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Vibrations induced by people

Dynamics of Structures

Dr. Francesco Vanin

Earthquake Engineering and Structural Dynamics Laboratory

Dr. Francesco Vanin

Program of Week 13-15

	Lecture (13:15 – 15:00)	(15:15 – 16:00)	(16:15 – 17:00)
Week 13	Machine-induced vibrations	Pascal Fleischer Trombik Ingenieure AG, Zürich	In-class exercise
Week 14	Footfall-induced vibrations	Assignment 4 TMD design	Footbridge demonstration
Week 15	Further structural dynamic problems		

Structural dynamics problems other than earthquake-induced shaking

- Machinery-induced vibrations
  - Machine foundations, bell towers, structure-borne sound
- Vibrations induced by traffic and construction activities
  - Roads, railways, construction works
  - Highspeed railway-bridge system
- Vibrations induced by people (footbridges, floors, high-diving platforms)
- Wind-induced vibrations

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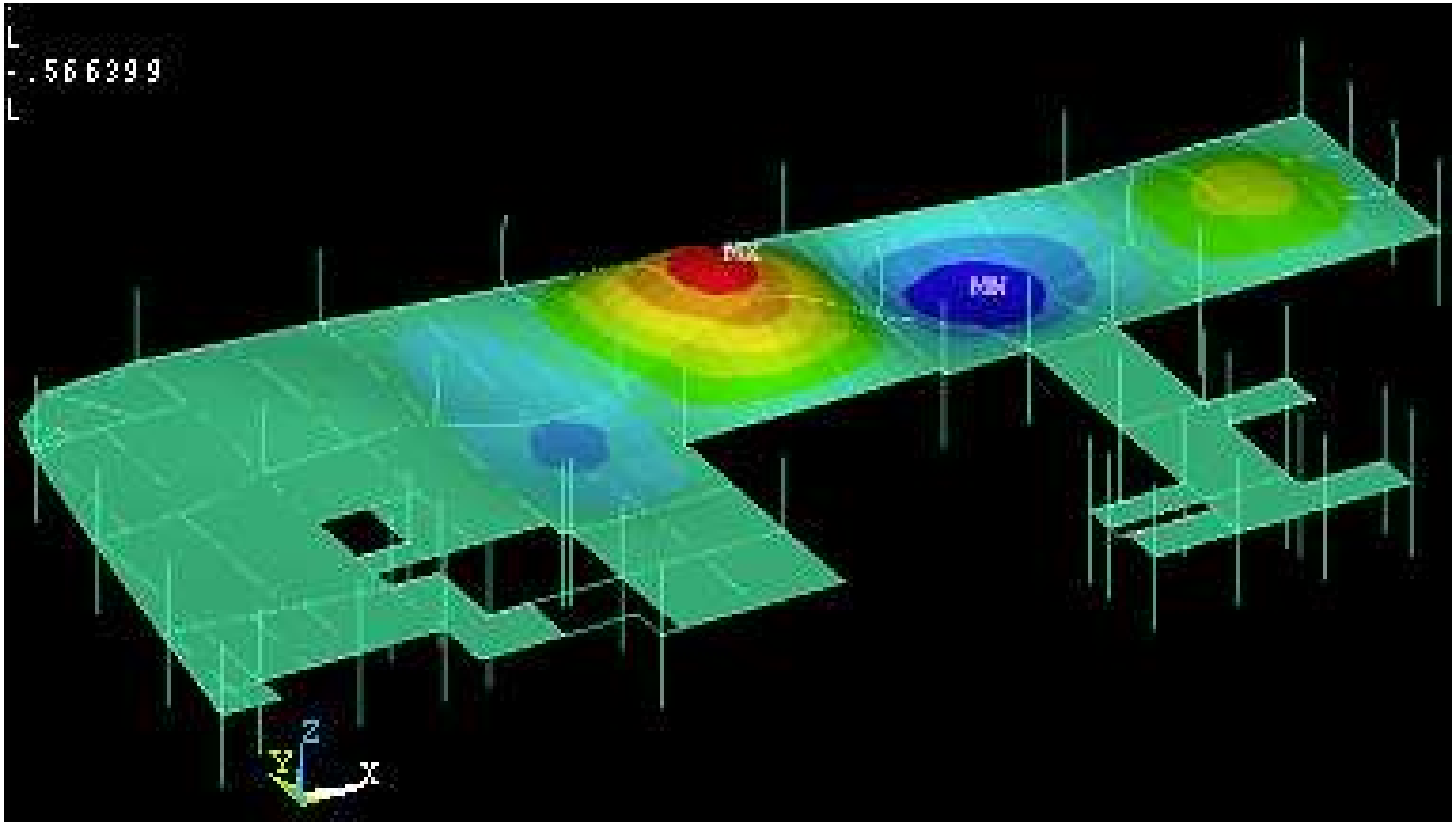
Footbridge – Millenium bridge

Footbridge

- https://www.youtube.com/watch?v=uWoiMMLIvco&list=PL8biE0hpPzEyZPPvb1MdD7ebUdP8t3dvB&index=1

Office floors



https://www.steelconstruction.info/File:G_Fig18.png

Structures that can be sensitive to footfall induced vibrations

Most common types of structures that can be sensitive to footfall induced vibrations:

- Footbridges
- Office floors
- Tribunes of a stadium
- Floors for sport or dance activities / concert halls
- High-diving platforms

Goal of today's lecture

- Characterise footfall induced vibrations
- Know how to estimate the maximum response of a structure due to footfall induced vibrations
- Know the effect of vibrations on structures, their content and users
- Know some remedies against vibration problems
- Know how to design a tuned mass damper

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Man-induced vibrations

Rhythmical body motions

- Walking
- Running
- Jumping / skipping
- Dancing
- Handclapping with body bouncing while standing
- Handclapping while seated
- Lateral body swaying

Impact body motions

- Heel impact
- Impact due to jumping off the ground
- Impact after jumping from an elevated position

@Bachmann et al. 1997

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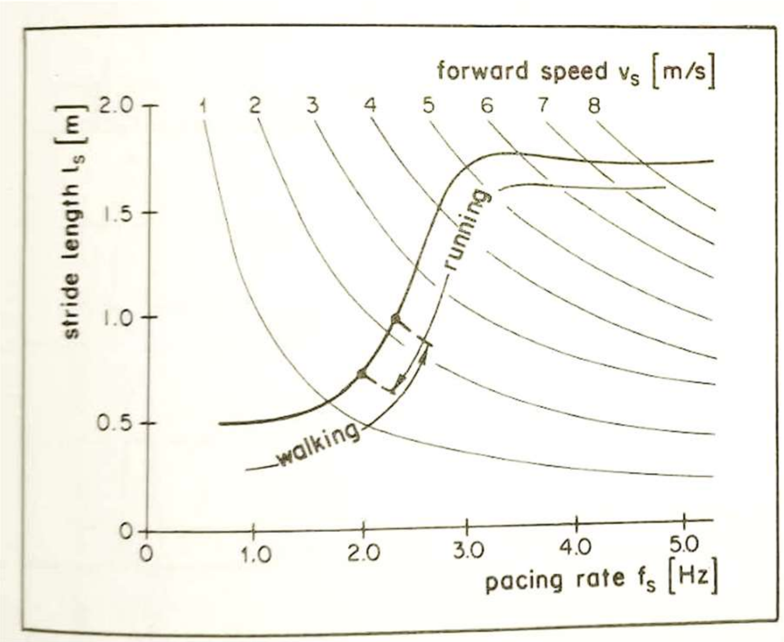
Activities and their typical frequencies

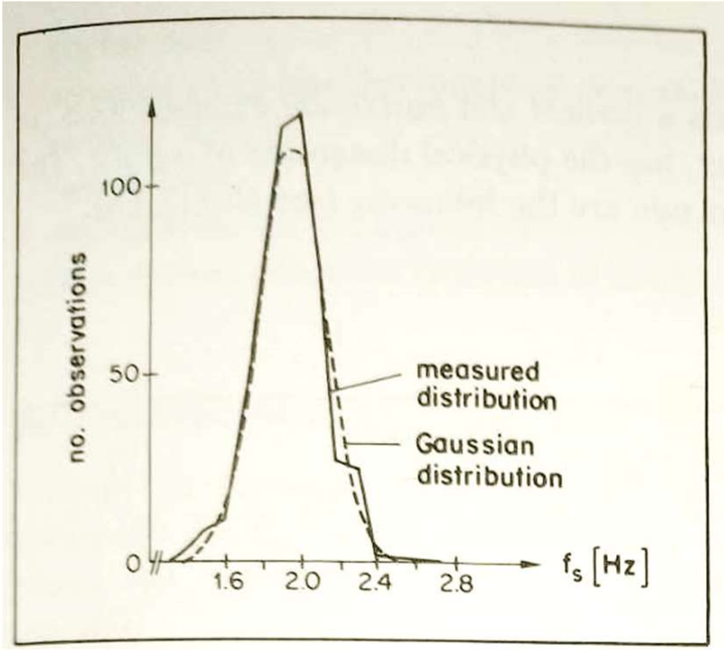
Representative types of activity			Range of applicability		
Designation	Definition	Design activity rate [Hz]	Actual activities	Activity rate [Hz]	Structure type
"walking"	walking with continuous ground contact	1.6 to 2.4	• slow walking (ambling) • normal walking • fast, brisk walking	~ 1.7 ~ 2.0 ~ 2.3	• pedestrian structures (pedestrian bridges, stairs, piers, etc.) • office buildings, etc.
"running"	running with discontinuous ground contact	2.0 to 3.5	• slow running (jog) • normal running • fast running (sprint)	~ 2.1 ~ 2.5 > 3.0	• pedestrian bridges on jogging tracks, etc.
"jumping"	normal to high rhythmical jumpingon the spot with simultaneous ground contact of both feet	1.8 to 3.4	• fitness training with jumping, skipping and running to rhythmical music • jazz dance training	~ 1.5 to 3.4 ~ 1.8 to 3.5	• gymnasia, sport halls • gymnastic training rooms
"dancing"	approximatly equivalent to "brisk walking"	1.5 to 3.0	• social events with classical and modern dancing (e.g. English Waltz, Rumba etc.)	~ 1.5 to 3.0	• dance halls • concert halls and other community halls without fixed seating
"hand clapping with body bouncing while standing"	rhythmical hand clapping in front of one's chest or above the head while bouncing vertically by forward and backward knee movement of about 50 mm	1.5 to 3.0	• pop concerts with enthusiastic audience	~ 1.5 to 3.0	• concert halls and spectator galleries with and without fixed seating and "hard" pop concerts
"hand clapping"	rhythmical hand clapping in front of one's chest	1.5 to 3.0	• classical concerts, "soft" pop concerts	~ 1.5 to 3.0	• concert halls with fixed seating (no "hard" pop concerts)
"lateral body swaying"	rhythmical lateral body swaying while being seated or standing	0.4 to 0.7	• concerts, social events		• spectator galleries

Table G.1: Representative types of activities and their applicability to different actual activities and types of structures

@Bachmann et al. 1997

Activities and their typical frequencies



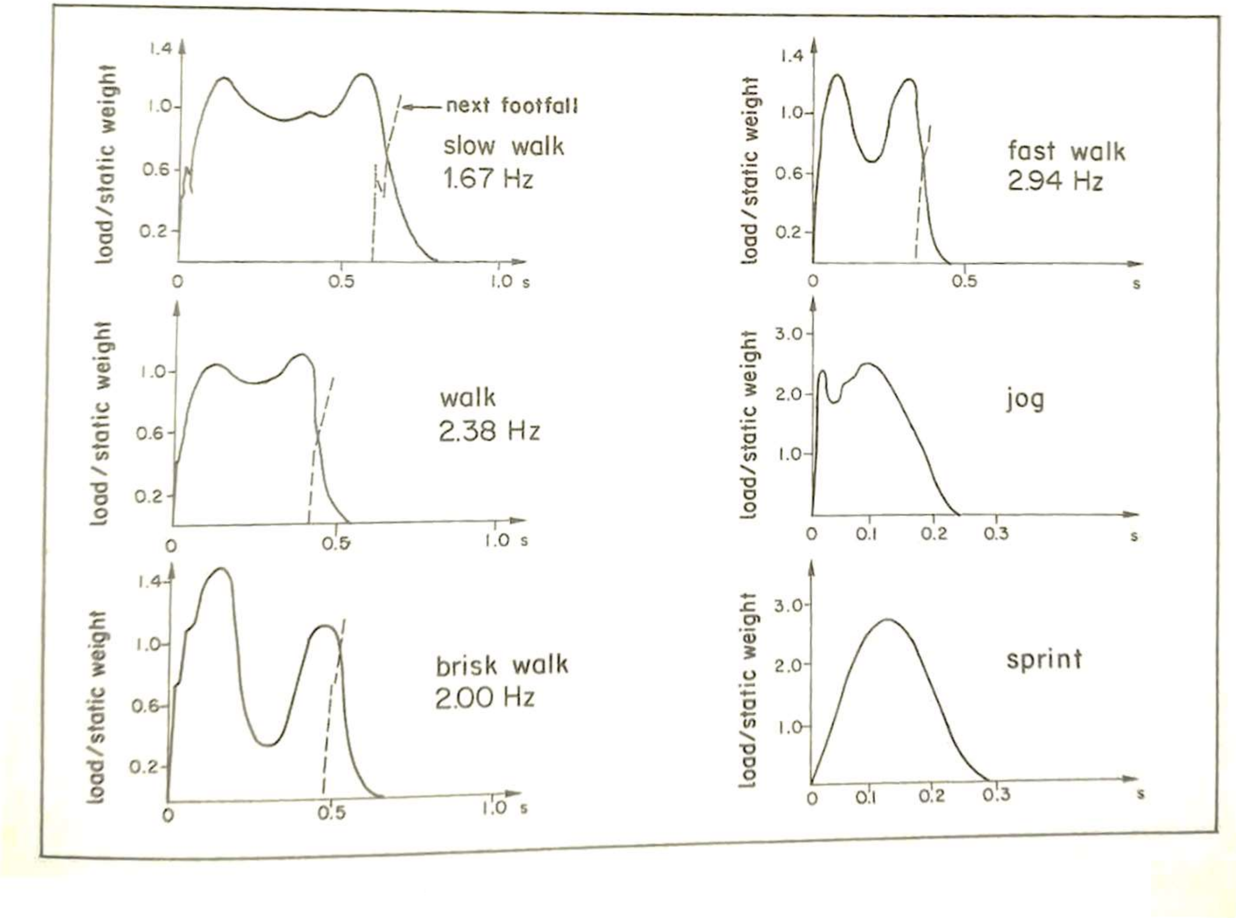


Typical pacing rates:

- Walking: 1.7 Hz – 2.3 Hz
- Running: 2.5 Hz – 3.2 Hz (150-190 steps per minute)

@Bachmann et al. 1987

Time function of the vertical load



- Continuous ground contact: Fourier series of several harmonics

- Discontinuous ground contact: semi-sinusoidal pulses

@Bachmann et al. 1987

Dynamic forces from rhythmical body motions

Forcing function due to a person's rhythmical body motion (continuous ground contact, possible extension to non-continuous ground contact):

- $F_p(t) = G + \sum_{i=1}^{n} G \cdot \alpha_i \cdot \sin(2\,\pi\, i\, f_p\, t\; - \phi_i),$
  where :

G	Weight of the person (standard value: 700 N)
α_i	Fourier coefficient of the i-th harmonic
$G \cdot \alpha_i$	Force amplitude of the i-th harmonic
f_p	Activity rate [Hz], i.e., walking / running / jumping / … frequency
ϕ_i	Phase lag of the i-th harmonic to the first harmonic
n	Total number of contributing harmonics

Dynamic forces from rhythmical body motions

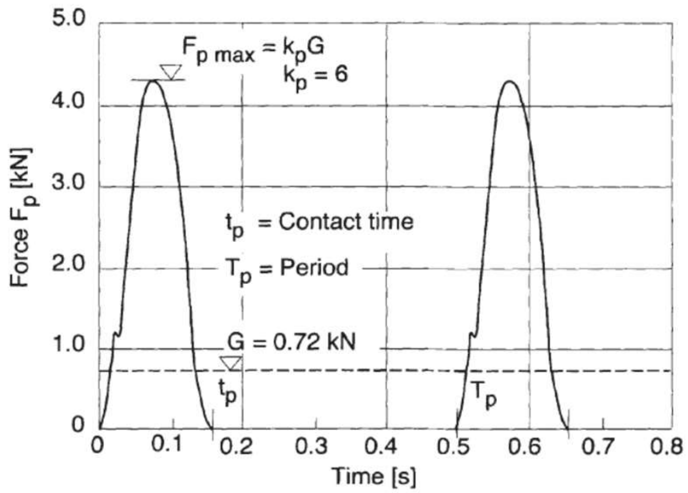


Figure G.2: Forcing function from jumping on the spot with both feet simultaneously at a jumping rate of 2 Hz [G.4]

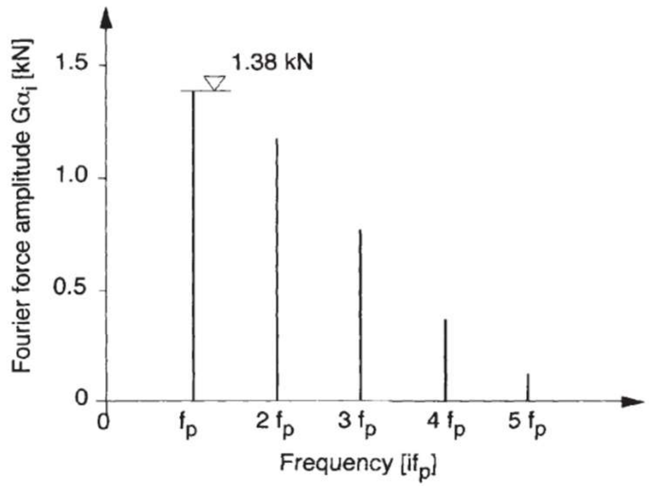


Figure G.3: Discrete Fourier amplitude spectrum for the forcing function from jur Figure G.2 up to the fifth harmonic

@Bachmann et al. 1997

Dynamic forces from rhythmical body motions

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Representative type of activity	Activity rate [Hz]		Fourier coefficient and phase lag α_1	α_2	ϕ_2	α_3	ϕ_3	Design density [persons/m^2]
"walking"	vertical	2.0	0.4	0.1	$\pi/2$	0.1	$\pi/2$	~ 1
		2.4	0.5					
	forward	2.0	0.2	0.1				
			$\alpha_{1/2} = 0.1$					
	lateral	2.0	$\alpha_{1/2} = 0.1$	$\alpha_{3/2} = 0.1$				
"running"		2.0 to 3.0	1.6	0.7		0.2		-
"jumping"	normal	2.0	1.8	1.3	*)	0.7	*)	in fitness training ~ 0.25
		3.0	1.7	1.1	*)	0.5	*)	(in extreme cases up to 0.5)
	high	2.0	1.9	1.6	*)	1.1	*)	
		3.0	1.8	1.3	*)	0.8	*)	*) $\phi_2 = \phi_3 = \pi(1 - f_p t_p)$
"dancing"		2.0 to 3.0	0.5	0.15		0.1		~ 4 (in extreme cases up to 6)
"hand clapping with body bouncing while standing"		1.6	0.17	0.10		0.04		no fixed seating ~ 4
		2.4	0.38	0.12		0.02		(in extreme cases up to ~ 6)
								with fixed seating ~ 2 to 3
"hand clapping"	normal	1.6	0.024	0.010		0.009		
		2.4	0.047	0.024		0.015		~ 2 to 3
	intensive	2.0	0.170	0.047		0.037		
"lateral body swaying"	seated	0.6	$\alpha_{1/2} = 0.4$	-		-		
	standing	0.6	$\alpha_{1/2} = 0.5$	-		-		~ 3 to 4

Table G.2: Normalized dynamic forces assigned to the representative types of activity defined in Table G.1

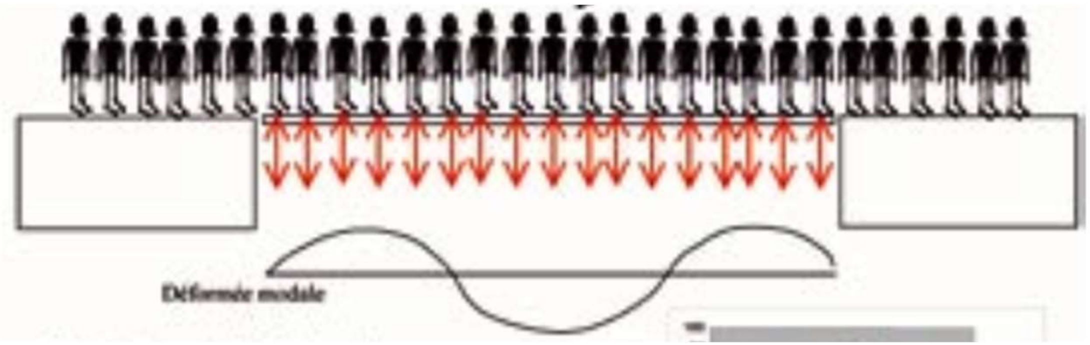
@Bachmann et al. 1997

Coordination of people

Man induced vibrations result often from the action of multiple people

- Non-synchronised activities: walking on a bridge/floor, running
- Synchronised activities: dancing, jumping (aerobics), lateral swaying, unconscious synchronization of crowds

The effect of a group of people is generally not directly proportional to the number of people, as a result of the lack of perfect synchronization (even for “synchronized” activities)



Coordination of people

Equivalent number of N coordinated people:

- Non-synchronised activities:
  - Hypotheses: casual phase of the action of different people, loads moving along a simple beam / multiple spans
  - Matsumoto: $N_{eq} = \sqrt{N}$
  - Guides SETRA-HIVOSS:

    Medium/low density of people (up to 0.5 P/m²) : $N_{eq} = 10.8\sqrt{\zeta N}$

    High density of people (0.8-1.0 P/m²) : $N_{eq} = 1.85\sqrt{N}$

Coordination of people

- Synchronised activities: coordination factors

Table A.2 — Recommended coordination factor $C(N)$ for evaluating the comfort of passive people for group size $N \geqslant 50$ for the activity "coordinated jumping"

Coordination	1st harmonic	2nd harmonic	3rd harmonic
high	0,80	0,67	0,50
medium	0,67	0,50	0,40
low	0,50	0,40	0,30
NOTE These values of $C(N)$ apply only to the serviceability limit state.			

@ISO 10137 (2007)

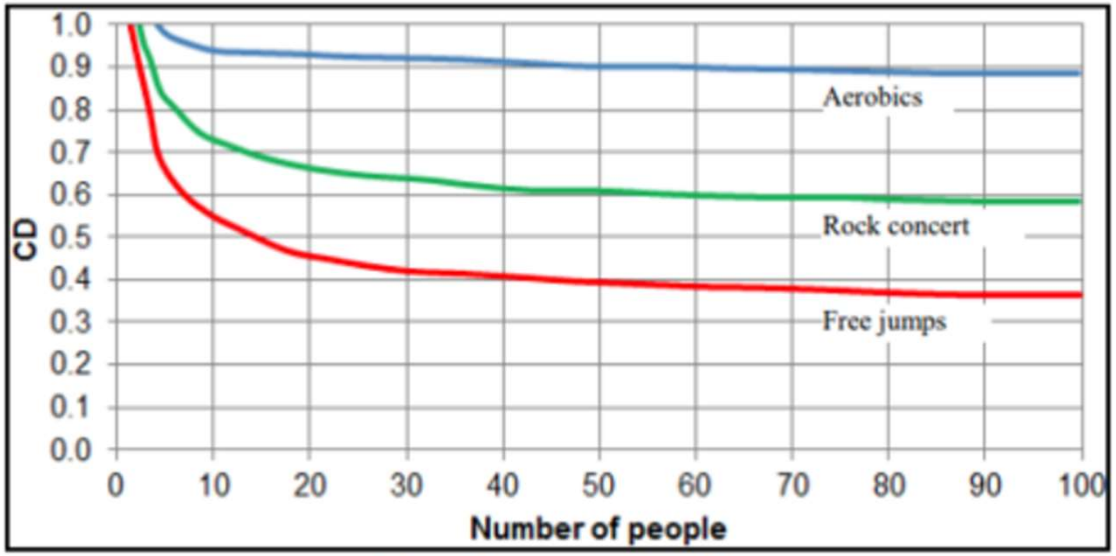


Figure 1. Phase coefficients [2].

@Da Silva et al. 2015

Footbridges

- Activity: walking, running
- Several people walking on bridge often walk in step
- Forcing function:
  - Short footbridge: transient, no steady-state reached
  - Long footbridge : steady state can be reached (most common case)

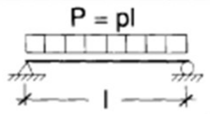
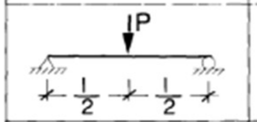
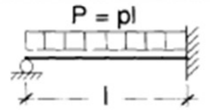
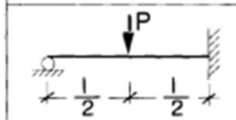
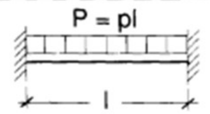
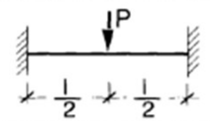
Equivalent substitute SDOF system

- Generalised mass:

$$\tilde{m} = \mu \int_0^L f^2(x)dx = \phi_M m_{tot}$$

$$\phi_M = \frac{1}{L}\int_0^L f^2(x)dx$$

(mass factor)

Loading and support conditions Reference point at l/2	Load factor ϕ_L	Mass factor ϕ_M		Effective beam stiffness k	Stiffness factor ϑ
		Lumped mass	Distributed mass		
	0.637	—	0.5	$\frac{384\ EI}{5 \cdot l^3}$	48.7
	1.0	1.0	0.5	$\frac{48\ EI}{l^3}$	48.7
	0.595	—	0.479	$\frac{185\ EI}{l^3}$	113.9
	1.0	1.0	0.479	$\frac{107\ EI}{l^3}$	113.9
	0.523	—	0.396	$\frac{384\ EI}{l^3}$	198.5
	1.0	1.0	0.396	$\frac{192\ EI}{l^3}$	198.5

Equivalent substitute SDOF system

- Generalised load:

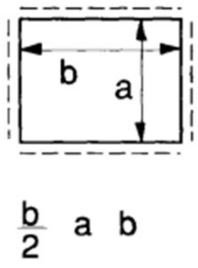
$$\tilde{p} = p\int_0^L f\,(x)dx = \phi_L pL$$

$$\phi_L = \frac{1}{L}\int_0^L f\,(x)dx$$

- Generalised stiffness:

$$\tilde{k} \cong \phi_L k$$

$$\omega_n{}^2 \cong \frac{\tilde{k}}{\tilde{m}}$$

Support conditions	$\frac{a}{b}$	Load factor ϕ_L	Mass factor ϕ_M	Effective plate stiffness k
	1.0	0.45	0.31	$\frac{271EI_0}{a^2}$
	0.9	0.47	0.33	$\frac{248EI_0}{a^2}$
	0.8	0.49	0.35	$\frac{228EI_0}{a^2}$
	0.7	0.51	0.37	$\frac{216EI_0}{a^2}$
	0.6	0.53	0.39	$\frac{212EI_0}{a^2}$
	0.5	0.55	0.41	$\frac{216EI_0}{a^2}$

Maximum acceleration response (single person crossing the bridge)

Upper bound estimate:

Assume that the force always acts at the point of maximum amplitude of the mode:

$$a_{max} = \omega_j^2 \cdot y \cdot \alpha \cdot \frac{1}{2\zeta} = \frac{\alpha G}{\widetilde{m}} \cdot \frac{1}{2\zeta}$$

- ω_j Structural frequency that is in resonance with the forcing function
- y Static deflection of the bridge at mid-span for the weight G of the person standing at the point of maximum amplitude of the mode (typical assumption G = 700 N)
- α Fourier coefficient of the relevant harmonic of the person walking or running. Relevant harmonic → the harmonic that causes resonance of the structure

This equation overestimate the maximum acceleration response because

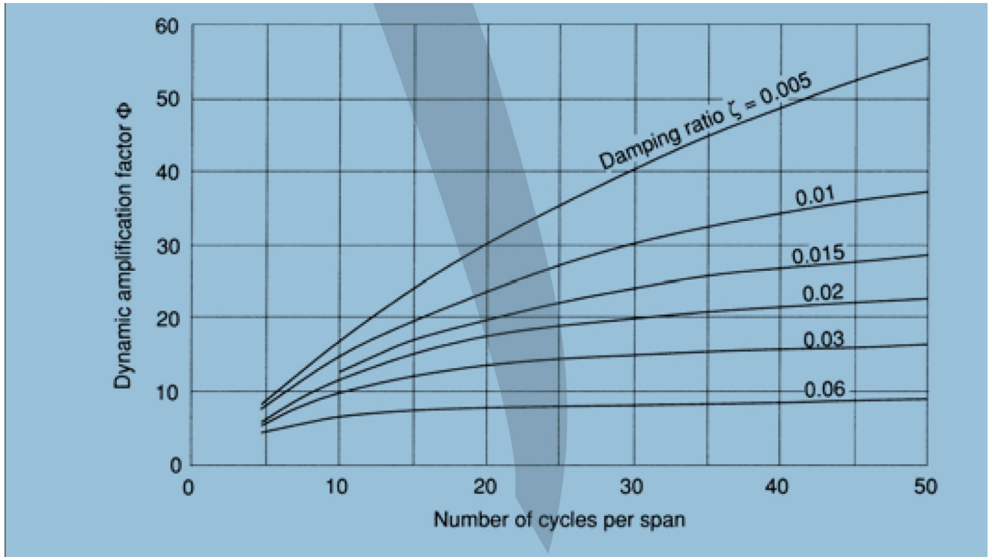
- The effectiveness of the pedestrian is reduced when the pedestrian is not at the location where the modal displacement is maximum
- The number of steps to cross the span is limited, steady state respose could be not reached

Maximum acceleration response (single person crossing the bridge)

Improved estimate:

Multiply the response with a dynamic amplification factor that accounts for the damping, the limited number of cycles due to a finite span and the fact that the force not always acts at the position where it is most effective:

$$a_{max} = \omega_j^2 \cdot y \cdot \alpha \cdot \phi = \frac{\alpha G}{\tilde{m}} \cdot \phi$$



Vibrations induced by people

- Leads rarely to fatigue or structural safety problems
- Can affect the serviceability of structures (comfort of the users, excessive déformations)
- Can cause forced synchronisation (particularly for lateral swaiying). Consequences can be serious (panic, increase of deformations)

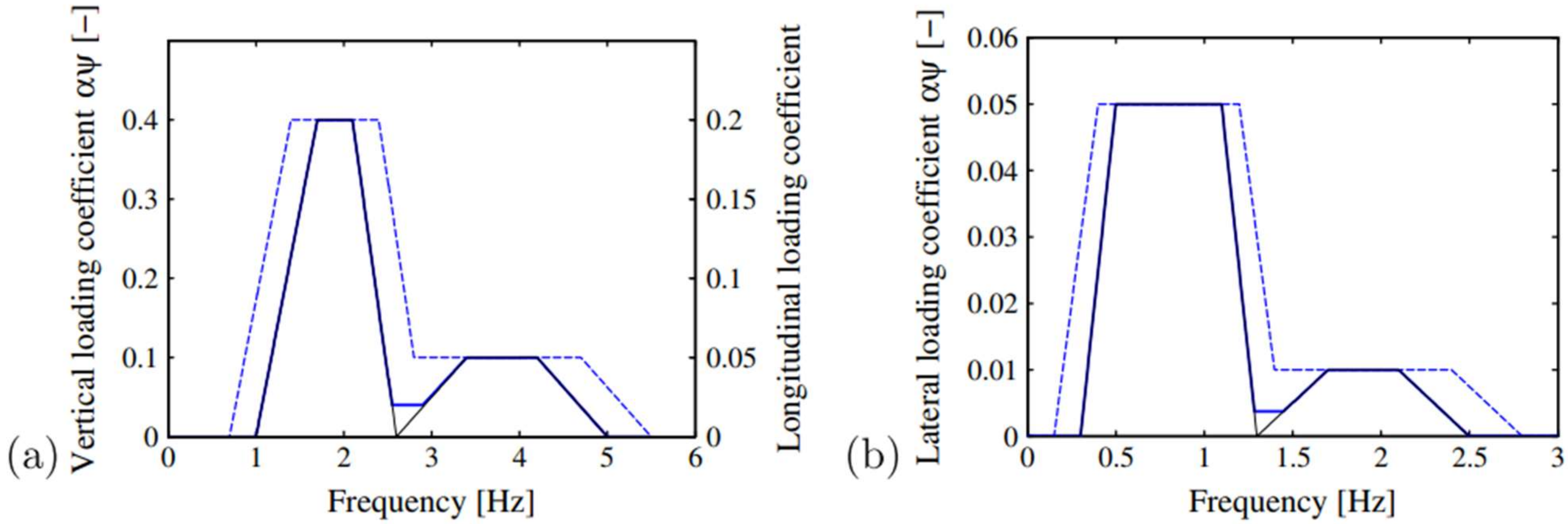
Comfort level of people

- Response of people to vibrations in buildings or of structures
  - Depends on
    - Activity of the person
    - Position of the person (laying down, standing up, driving a car, …)
    - Frequency, magnitude, duration, variability, … of the acceleration
    - Structural appearance, familiarity with vibrations, height above ground, …
    - …
- Effects of vibrations on human occupants can be divided into 5 classes
  - Class a: Influences below human perception threshold
  - Class b: Basic threshold effects
  - Class c: Intrusion, alarm and fear
  - Class d: Interference with activities
  - Class e: Possibility of injury or risk
- The comfort level correlates with peak accelerations in the low frequency range (1-10 Hz) and with the peak velocity in the higher frequency range (10-100 Hz)
- A often used limit acceleration value is 1.0 m/s^2.

@ISO 10137:2007

Dynamic assessment of a footbridge (HIVOSS guidelines)

- Step 1: evaluation of natural frequencies (including mass of the people)
- Step 2: check for critical range of natural frequencies



@Van Nimmen et al. 2014

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Dynamic assessment of a footbridge (HIVOSS guidelines)

- Step 3: definition of a combination of design situation (traffic, frequency of the scenario) and a level of confort

Comfort class	Degree of comfort	Vertical a_{limit}	Lateral a_{limit}
CL 1	Maximum	< 0,50 m/s²	< 0,10 m/s²
CL 2	Medium	0,50 – 1,00 m/s²	0,10 – 0,30 m/s²
CL 3	Minimum	1,00 – 2,50 m/s²	0,30 – 0,80 m/s²
CL 4	Unacceptable discomfort	> 2,50 m/s²	> 0,80 m/s²

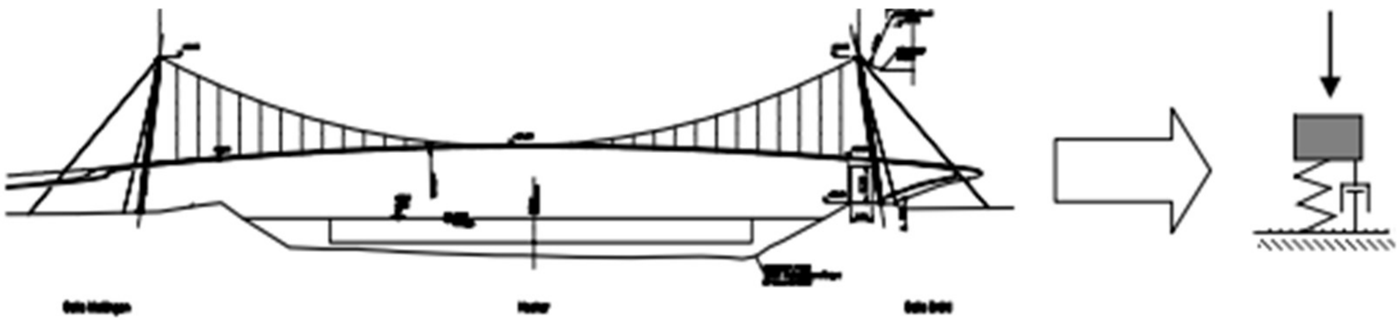
d = 0,2 P/m²	Weak traffic 	Comfortable and free walking Overtaking is possible Single pedestrians can freely choose pace
d = 0,5 P/m²	Dense traffic 	Still unrestricted walking Overtaking can intermittently be inhibited
d = 1,0 P/m²	Very dense traffic 	Freedom of movement is restricted Obstructed walking Overtaking is no longer possible

@HIVOSS

Dynamic assessment of a footbridge (HIVOSS guidelines)

- Step 4: assessment of structural damping
- Step 5: determination of the maximum acceleration response <u>for every critical mode independently</u>

Construction type	Minimum ξ	Average ξ
Prestressed concrete	0,50%	1,0%
Composite steel-concrete	0,30%	0,60%
Steel	0,20%	0,40%
Timber	1,0%	1,5%
Stress-ribbon	0,70%	1,0%



@HIVOSS

Solutions for reducing footfall-induced vibrations in footbridges

- Stiffening – often very expensive and intrusive with regard to the appearance of the footbridge
- Increased damping – e.g. through a thick layer of high viscosity asphalt but this is often not very practical and the outcome uncertain
- Tuned Mass Damper (TMD) – can be very effective if the structural damping is low and the mass is limited; anchorage points for the TMD can be foreseen in the construction stage of the footbridge and the TMD installed later, if needed.

@Bachmann et al. 1997

TMD

https://www.youtube.com/watch?v=lhNjfNUOUo8&list=PPSV

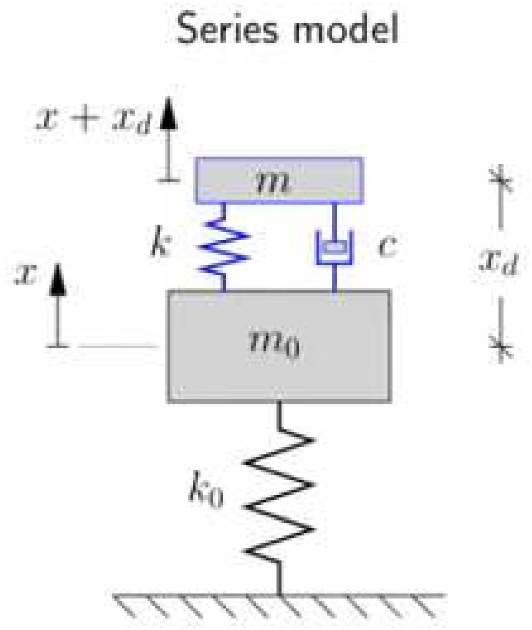
TMDs in the Millenium Bridge

Vertical TMD

Horizontal TMD

https://www.gerb.com/wp-content/uploads/2022/10/GERB_Tuned-Mass-Dampers_EN-2022-09.pdf

TMD - Concept



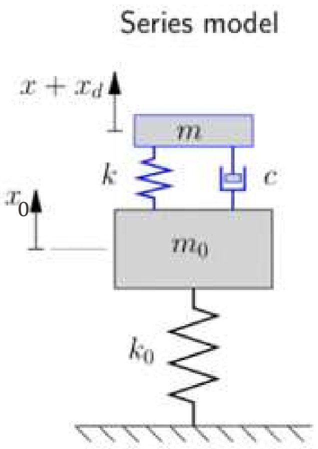
@ Krenk S, Hogsberg J (2020)

- Lightly damped structures develop large amplitude vibrations for loads with a frequency near structural resonance.
- Remedy: Add a SDOF to the structure that vibrates along the axis of the structure in which the vibration amplitudes should be reduced.
- The TMD is tuned to reduce the vibrations of the steady state response.
- The TMD is a *passive* vibration control system.

Design approach

- Choose the mass such that the effect on the structure is sufficiently large.
- Tune the frequency of the TMD to the structural mode that causes the vibrations of the structure
- Add damping to the TMD to remove energy from the system and increase the robustness of the tuning.
- Note: The damping must not be too large

TMD – Assumptions and notations



@ Krenk S, Hogsberg J (2020)

Structure

- The vibrations that should be reduced are linked to the stuctural mode j
- m_0, k_0 and ω_0 are the modal mass, modal stiffness and frequency of the jth mode of the structure
- The damping coefficient of the structure is assumed to be zero – the efficiency of the system is reduced for structures with higher damping

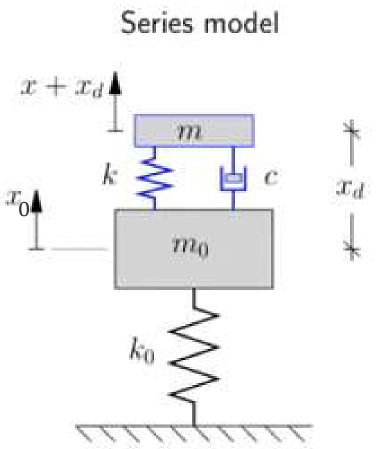
TMD

- The TMD is a SDOF with mass m, stiffness k, damping c and frequency ω_d

Forcing function

- The forcing function is v

TMD – Solution of the steady-state response



$$\frac{x_0}{f} = \frac{(-m\omega^2 + i\omega c + k)}{[-\omega^2(m_0 + m) + k_0][-m\omega^2 + i\omega c + k] - (m\omega^2)^2}$$

$$\frac{x_d}{f} = \frac{m\omega^2}{[-\omega^2(m_0 + m) + k_0][-m\omega^2 + i\omega c + k] - (m\omega^2)^2}$$

Rewrite using the following normalised terms:

- Mass ratio $\mu = m/m_0$
- Damping ratio of TMD $\zeta_d = c/2\sqrt{km}$
- Frequency of structure $\omega_0^2 = k_0/m_0$
- Frequency of TMD $\omega^2 = k/m$

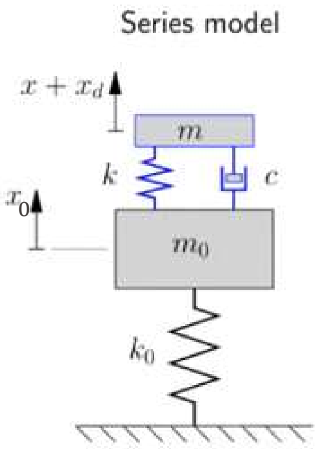
DAF

$$\frac{x_0}{f/k_0} = \frac{\omega_0^2[\omega_d^2 - \omega^2 + 2i\zeta_d\omega_d\omega]}{\omega^4 - [\omega_0^2 + (1+\mu)\omega_d^2]\omega^2 + \omega_0^2\omega_d^2 + 2i\zeta_d\omega_d\omega[\omega_0^2 - (1+\mu)\omega^2]}$$

$$\frac{x_d}{f/k_0} = \frac{\omega_0^2\omega^2}{\omega^4 - [\omega_0^2 + (1+\mu)\omega_d^2]\omega^2 + \omega_0^2\omega_d^2 + 2i\zeta_d\omega_d\omega[\omega_0^2 - (1+\mu)\omega^2]}$$

@ Krenk S, Hogsberg J (2020)

TMD – Solution of the steady-state response



$$\frac{x_0}{f/k_0} = \frac{\omega_0^2[\omega_d^2 - \omega^2] + 2i\zeta_d\omega_d\omega\omega_0^2}{\omega^4 - [\omega_0^2 + (1+\mu)\omega_d^2]\omega^2 + \omega_0^2\omega_d^2 + 2i\zeta_d\omega_d\omega[\omega_0^2 - (1+\mu)\omega^2]} = \frac{A + 2i\zeta_d B}{C + 2i\zeta_d D}$$

$$\frac{x_d}{f/k_0} = \frac{\omega_0^2\omega^2}{\omega^4 - [\omega_0^2 + (1+\mu)\omega_d^2]\omega^2 + \omega_0^2\omega_d^2 + 2i\zeta_d\omega_d\omega[\omega_0^2 - (1+\mu)\omega^2]} = \frac{\omega_0^2\omega^2}{C + 2i\zeta_d D}$$

We are interested in $|x_0|$ and $|x_d|$. To obtain the magnitude of these complex ratios, we recall:

$$\frac{a+ib}{c+id} = \frac{\rho_1 e^{i\phi_1}}{\rho_2 e^{i\phi_2}} = \frac{\rho_1}{\rho_2} e^{i(\phi_1 - \phi_2)}$$

With $\rho_1 = \sqrt{a^2 + b^2}$ and $\rho_2 = \sqrt{c^2 + d^2}$

Hence:

$$\frac{|x_0|}{f/k_0} = \frac{\sqrt{A^2 + (2\zeta_d)^2 B^2}}{\sqrt{C^2 + (2\zeta_d)^2 D^2}}$$

$$\frac{|x_d|}{f/k_0} = \frac{\omega_0^2\omega^2}{\sqrt{C^2 + (2\zeta_d)^2 D^2}}$$

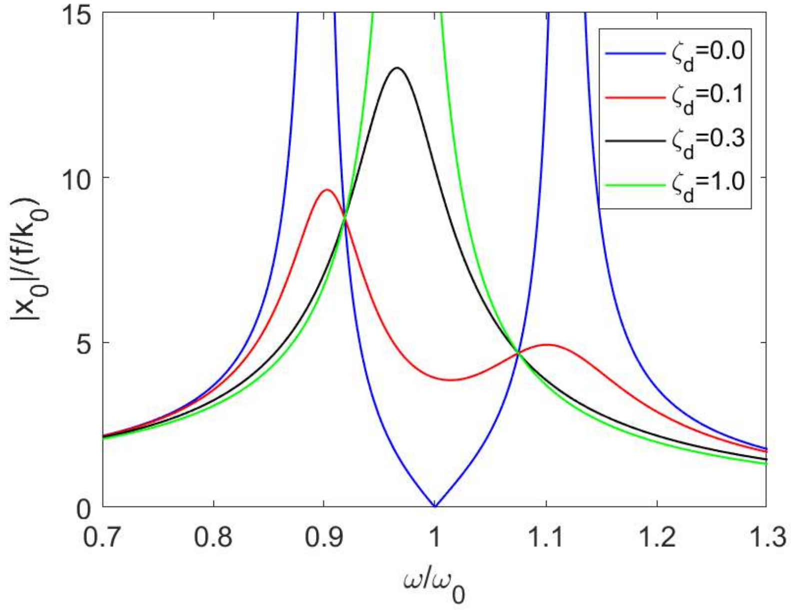
@ Krenk S, Hogsberg J (2020)

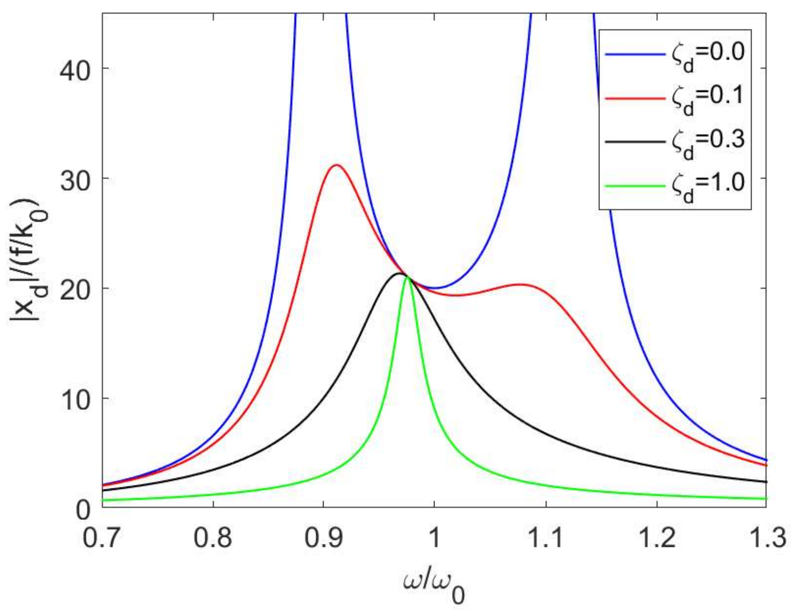
Dr. Francesco Vanin

TMD – let's plot the solution

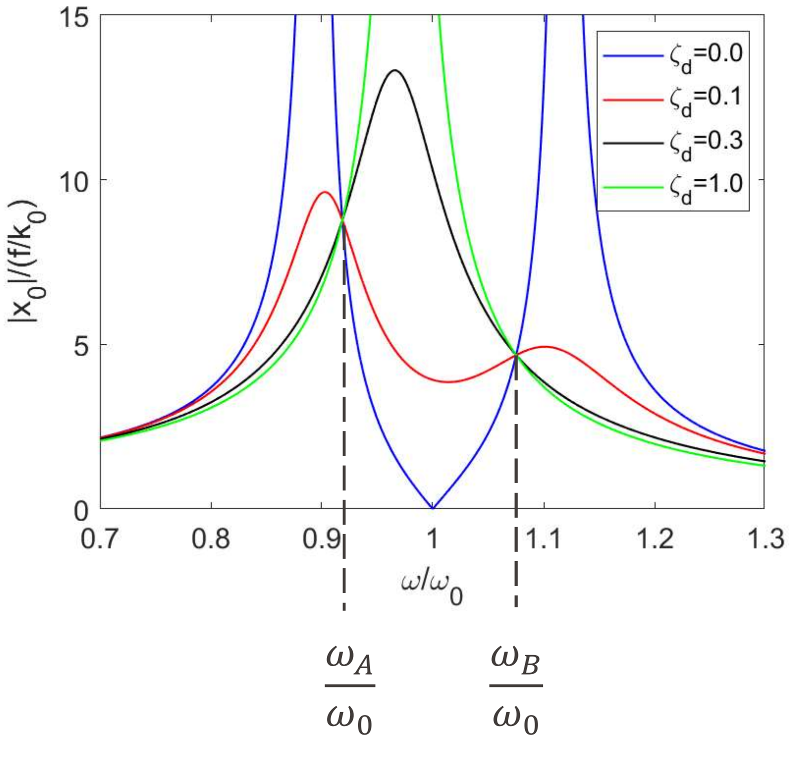
Plot the maximum absolute displacement of the structure and of the damper for a chosen mass ratio, a chosen TMD frequency and various damping ratios of the TMD

Here: μ=0.05, $\frac{\omega_d}{\omega_0} = 1.00$





TMD – Neutral points



Neutral points ω_A and ω_B

frequencies for which the response is independent of the damping ratio ζ_d

At the neutral points ω_A and ω_B (for all values of ζ_{d1} and ζ_{d2}):

$$\frac{A^2 + (2\zeta_{d1})^2 B^2}{C^2 + (2\zeta_{d1})^2 D^2} = \frac{A^2 + (2\zeta_{d2})^2 B^2}{C^2 + (2\zeta_{d2})^2 D^2}$$

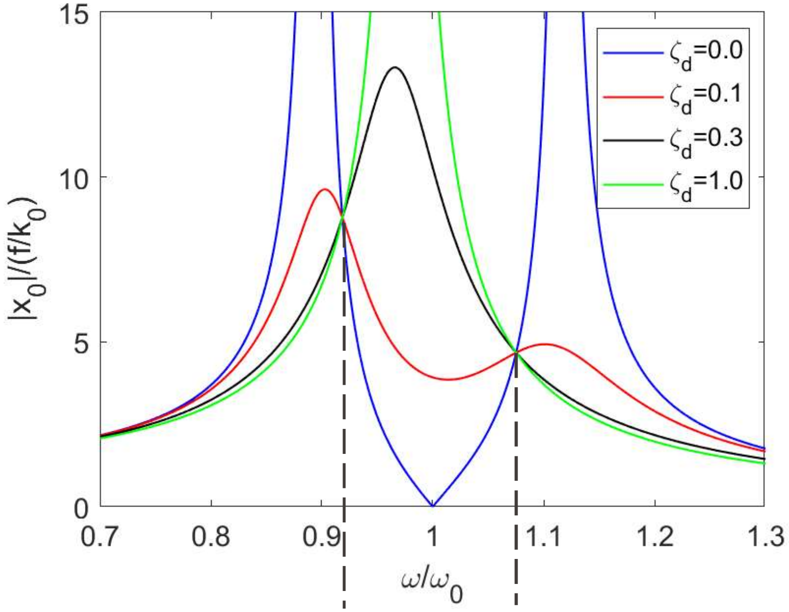
This holds if $A^2 = \frac{B^2 C^2}{D^2}$ or $AD = \pm BC$

ω_A^2 and ω_B^2 are the roots of the following quadratic equation in ω^2

$$(2+\mu)\left(\frac{\omega}{\omega_0}\right)^2\left(\frac{\omega}{\omega_d}\right)^2 - 2\left[\left(\frac{\omega}{\omega_d}\right)^2 + (1+\mu)\left(\frac{\omega}{\omega_0}\right)^2\right] + 2 = 0$$

TMD – Choice of stiffness of TMD

$\omega_d = \omega_0$

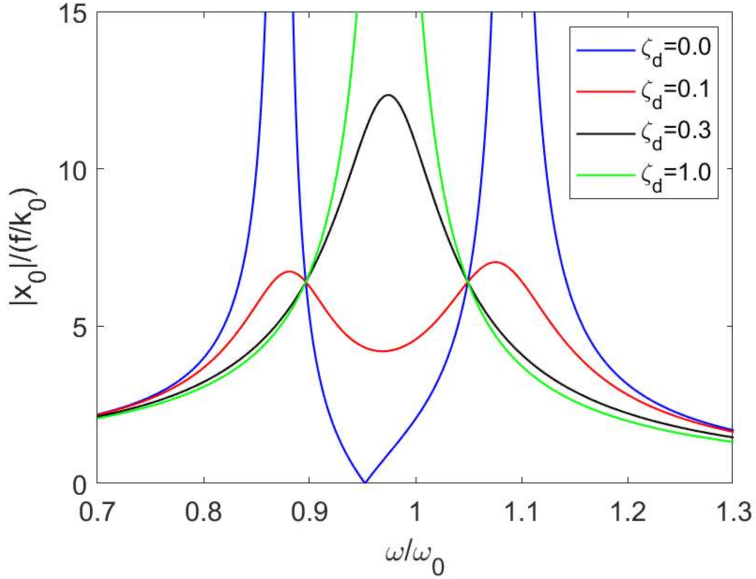


$\frac{\omega_A}{\omega_0}$ $\frac{\omega_B}{\omega_0}$

Design approach for stiffness of TMD:

Choose ω_d such that $|x_0|$ at the neutral points is equal

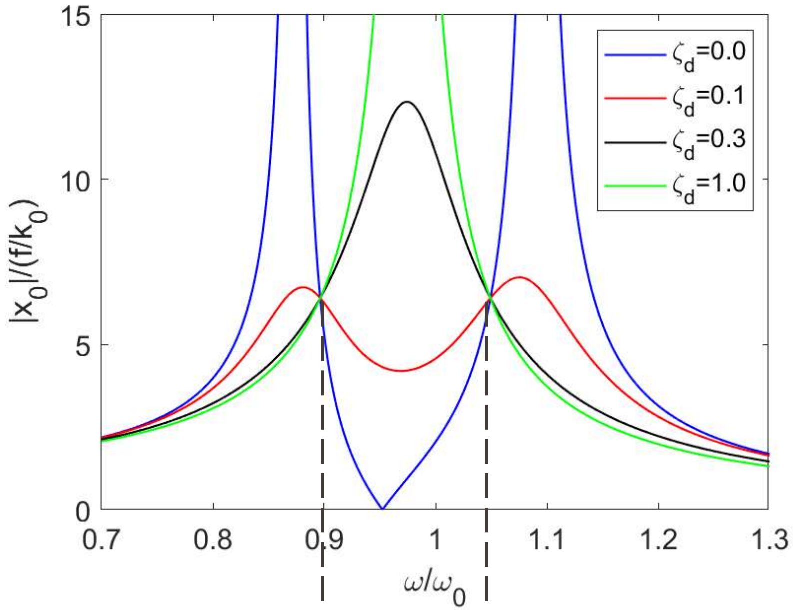
One obtains the simple equation: $\omega_d = \frac{\omega_0}{1+\mu}$



$$\omega_d = \frac{\omega_0}{1+\mu} = 0.95\omega_0$$

TMD – Choice of mass of TMD

$$\omega_d = \frac{\omega_0}{1+\mu} = 0.95\omega_0$$



$$\frac{\omega_A}{\omega_0} \quad \frac{\omega_B}{\omega_0}$$

Design approach for mass of TMD:

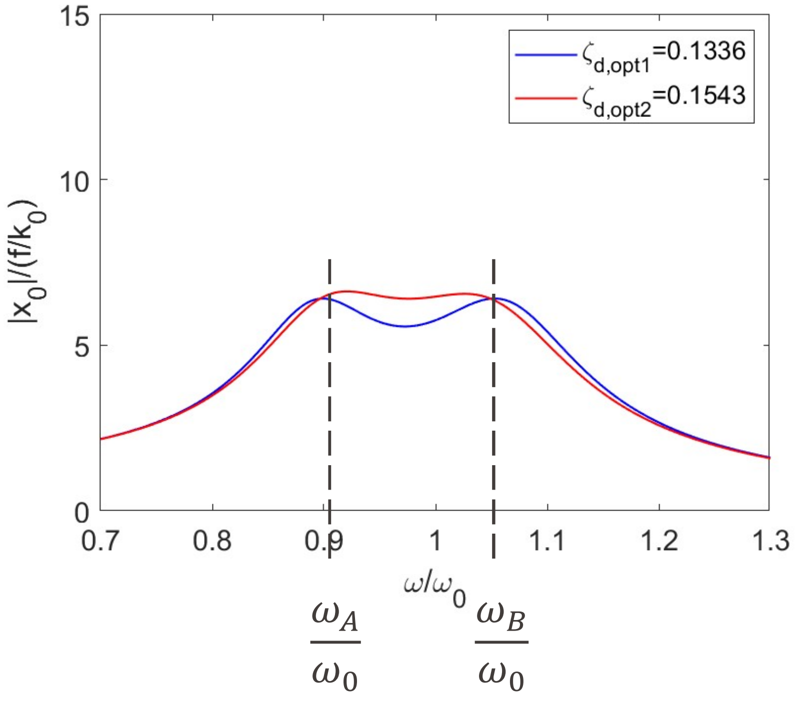
The dynamic aplification factor (DAF) at these neutral points is $\frac{|x_0|}{f/k_0} = \sqrt{\frac{2+\mu}{\mu}} \cong \sqrt{\frac{2}{\mu}}$

Choose the mass such that the DAF is sufficiently small:

μ	**DAF**
0.01	14.2
0.05	6.4
0.10	4.6

TMD – Choice of damping ratio of TMD

$$\omega_d = \frac{\omega_0}{1+\mu} = 0.95\omega_0$$



Design approach for damping ratio of TMD:

Choose ζ_d such that the response regime between the neutral points is approximately flat.

Robust response if frequency of forcing function various (here: walking frequency)

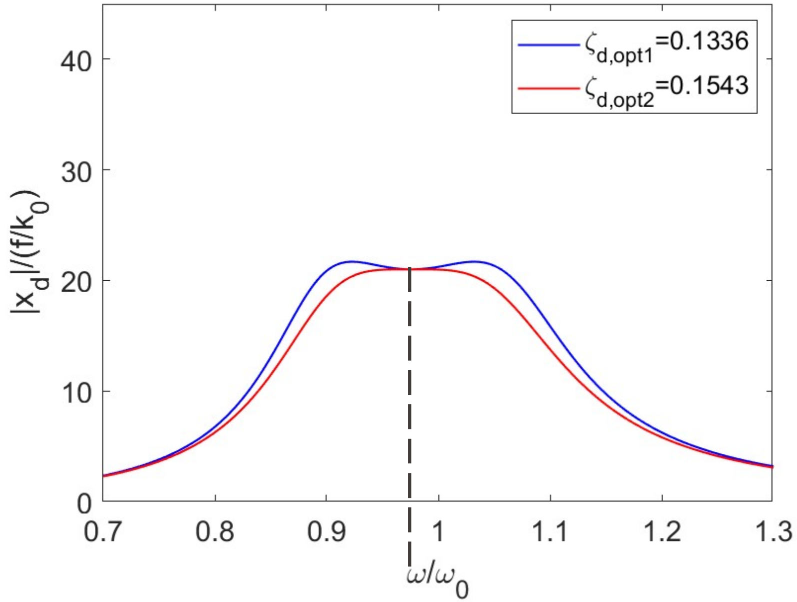
Various proposals for optimum damping ratios have been made:

Den Hartog (1956) $\zeta_{d,opt1} = \sqrt{\frac{3}{8} \cdot \frac{\mu}{\mu+1}}$

Krenk and Hogsberg (2020) $\zeta_{d,opt2} = \sqrt{\frac{1}{2} \cdot \frac{\mu}{\mu+1}}$

TMD – Amplitude of TMD movement

$$\omega_d = \frac{\omega_0}{1+\mu} = 0.95\omega_0$$



$$\frac{\omega_\infty}{\omega_0}$$

The amplitude of the TMD movement is important when designing the TMD fastening to the structure:

At the locked damping frequency $\omega_\infty = \frac{\omega_0}{\sqrt{1+\mu}}$:

$$\frac{|x_d|}{f/k_0} = \frac{1+\mu}{\mu}$$

TMD design in a nutshell

- Choose an acceptable DAF
- Compute the required mass ratio of the TMD: $\mathrm{DAF} = \frac{|x_0|}{f/k_0} \cong \sqrt{\frac{2}{\mu}}$ → $\mu = \frac{2}{DAF^2}$
- Compute the optimum frequency of the TMD: $\omega_d = \frac{\omega_0}{1+\mu}$
- Compute the corresponding stiffness of the TMD: $k_d = m_d \omega_d^2$
- Compute the optimum damping of the TMD: for ex.: $\zeta_{d,opt1} = \sqrt{\frac{3}{8} \cdot \frac{\mu}{\mu+1}}$
- Compute the maximum displacement of the TMD (if $\zeta_d \cong \zeta_{d,opt}$): $\frac{|x_d|}{f/k_0} \cong \frac{1+\mu}{\mu}$

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