Assignment 4 – Solution

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Goal: The goal of this project is to estimate the structural response of the footbridge outside the GC building with and without activated TMD due to a person jumping at midspan of the footbridge and due to a person walking across the bridge.

Task 1: Natural frequency

Estimate the natural frequency of the footbridge from the construction drawings. State any assumptions you are making.

Assumptions

Material assumptions

We will consider a S355 Steel to be used for the members of the bridge. The Young's modulus can therefore be estimated to be of $E = 210'000\,\mathrm{MPa}$. Further, the material density can be estimated at $\rho = 8000\,kg/m^3$ which corresponds to a unit weight of $\gamma = 80\,kN/m^3$. The same unit weight will be used for the bars on top of the structure as well as the elements forming the walking surface.

Structural assumptions

The bridge is made of two simple bridge girders working in parallel as a simply supported beam. The elements of the walking surface do not add any stiffness to the structure and are only for utilization purpose. Therefore, the structural system is a simply supported beam made of two parallel girders HEA 140 with a span of $L = 10\,m$ between the two supports.

Oscillating system

In this assignment only the first mode of vibration will be analysed since it is the one with the lowest frequency and therefore the easiest to activate for people using the bridge. Of course, there are other modes with double or even triple bending of the girders or twisting (see also the video of the timber structure in week 12s class), but they are not of interest for this assignment.

Other assumptions

The mass from the Tuned Mass Damper (TMD) will be neglected for this first task, as well as the mass from any connections and the IPET 180 and the transversal beams at the support section. For simplicity the gravity acceleration is assumed to be $g = 10\,m/s^2$. The mass of the people as well as the mass of the inactivated TMD are neglected.

Calculation of the mass of the system

ROR 51 x 5 bars

The first step is to calculate the total mass of the structure. The first elements are the ROR elements fixed to the bridge every 2 meters. The section is of type ROR 51 x 5 with a length of $l_{ROR} = 1.33\,m$:

- Linear mass of the element $m_{ROR} = 5.67\,kg/m$ (SZS C4/C5)
- Mass per bar $M_{ROR} = m_{ROR} * l_{ROR} = 5.67 * 1.33 = 7.54\,kg$
- Total mass of bars $M_{ROR,tot} = 12 * 7.54 = 90.49\,kg \approx 90\,kg$

Girders HEM 140

There are two girders of cross section type HEM 140 as the main structure of the foot bridge. Their total length is of $l_{HEM} = 10.6\ m$:

- Linear mass of the element $m_{HEM} = 63.2\ kg/m$ (SZS C4/C5)
- Mass per beam, $M_{HEM} = m_{HEM} * l_{HEM} = 63.2 * 10.6 = 669.92\ kg$
- Total mass of beams $M_{HEM,tot} = 2 * 669.92 = 1339.84\ kg\ \approx 1340\ kg$

Steel plates of walking surface

The plates to form the walking surface are made from the same steel and can be estimated to be approximately double the thickness as the HEM 140 flanges. So, $t_{plate} \approx 40\ mm$. The width of the element is estimated to $w_{plate} \approx 1.0\ m$ and they have the same total length as the girders, $l_{plates,tot} = 10.6\ m$. Hence:

- Linear mass of the element $m_{plate} = 320\ kg/m$ (SZS C4/C5)
- Total mass of plates $M_{plate,tot} = m_{ROR} * l_{ROR} = 320 * 10.6 = 3392\ kg$

Total mass

The total mass of the structure is as follows:

$$M_{tot} = M_{ROR,tot} + M_{HEM,tot} + M_{plate} = 90 + 1340 + 3392 = 4822 kg \approx 5000\ kg^{*}$$

Note that since we neglected some elements, we are rounding the mass up to 5000 kg. This allows for a simpler calculation later.

Calculation of the frequency

Finally, we can calculate the circular frequency, as well as the eigen frequency and the corresponding period (for more information see the explanation for systems with distributed mass):

- $\omega_n = \left(\frac{k_n \cdot \pi}{L}\right)^2 \sqrt{\frac{EI}{\mu}}$

Moment of inertia of a section HEM 140: $I_y = 32.6 \cdot 10^6\ mm^4 = 32.6 \cdot 10^{-6}\ m^4$

Distributed mass: $\mu = \frac{M}{L} = \frac{5000\ kg}{10\ m} = 500\ kg/m'$

- First mode: $\omega_1 = \left(\frac{1 \cdot \pi}{10}\right)^2 \sqrt{\frac{210 \cdot 10^9 \cdot 2 \cdot 32.6 \cdot 10^{-6}}{500}} = 16.4\ s^{-1}$
- $f_n = \frac{\omega_n}{2\pi} = 2.61\ Hz$
- $T_n = \frac{1}{f_n} = 0.38\ s$

Task 2: Response without tuned mass damper (TMD)

Assumptions

- The damping ratio of a steel structure can be assumed between 2% and 4%. In this case it has been decided to use a damping ratio of $\zeta = 3\%$.

Load case 1:

For the first load case the formula suggested in slide 16 lecture 12 can be used to calculate the maximum acceleration:

$$a_{max} = \omega_j^2 \cdot y \cdot\ \alpha \cdot \frac{1}{2\zeta}$$

Where:

- ω_j is the structural frequency that is in resonance with the forcing function. In this case $\omega_j = \omega_n = 16.41\ Hz$.
- y is the static deflection of the bridge at mid-span for the weight G of the person standing at the point of maximum amplitude of the mode.

$$\delta_{static} = \frac{G \cdot L^3}{48 \cdot EI} = \frac{800\ N\ \cdot\ (10'000\ mm)^3}{48\ \cdot 1.3818 * 10^{13} Nmm^2} = 1.2\ mm$$

- α is the Fourier coefficient of the relevant harmonic of the person jumping. Referring to the table of Bachmann et al. 1997 on slide 14, we take $\alpha = 1.8$.

Therefore, the maximum acceleration can be calculated as:

$$a_{max} = \omega_j^2 \cdot y \cdot\ \alpha \cdot \frac{1}{2\zeta} = (16.41\ s^{-1})^2 * 0.0012\ m * 1.8 * \frac{1}{2 * 0.03} = \mathbf{9.74\ m/s^2}$$

Load case 2:

In the second load case the person is not jumping in the same frequency as the bridge but walking across it. With the formula given in slide 17 of lecture 12 we can estimate the maximum acceleration in an easy manner:

$$a_{max} = \omega_j^2 \cdot y \cdot \alpha \cdot \phi$$

Where:

- ω_j is the structural frequency that is in resonance with the forcing function. In this case $\omega_j = \omega_n = 16.41\ Hz$.
- y is the static deflection of the bridge at mid-span for the weight G of the person standing at the point of maximum amplitude of the mode. $F_{p,max}/k^* = \frac{800N}{673'000\ N/m} = 0.0012\ m = 1.2\ mm$
- α is the Fourier coefficient of the relevant harmonic of the person walking. Referring to the table of Bachmann et al. 1997 on slide 14, we take $\alpha = 0.5$.[AB1][KB2]
- ϕ is the dynamic amplification factor. Assuming the average step length of a person to be around 65-75 cm the person will need around 15 steps to go over the bridge. Using the diagram on slide 17 of lecture 12 we get $\phi \approx 25$[AB3][KB4].

Therefore, the maximum acceleration can be calculated as:

$$a_{max} = \omega_j^2 \cdot y \cdot \alpha \cdot \phi = (16.41 s^{-1})^2 * 0.0012\ m * 0.5 * 13 = \mathbf{2.10\ m/s^2}$$ [AB5]

Discussion

Comparing the two load cases we can see that the acceleration of the second load case is significantly smaller than the one of the first load case. This can be explained by two things:

1. **Placement of the force:** In the first load case the person is jumping at mid-span where the acceleration will be the highest and where the impact of the excitation is the biggest. In the second load case the person is crossing the bridge and not staying in one place. This results in the impact force not always being applied to the mid-span point (only approximately once). Therefore, the impact is not “optimal” for this first mode of vibration of the structure and results therefore in less acceleration at mid-span.
2. **Alpha factor:** In the first load case the alpha factor is of 1.8 while the alpha factor is of only 0.5 for the second load case. Hence, also the acceleration varies accordingly.

Task 3: Response with TMD

The goal of the last task is to derive the behaviour of the foot bridge in the case a tuned mass damper (TMD) is added to the structure. The TMD has the following properties:

- Mass: $m_d = 140\ kg$
- Frequency: $f_d = 2.44\ Hz$
- Damping ratio $\zeta_d = 12\%$
- Mass ratio $\mu = \frac{m_d}{m^*} = \frac{140\ kg}{2500\ kg} = 0.056$

Let's first verify the design of the TMD:

We choose the frequency of the TMD such that: $\omega_d = \frac{\omega}{1+\mu}$ or we can also formulate it as $f_d = \frac{f}{1+\mu} = \frac{2.61Hz}{1+0.056} = 2.47\ Hz\ \approx 2.44\ Hz \rightarrow$ OK! Therefore, we know that the frequency of the tuned mass damper has been chosen to have the same $|x_0|$ at the neutral points. This means we can calculate the dynamic amplification factor with the TMD as:

$$DAF_{TMD} = \sqrt{\frac{2+\mu}{\mu}} = \sqrt{\frac{2+0.056}{0.056}} = 6.06 = \frac{|x_0|}{f/k^*}$$

The dynamic amplification factor without the TMD is calculated as:

$$DAF_{without\ TMD} = \frac{1}{2\,\zeta} = 16.67$$

The maximum acceleration of the system with the tuned mass damper for load case 1 can therefore be calculated as:

$$\ddot{u}_{max,TMD} = a_{max}\frac{DAF_{TMD}}{DAF_{without\ TMD}} = 9.74\,m/s^2\,\frac{6.06}{16.67} = 3.54\,m/s^2$$ [KB6]

This corresponds to a fraction of only 36% of the system without the damper, meaning that the maximum acceleration of the bridge at mid span for the harmonic jumping force has been reduced by 64% (or by a factor of 2.75). This shows how efficient the addition of a TMD is.