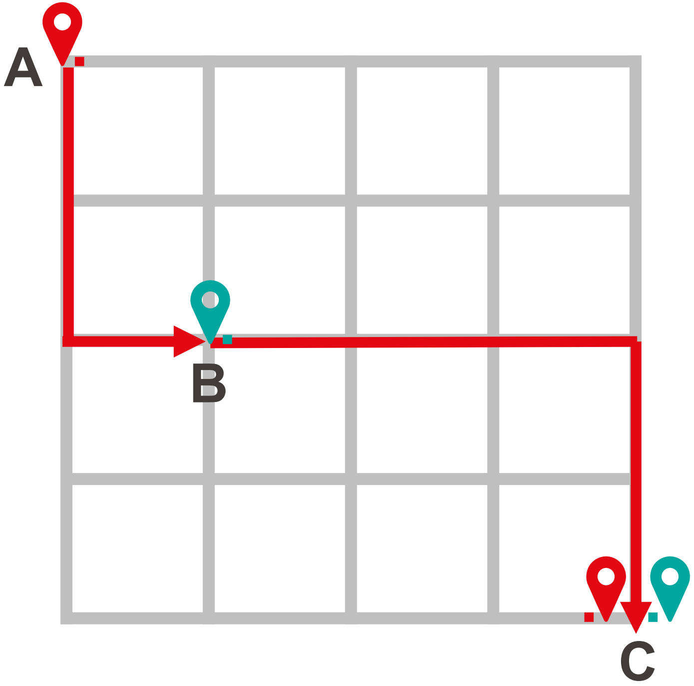
w/ sharing $\tilde{q}_{AB} = 100, \tilde{q}_{BC} = 100$

	AB	AC	BC
q	0	100	0
q̃	100	0	100

- ***Q: How to express the demand segmentation in a compact way?***

Ride-sharing between different OD

- Segmentation of trips in ride-sharing



rider demand $q_{BC}^{\text{pax}} = 100$

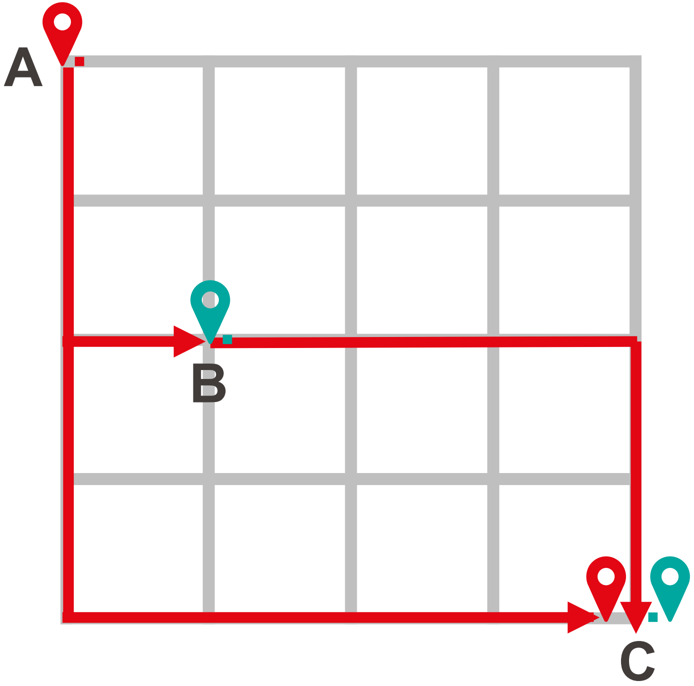
w/o sharing $q_{AC} = 100$

w/ sharing $\tilde{q}_{AB} = 100, \tilde{q}_{BC} = 100$

	Γ			$\mathbf{q}$		$\tilde{\mathbf{q}}$
AB	0	1	0	0		100
AC	0	0	0	100	=	0
BC	0	1	0	0		100

Ride-sharing between different OD

- Segmentation of trips in ride-sharing



rider demand $q_{BC}^{\text{pax}} = 50$

w/o sharing $q_{AC} = 100$

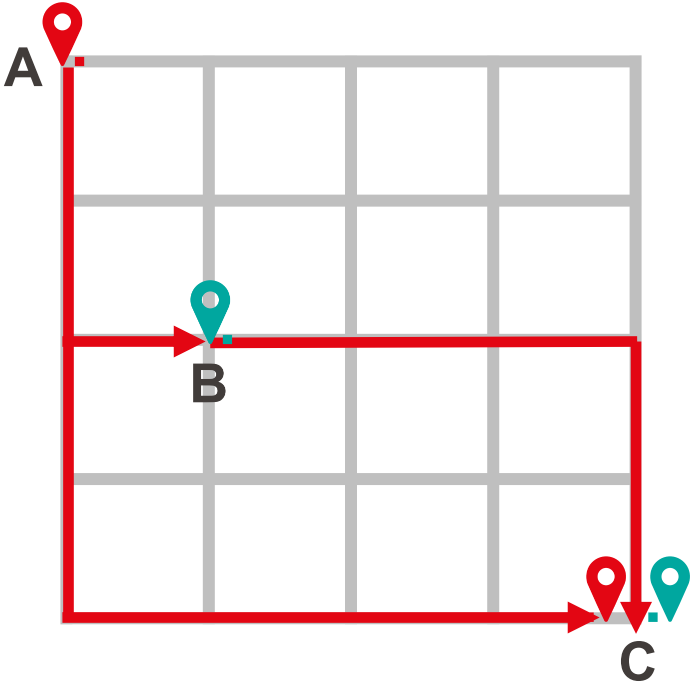
w/ sharing $\tilde{q}_{AB} = 50, \tilde{q}_{AC} = 50, \tilde{q}_{BC} = 50$

	Γ			$\mathbf{q}$		$\tilde{\mathbf{q}}$
AB	0	0.5	0	0		50
AC	0	0.5	0	100	=	50
BC	0	0.5	0	0		50

*** values from origin sums up to 1**

Ride-sharing between different OD

- Segmentation of trips in ride-sharing



rider demand $q_{BC}^{\text{pax}} = 50$

w/o sharing $q_{AC} = 100$

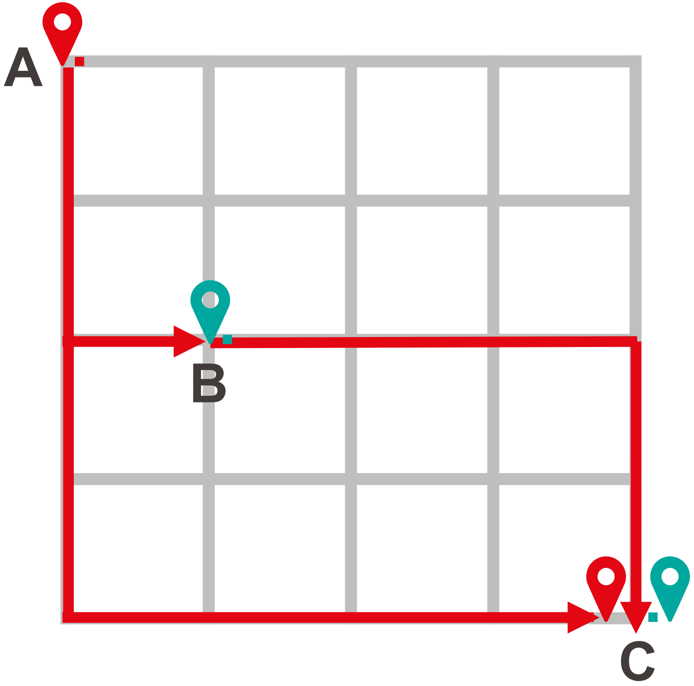
w/ sharing $\tilde{q}_{AB} = 50, \tilde{q}_{AC} = 50, \tilde{q}_{BC} = 50$

	Γ			$\mathbf{q}$		$\tilde{\mathbf{q}}$
AB	0	0.5	0	0		50
AC	0	0.5	0	100	=	50
BC	0	0.5	0	0		50

*** values to destination sums up to 1**

Ride-sharing between different OD

- Segmentation of trips in ride-sharing



rider demand $q_{BC}^{\text{pax}} = 50$

w/o sharing $q_{AC} = 100$

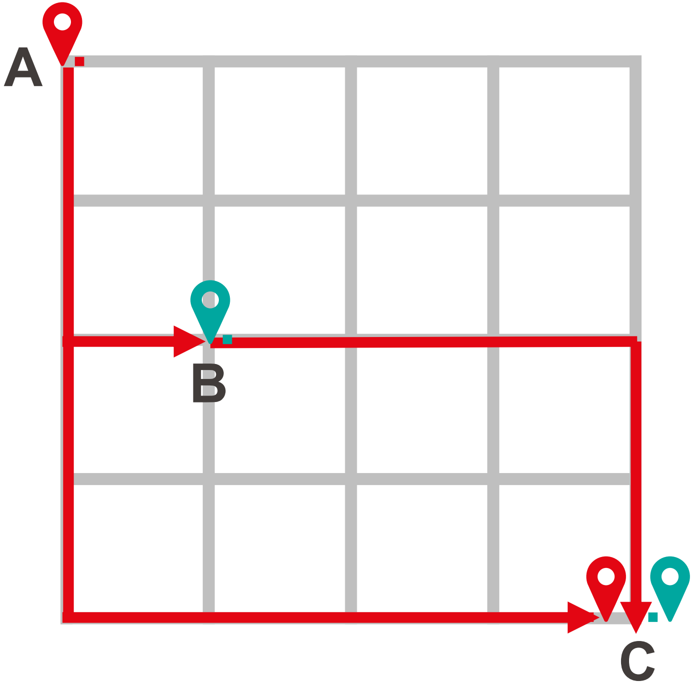
w/ sharing $\tilde{q}_{AB} = 50, \tilde{q}_{AC} = 50, \tilde{q}_{BC} = 50$

	Γ			$\mathbf{q}$		$\tilde{\mathbf{q}}$
AB	0	0.5	0	0		50
AC	0	0.5	0	100	=	50
BC	0	0.5	0	0		50

*** inflow equals outflow at detour node**

Ride-sharing between different OD

- Segmentation of trips in ride-sharing



rider demand $q_{BC}^{\text{pax}} = 50$

w/o sharing $q_{AC} = 100$

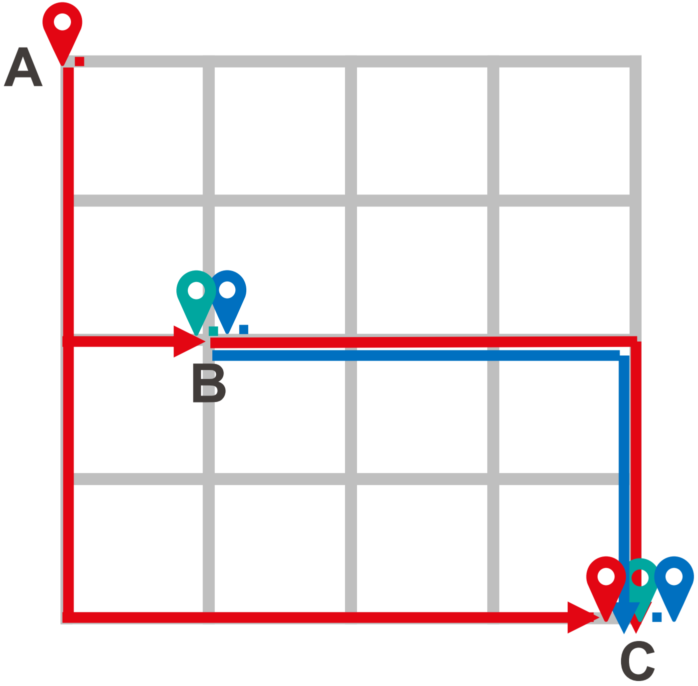
w/ sharing $\tilde{q}_{AB} = 50, \tilde{q}_{AC} = 50, \tilde{q}_{BC} = 50$

	Γ			$\mathbf{q}$		$\tilde{\mathbf{q}}$
AB	0	0.5	0	0		50
AC	0	0.5	0	100	=	50
BC	0	0.5	0	0		50

*** rider demand is satisfied**

Ride-sharing between different OD

- Segmentation of trips in ride-sharing



rider demand $q_{BC}^{\text{pax}} = 100$

w/o sharing $q_{AC} = 100, q_{BC} = 50$

w/ sharing $\tilde{q}_{AB} = 50, \tilde{q}_{AC} = 50, \tilde{q}_{BC} = 50$

	Γ			$\mathbf{q}$		$\tilde{\mathbf{q}}$
AB	0	0.5	0	0		50
AC	0	0.5	0	100	=	50
BC	0	0.5	1	50		100

*** riders can be served by drivers between different ODs**

Ride-sharing between different OD

- Static traffic assignment + different OD ride-sharing
  - Subproblem 1: Solve demand segmentation Γ
    - centralized matching

$$\min_{\Gamma} \quad \sum_{w,w'} \Gamma_{ww'} q_{w'} \mu_w^*$$

$$s.t. \quad \sum_{w'} \Gamma_{w^r w'} q_{w'} = q_w^r, \ \forall w \in W^r$$

$$\sum_{w(r,\cdot)} \Gamma_{ww'} = 1, \qquad \forall w' = (r,s)$$

$$\sum_{w(\cdot,s)} \Gamma_{ww'} = 1, \qquad \forall w' = (r,s)$$

$$\Gamma_{w_1 w'} = \Gamma_{w_2 w'}, \qquad \forall w' = (r,s), w_1 = (r,i), w_2 = (i,s)$$

$$\mathbf{0} \le \Gamma \le \mathbf{1}$$

* rider demand is satisfied

* flow conservation at origin/destination/detour point

  - μ_w^*: min path cost by Subproblem 2
  - Subproblem 2: Solve traffic equilibrium with adjusted demand

$$\tilde{\mathbf{q}} = \Gamma \mathbf{q}$$

Ride-sharing between different OD

- Static traffic assignment + different OD ride-sharing
  - Subproblem 1: Solve demand segmentation Γ
    - centralized matching

$$\min_{\Gamma} \quad (\Gamma \mathbf{q})^T \boldsymbol{\mu}^* = {\boldsymbol{\mu}^*}^T \Gamma \mathbf{q}$$

$$s.t. \quad B^{\mathrm{pax}}(\Gamma \mathbf{q}) = \mathbf{q}^r$$

$$B^{\mathrm{r}}\,\Gamma = \mathbf{1}$$

$$B^{\mathrm{s}}\Gamma = \mathbf{1}$$

$$B^{\mathrm{d}}\Gamma = \mathbf{0}$$

$$\mathbf{0} \leq \Gamma \leq \mathbf{1}$$

*** linear programming given $\boldsymbol{\mu}^*$**

- $\boldsymbol{\mu}^*$: min path cost by Subproblem 2
- $B^{\mathrm{pax}}, B^{\mathrm{r}}, B^{\mathrm{s}}, B^{\mathrm{d}}$ are all binary matrices

  - Subproblem 2: Solve traffic equilibrium with adjusted demand

$$\tilde{\mathbf{q}} = \Gamma \mathbf{q}$$

Questions?

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  - Empty vehicle movements
- First-mile service
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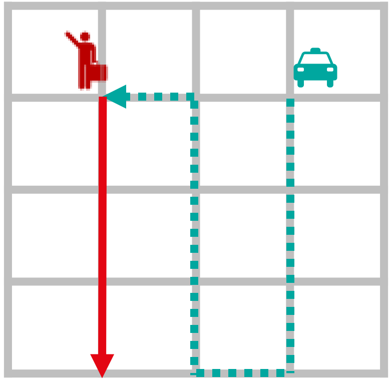
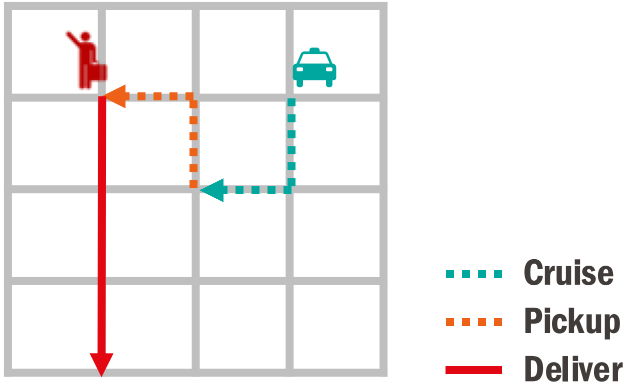
Empty ride-hailing vehicle movements

- Cruising and pickup in different ride-hailing services

street-hailing



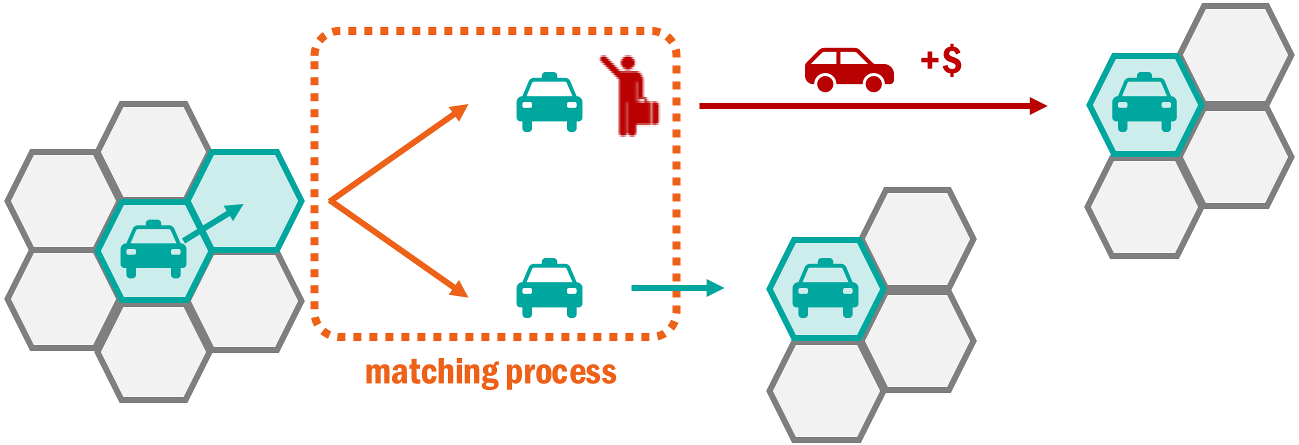
e-hailing



Empty ride-hailing vehicle movements

- Cruising and pickup in different ride-hailing services
  - Instead of following the shortest path as in regular trips, drivers make sequential decisions on the search location
  - The search decisions can be modeled as a Markov decision process (MDP)
    - solution of MDP is expressed a policy

    “Where should I go to find passengers when I’m located in A (at time t)?”



Empty ride-hailing vehicle movements

- Static traffic assignment + ride-hailing
  - Simplifications
    - all travel demands are served by ride-hailing vehicles
    - demand is perfectly known and thus empty vehicles move to passenger trip origins in the most efficient way
  - Demand matching
    - Passenger demand $\mathbf{q}_{\text{pax}}$ and empty vehicle trips $\mathbf{q}_{\text{veh}}$

$$\sum_{w:(\cdot,i)\in W} q_w^{\text{veh}} \geq \sum_{w:(i,\cdot)\in W} q_w^{\text{pax}}, \qquad \forall i \in N$$

*** empty vehicles arriving at each origin are more than departing passengers**

$$\sum_{w:(\cdot,i)\in W} q_w^{\text{pax}} \geq \sum_{w:(i,\cdot)\in W} q_w^{\text{veh}}, \qquad \forall i \in N$$

*** vehicles ending trips at each destination are more than relocating empty vehicles**

Empty ride-hailing vehicle movements

- Static traffic assignment + ride-hailing
  - Subproblem 1: Solve empty vehicle trips $\mathbf{q}_{\text{veh}}$
    - centralized rebalancing

$$\min_{\mathbf{q}_{\text{veh}}} \sum_{w \in W} q_w^{\text{veh}} \mu_w^*$$

$$s.t. \quad \sum_{w:(\cdot,i)\in W} q_w^{\text{veh}} \geq \sum_{w:(i,\cdot)\in W} q_w^{\text{pax}}, \ \forall i$$

$$\sum_{w:(\cdot,i)\in W} q_w^{\text{pax}} \geq \sum_{w:(i,\cdot)\in W} q_w^{\text{veh}}, \ \forall i$$

$$\mathbf{q}_{\text{veh}} \geq \mathbf{0}$$

- μ_w^*: min path cost by Subproblem 2

  - Subproblem 2: Solve traffic equilibrium with aggregate demand

$$\mathbf{q} = \mathbf{q}_{\text{pax}} + \mathbf{q}_{\text{veh}}$$

Empty ride-hailing vehicle movements

- Static traffic assignment + ride-hailing
  - Subproblem 1: Solve empty vehicle trips $\mathbf{q}_{\text{veh}}$
    - centralized rebalancing

$$\min_{\mathbf{q}_{\text{veh}}} \quad \mathbf{q}_{\text{veh}}^T \boldsymbol{\mu}^*$$
$$s.t. \quad B^{\text{s}} \mathbf{q}_{\text{veh}} \geq B^{\text{r}} \mathbf{q}_{\text{pax}}$$
$$B^{\text{r}} \mathbf{q}_{\text{veh}} \leq B^{\text{s}} \mathbf{q}_{\text{pax}}$$
$$\mathbf{q}_{\text{veh}} \geq \mathbf{0}$$

*** linear programming given $\boldsymbol{\mu}^*$**

- $\boldsymbol{\mu}^*$: min path cost by Subproblem 2
- $B^{\text{r}}, B^{\text{s}}$ are both binary matrices

  - Subproblem 2: Solve traffic equilibrium with aggregate demand

$$\mathbf{q} = \mathbf{q}_{\text{pax}} + \mathbf{q}_{\text{veh}}$$

- ***Q: What if the passenger trips are also centralized controlled?***

Empty ride-hailing vehicle movements

- Static traffic assignment + ride-hailing
  - Centralized trip routing and rebalancing (e.g., robotaxi)

$$\min_{\mathbf{f},\, \mathbf{q}_{\mathrm{veh}}} \quad \mathbf{x}^T \mathbf{t}(\mathbf{x})$$

$$s.t. \quad \Lambda \mathbf{f} = \mathbf{q}_{\mathrm{pax}} + \mathbf{q}_{\mathrm{veh}}$$

$$\Delta \mathbf{f} = \mathbf{x}$$

$$\mathbf{f} \geq \mathbf{0}$$

$$B^{\mathrm{s}} \mathbf{q}_{\mathrm{veh}} \geq B^{\mathrm{r}} \mathbf{q}_{\mathrm{pax}},$$

$$B^{\mathrm{r}} \mathbf{q}_{\mathrm{veh}} \leq B^{\mathrm{s}} \mathbf{q}_{\mathrm{pax}},$$

$$\mathbf{q}_{\mathrm{veh}} \geq \mathbf{0}$$

*** original SO problem**

Questions?

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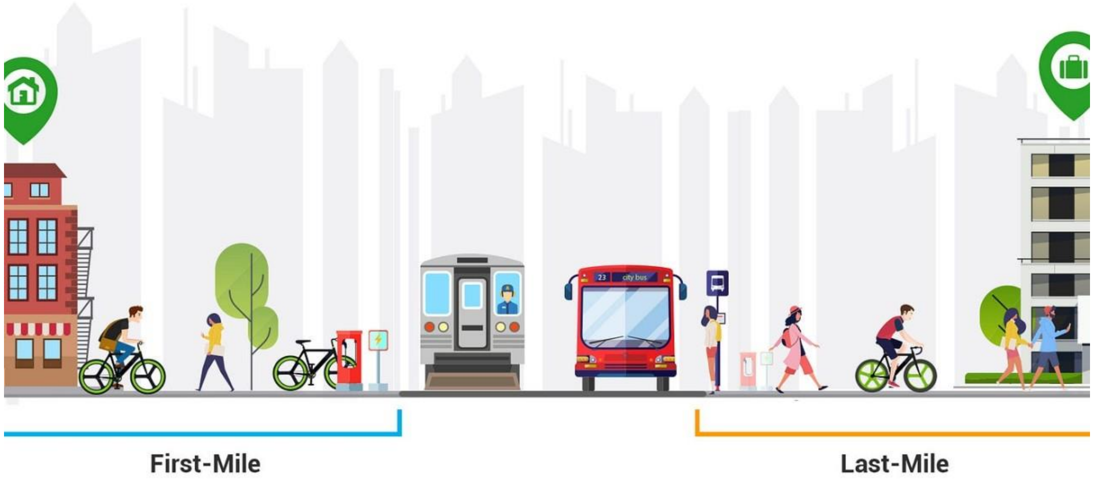
First-mile service

- On-demand mobility services that connect individual trip origins to mass transit hubs
  - Shuttles and microtransit
  - Ride-hailing
  - Bike-sharing and scooter sharing



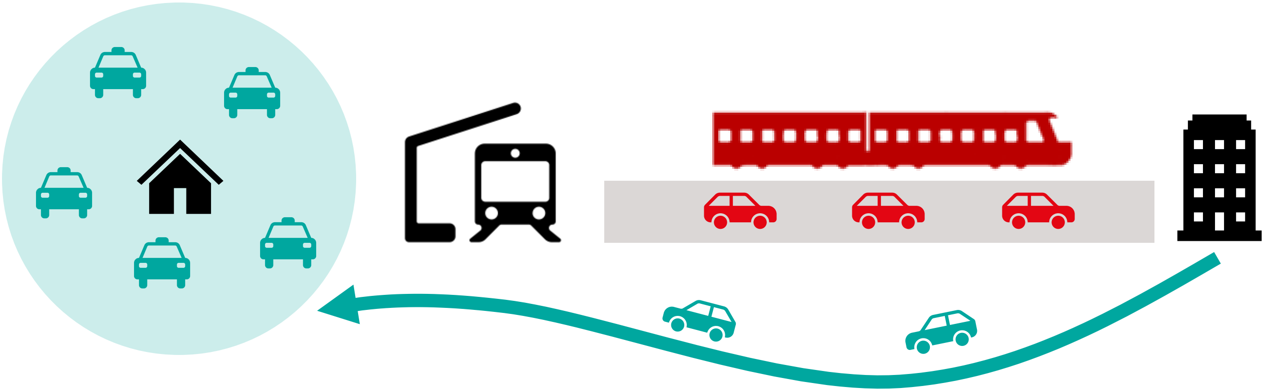
First-mile service

- On-demand mobility services that connect individual trip origins to mass transit hubs
  - Shuttles and microtransit
  - Ride-hailing
  - Bike-sharing and scooter sharing

- Incentives of stakeholders
  - City
    - reduce traffic congestion and promote public transit
  - Mass transit operator
    - encourage ridership
  - Travelers
    - lower travel cost
  - ***Q: How about mobility service operator (e.g., Uber)?***

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service

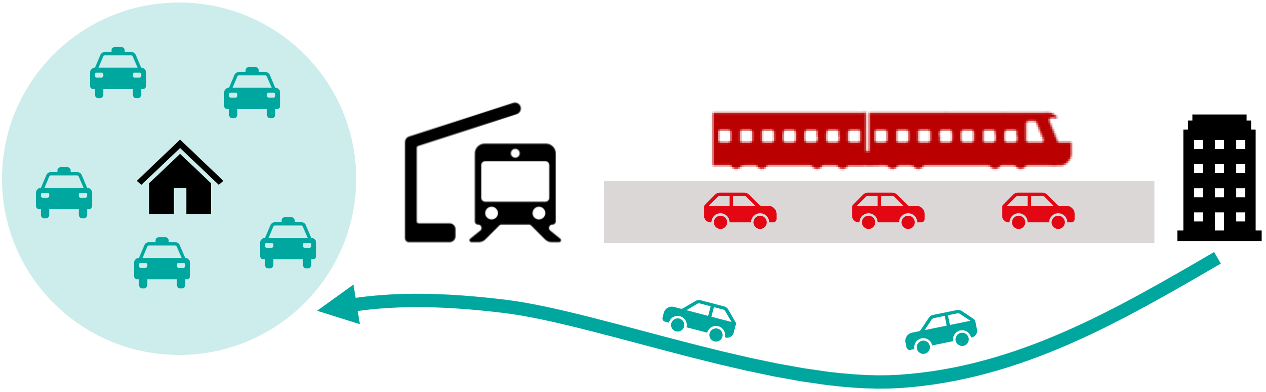


  - All travelers go to the city center
    - option 1: door-to-door ride-hailing
    - option 2: first-mile ride-hailing + train

- ***Q: What are the trade-offs in mode choice and service operations?***

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service

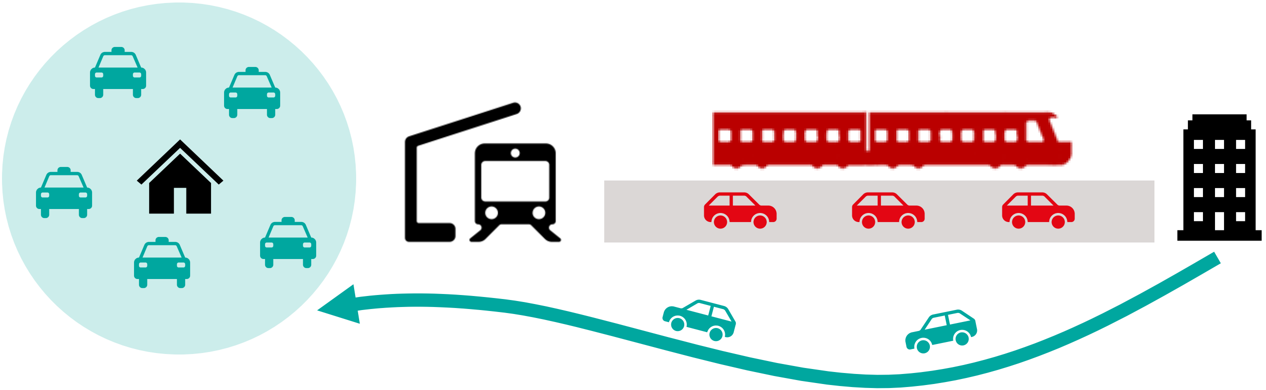


  - Path cost
    - Door-to-door $c_{\mathrm{D2D}} = c_0^{\mathrm{D2D}} + w(V) + t_0 + t_{\mathrm{highway}}(q_{\mathrm{D2D}})$
    - First-mile $c_{\mathrm{1st}} = c_0^{\mathrm{1st}} + w(V) + t_0 + t_{\mathrm{train}}$

*** trip fare, designed by operator**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service

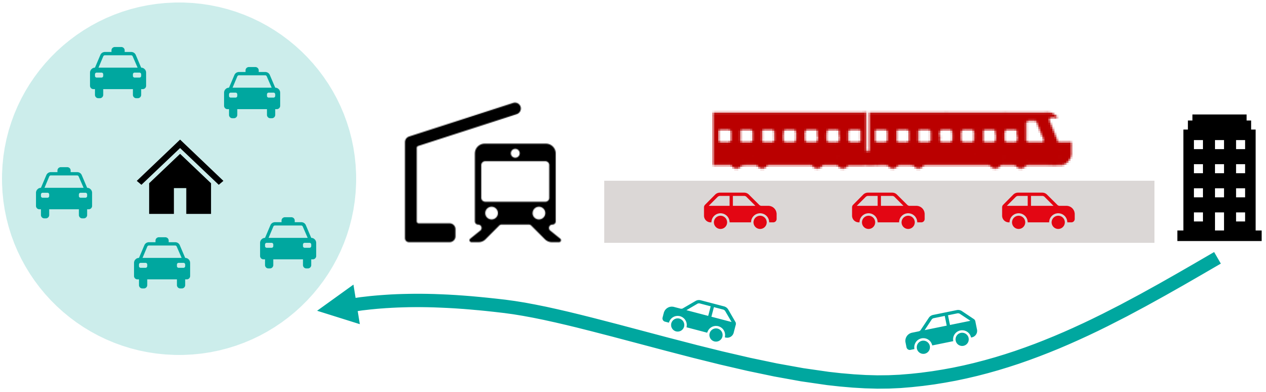


  - Path cost
    - Door-to-door $c_{\mathrm{D2D}} = c_0^{\mathrm{D2D}} + w(V) + t_0 + t_{\mathrm{highway}}(q_{\mathrm{D2D}})$
    - First-mile $c_{1\mathrm{st}} = c_0^{1\mathrm{st}} + w(V) + t_0 + t_{\mathrm{train}}$

*** waiting time, dependent of empty vehicles V**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service

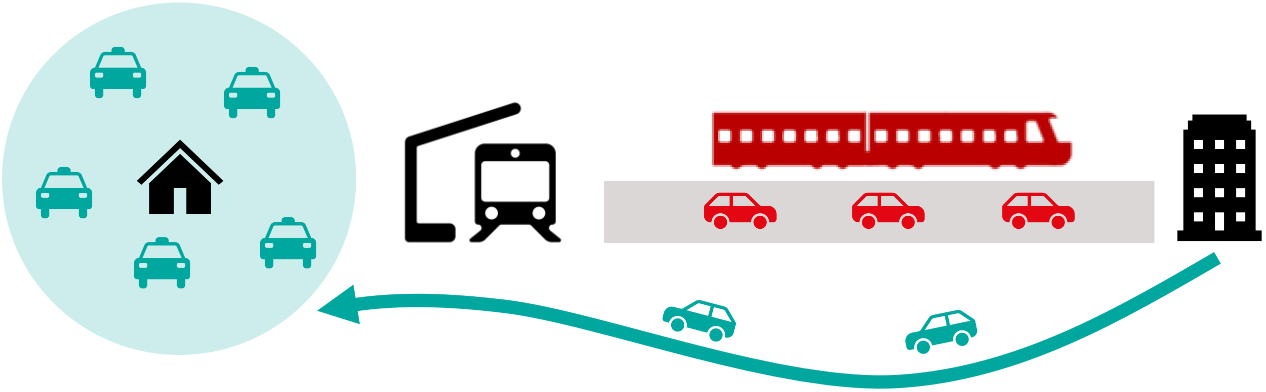


- Path cost
  - Door-to-door $c_{\mathrm{D2D}} = c_0^{\mathrm{D2D}} + w(V) + t_0 + t_{\mathrm{highway}}(q_{\mathrm{D2D}})$
  - First-mile $c_{\mathrm{1st}} = c_0^{\mathrm{1st}} + w(V) + t_0 + t_{\mathrm{train}}$

*** travel time to train station, assumed fixed**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service

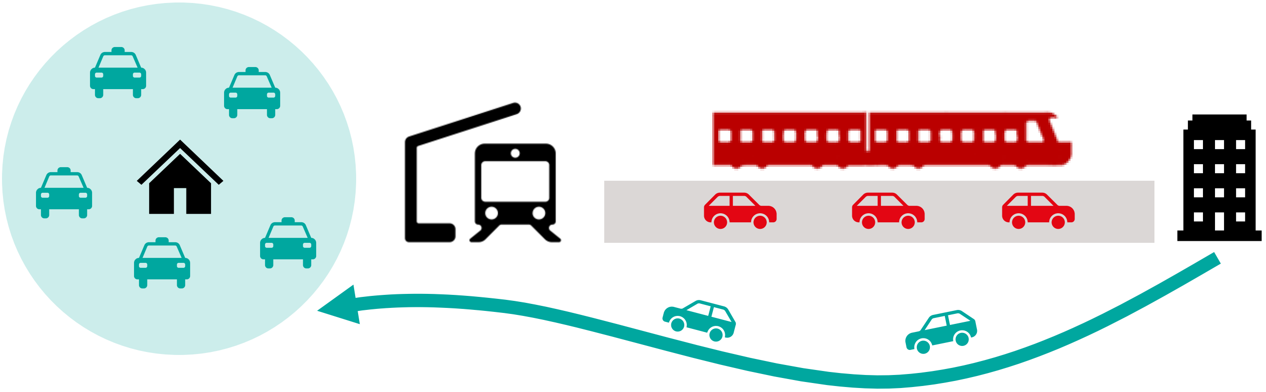


  - Path cost
    - Door-to-door $c_{\mathrm{D2D}} = c_0^{\mathrm{D2D}} + w(V) + t_0 + t_{\mathrm{highway}}(q_{\mathrm{D2D}})$
    - First-mile $c_{\mathrm{1st}} = c_0^{\mathrm{1st}} + w(V) + t_0 + t_{\mathrm{train}}$

*** travel time on highway, dependent of D2D demand**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service

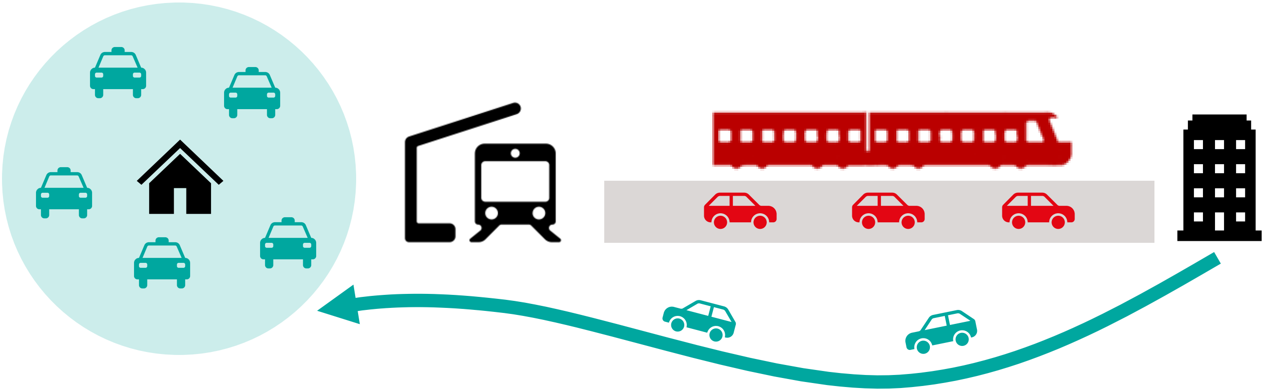


  - Path cost
    - Door-to-door $c_{\mathrm{D2D}} = c_0^{\mathrm{D2D}} + w(V) + t_0 + t_{\mathrm{highway}}(q_{\mathrm{D2D}})$
    - First-mile $c_{\mathrm{1st}} = c_0^{\mathrm{1st}} + w(V) + t_0 + t_{\mathrm{train}}$

*** travel time on train, assumed fixed**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service



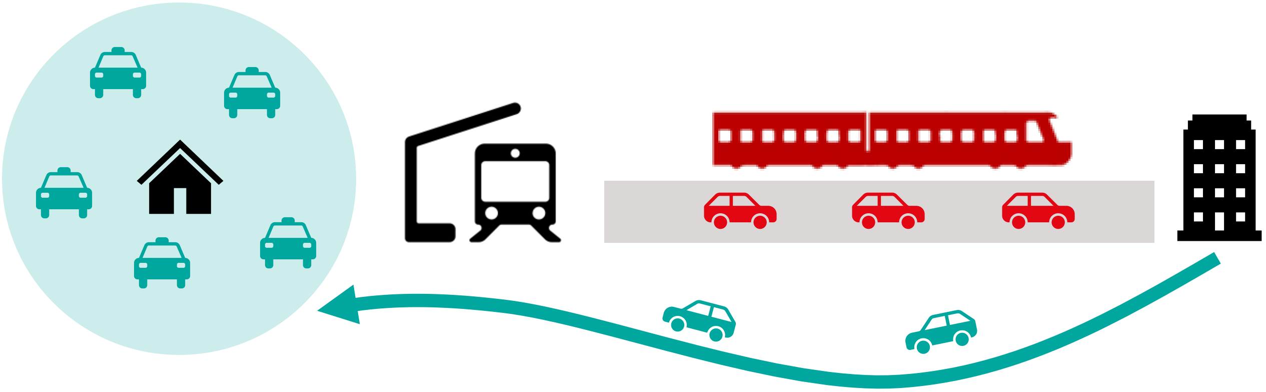
  - Ride-hailing vehicle supply

$$N = V + (q_{\mathrm{D2D}} + q_{1st})(w(V) + t_0) + q_{\mathrm{D2D}}(t_{\mathrm{highway}}(q_{\mathrm{D2D}}) + t_r)$$

*** total vehicle time**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service



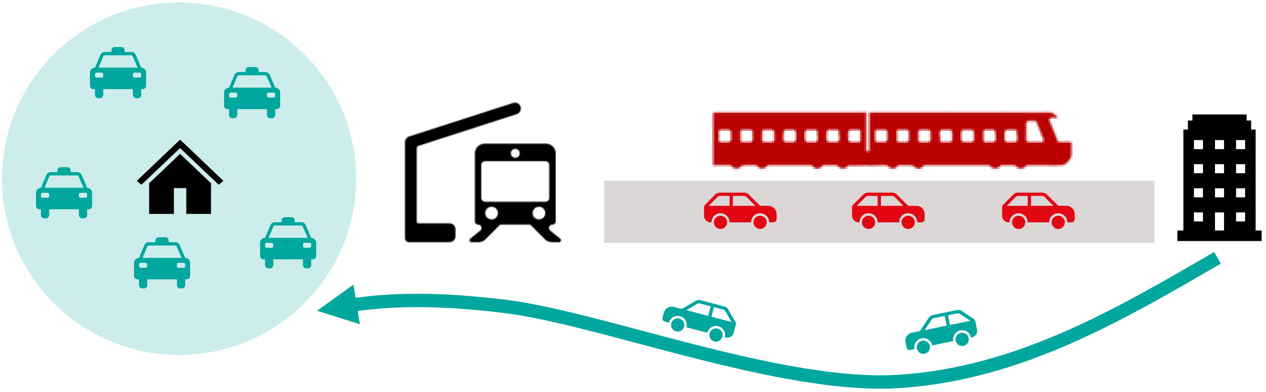
  - Ride-hailing vehicle supply

$$N = V + (q_{\mathrm{D2D}} + q_{1st})(w(V) + t_0) + q_{\mathrm{D2D}}(t_{\mathrm{highway}}(q_{\mathrm{D2D}}) + t_r)$$

*** empty vehicle time**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service



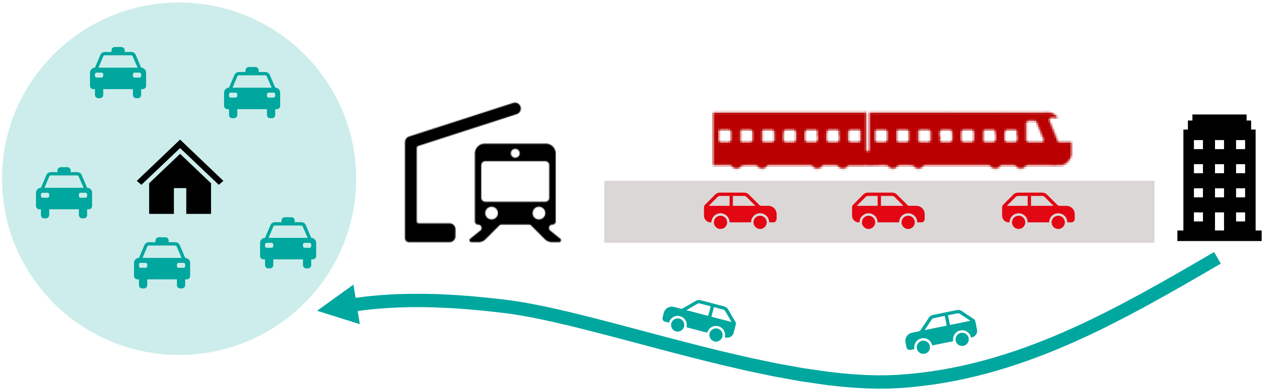
  - Ride-hailing vehicle supply

$$N = V + (q_{\mathrm{D2D}} + q_{1st})(w(V) + t_0) + q_{\mathrm{D2D}}(t_{\mathrm{highway}}(q_{\mathrm{D2D}}) + t_r)$$

*** pickup and first-mile travel time**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service



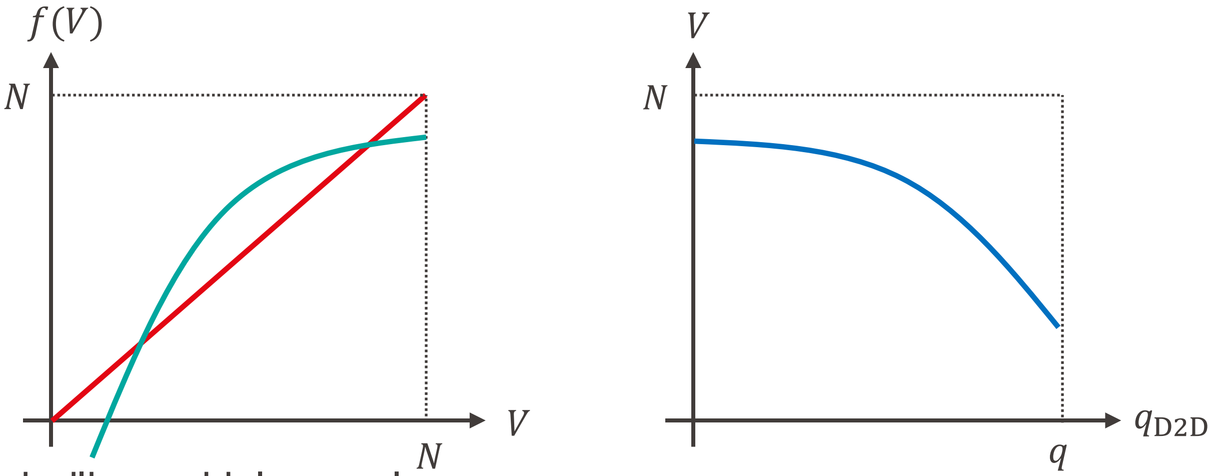
  - Ride-hailing vehicle supply

$$N = V + (q_{\mathrm{D2D}} + q_{1st})(w(V) + t_0) + q_{\mathrm{D2D}}(t_{\mathrm{highway}}(q_{\mathrm{D2D}}) + t_r)$$

*** highway travel and return time**

First-mile service

- Static traffic assignment + first-mile service
  - A single OD network with ride-hailing service



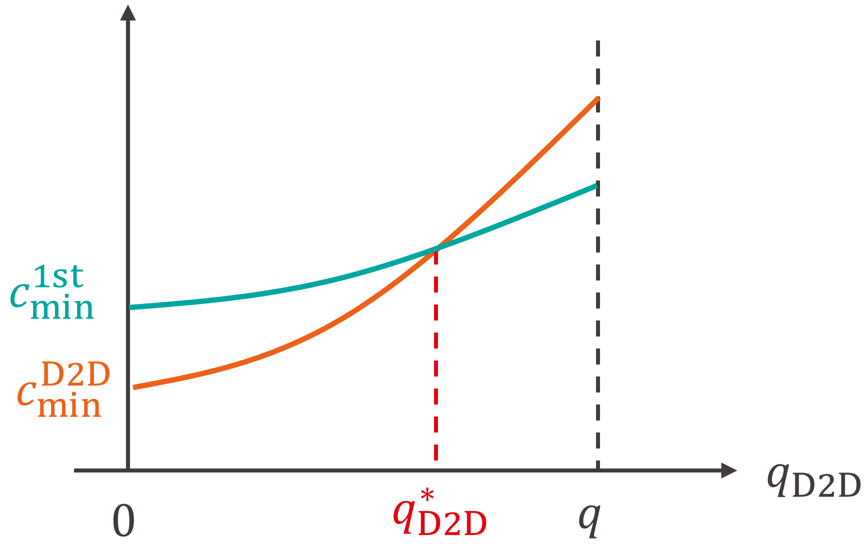
  - Ride-hailing vehicle supply

$$N = V + (q_{\mathrm{D2D}} + q_{1st})(w(V) + t_0) + q_{\mathrm{D2D}}(t_{\mathrm{highway}}(q_{\mathrm{D2D}}) + t_r)$$

$$\Rightarrow \quad V = N - (q_{\mathrm{D2D}} + q_{1st})(w(V) + t_0) - q_{\mathrm{D2D}}(t_{\mathrm{highway}}(q_{\mathrm{D2D}}) + t_r)$$

First-mile service

- Static traffic assignment + first-mile service
  - Equilibrium conditions



$$c_{\min}^{\text{D2D}} = c_0^{\text{D2D}} + w(V_{\max}) + t_0 + t_{\text{highway}}(0)$$

$$c_{\min}^{\text{1st}} = c_0^{\text{1st}} + w(V_{\max}) + t_0 + t_{\text{train}}$$