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Exercise #3: Response to pulses and approximate analysis for short pulses

Problem 1

The 24-m-high full water tank shown in the figure is subjected to the force $p(t)$ shown in the figure caused by an aboveground explosion. The weight of the water when the tank is full is 160kN. The damping coefficient was measured from a free vibration test and was found to be, $c = 0.0063kN - sec/mm$ and its lateral stiffness $k = 0.5\,kN/mm$. Determine the following:

1. The natural vibration period and damping ratio of the structure with the tank full;
2. The maximum base shear and bending moment at the base of the tower supporting the tank.

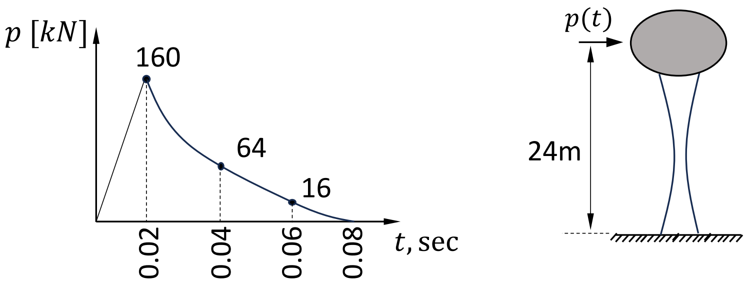


Figure 3.1

Problem 1 - Solution

1. For this water tank, when it is assumed to be full, the corresponding mass is as follows:

$$m = \frac{W}{g} = \frac{160kN}{9810\,mm/s^2} = 0.016kN - \sec^2/mm$$

Therefore, the period of vibration can be estimated as follows:

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.016kN - \sec^2/mm}{0.5\;kN/mm}} = 1.13sec$$

Regarding the damping ratio, it is known that:

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{0.0063kN - sec/mm}{2\sqrt{\frac{0.5kN}{mm} \cdot 0.016kN - \sec^2/mm}} = 0.035 = 3.5\%$$

2. We should first check what is the ratio, t_d/T_n

$\frac{t_d}{T_n} = \frac{0.08}{1.13} = 0.07$

Because $t_d/T_n < 0.25$, the forcing function may be treated as a pure impulse of magnitude,

$$I = \int_0^{0.08} p(t)\,dt$$

NOTE: essentially if we integrate the acting force diagram versus time, we can estimate the amplitude of the equivalent impulse force.

The above integration cannot be done analytically as the function p(t) is given at discrete points. As such, we can apply the trapezoidal rule (or other) to do the integration. Here, the integration rule is applied:

$$I = \int_0^{0.08} p(t)\,dt = \frac{0.02}{2}(0 + 2(160) + 2(64) + 2(16) + 0) = 4.80kN - sec$$

As the response has been transformed into a pure impulse, the displacement history of an SDF system under an impulse load, I causes free vibration of the SDF system due to the initial velocity, $\dot{u}(\tau) = I/m$ and initial displacement $u(\tau) = 0$ (see Slide 21); hence, the corresponding displacement after neglecting damping is as follows:

$$u(t) = \frac{I}{m\omega_n}\sin\left[\omega_n(t-\tau)\right] \text{ for } t \geq \tau$$

In slide 21 of the lecture notes, we looked at the case of a unit impulse (its amplitude is equal to 1). However, in our case the amplitude is equal to I.

In our case, $\tau = 0$, therefore,

$$u(t) = \frac{I}{m\omega_n}\sin(\omega_n t) \text{ for } t \geq 0$$

The above function maximizes when,

$$\frac{d(u(t))}{dt} = 0 \Longrightarrow \cos(\omega_n t) = 0 \Longrightarrow \omega_n t = \frac{\pi}{2} \Longrightarrow t = \frac{\pi}{2\omega_n}$$

Substituting to the equation above, the maximum displacement may be computed as follows,

$$u_o = \frac{I}{m\omega_n}\sin\left(\omega_n \frac{\pi}{2\omega_n}\right) = \frac{I}{m\omega_n}\cdot 1 = \frac{I}{m\omega_n} = \frac{I}{(k/\omega_n^2)\omega_n} = \frac{I}{(k/\omega_n)} = \frac{I}{k}\frac{2\pi}{T_n}$$

NOTE: $\omega_n = \sqrt{k/m}$

Neglecting the effect of damping, the maximum displacement in this case may be estimated as follows:

$$u_o = \frac{I}{k}\frac{2\pi}{T_n} = \frac{4.80}{0.5}\frac{2\pi}{1.13} = 53mm$$

To be able to compute the associated shear force and moment, the equivalent static force, f_{So} acting on the system should be estimated. This may be done as follows,

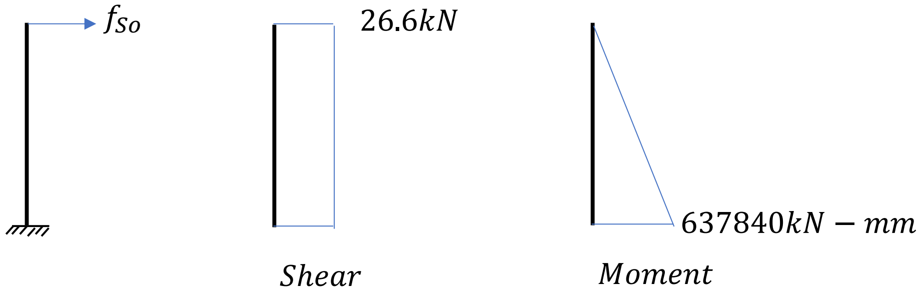
$$f_{So} = ku_o = \frac{0.5kN}{mm}\cdot 53mm = 26.6kN$$

As such, the base shear is simply calculated by equilibrium as shown in the figures below and,

$$V_b = f_{So} = 26.6kN$$

The moment at the base,

$$M_b = V_b h = 26.6kN \cdot 24000mm = 637840kN - mm$$



Problem 2

A one-storey building, idealized as a $h = 3.6$-m-high frame with two columns hinged at the base and a rigid beam, has a natural period of $0.5\ sec$. Each column has a rectangular cross section of $270\ mm\ x\ 100\ mm$. The Young's modulus of the material is $30GPa$. Neglecting damping, determine the maximum response of this frame due to a rectangular pulse force of amplitude $16kN$ and duration $t_d\ =\ 0.2sec$. Assume that the system is elastic. The response quantities of interest to be calculated are:

1. the displacement at the top of the frame; and
2. the maximum bending stress in the columns.

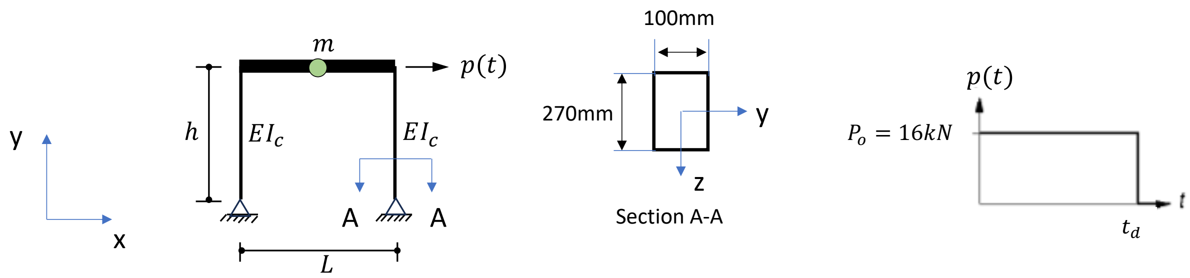


Figure 3.2

Problem 2 – Solution

The natural period of vibration is given ($T_n = 0.5$ sec). As such, we can compute the dynamic amplification factor, R_d. With that we can conduct simplified analysis of short pulses.

$$R_d = \frac{u_o}{(u_{st})_o} = 2\sin\left(\frac{\pi t_d}{T_n}\right) = 2\sin(0.4\pi) = 1.902$$

The corresponding lateral stiffness of the SDF system may be computed as follows:

$$K_{SDF} = 2K_{col} = 2\left(\frac{3EI_c}{h^3}\right) = 6\cdot\frac{(30\,kN/mm^2)\cdot 164025000mm^4}{3600mm^4} = 0.63\,\frac{kN}{mm}$$

NOTE: For the above calculation, the second moment of area of the column cross section should be computed (rectangular):

$$I_c = \frac{100\cdot 270^3}{12} = 164025000mm^4$$

As such, the peak displacement of the steady state response may be estimated as follows:

$$(u_{st})_o = \frac{p_o}{K_{SDF}} = \frac{16kN}{0.63\,kN/mm} = 25.3mm$$

The peak displacement of the SDF response may be estimated as follows:

$$u_o = R_d(u_{st})_o = 1.902\cdot 25.3mm = 48.09mm$$

Finally, the bending moment of the cross section (from Statics 2):

$$M = \frac{3EI_c}{h^2}u_o = \left(\frac{3\cdot 30\,kN/mm^2\cdot 164025000mm^4}{3600^2mm^2}\right)48.09mm = 54777.6kN - mm$$

Alternatively, we can find the bending moment from the equivalent static force,

$$f_{So} = p_oR_d = 16(1.902) = 30.4kN$$

Because both columns are identical, the force $f_{So}$is equally distributed to the those; and the corresponding moment is as follows:

$$M = \frac{f_{So}}{2}h = \frac{30.4kN}{2}3600mm = 54777.6kN - mm$$

The corresponding longitudinal stress due to flexure can then be calculated from principles of structural mechanics as follows:

$$\sigma = \frac{M}{I_c} \cdot \frac{b}{2} = \frac{54777.6kN - mm}{164025000mm^4} \cdot \frac{270mm}{2} = 0.045 \frac{kN}{mm^2} = 45MPa$$