Water Resources Engineering and Management

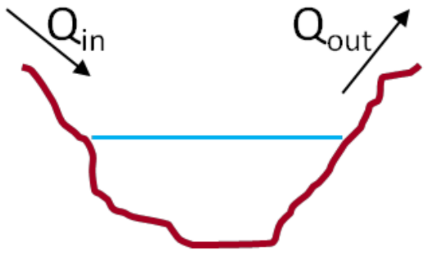
Exercises Lecture 8: Exercises for exam simulation

07/04/2025

Exercise 1: Reservoir dynamics

The table illustrates the daily cycle of input and output discharges of a reservoir.

Time interval	Duration [h]	Input discharge Q_{in} [m^3/h]	Output discharge Q_{out} [m^3/h]
1 (0 -6)	6	10	30
2 (6 -16)	10	20	0
3 (16-24)	8	0	10



The volume rating curve of the reservoir is $V = a\,y^2$, where V [m^3] is the volume stored in the reservoir, y [m] is the water level with respect to the bottom of the reservoir, and a=1 m is a constant.

QUESTIONS

- Compute the mean input discharge.
- Assuming that during the daily cycle the minimum level is equal to 0, compute the level of the water at the beginning of the cycle and the maximum level reached during the day.
- Compute the lake area when the level is equal to 10 meters

Exercise 1: Reservoir dynamics - SOLUTION

- Compute the mean input discharge

$$Q_{avg} = \frac{6 \cdot 10 + 10 \cdot 20}{24} = 10.83\,\mathrm{m}^3/\mathrm{h}$$

- Assuming that during the daily cycle the minimum level is equal to 0, compute the level of the water at the beginning of the cycle and the maximum level reached during the day

Time [h]	Q_{in} [m^3/h]	Q_{out} [m^3/h]	V_{in} [m^3]	V_{out} [m^3]	$V - V_0$ [m^3]
0	-	-	0	0	0
6	10	30	60	180	-120
16	20	0	260	180	+80
24	0	10	260	260	0

$$V_0 = -\min(V - V_0) = 120\,\mathrm{m}^3 \rightarrow y_0 = \sqrt{120} = 10.95\,\mathrm{m}$$

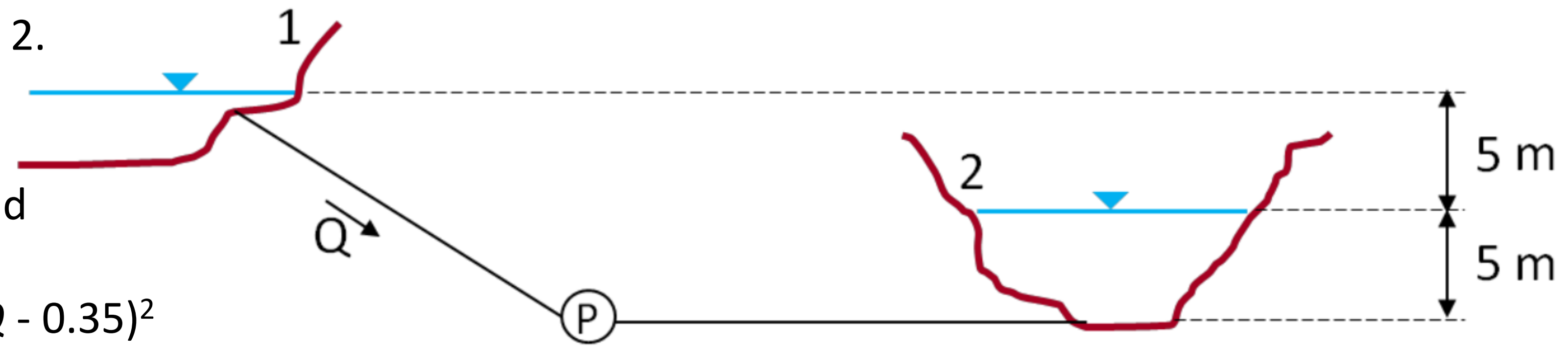
$$V_{max} = \max(V - V_0) - \min(V - V_0) = 200\,\mathrm{m}^3 \rightarrow y_0 = \sqrt{200} = 14.14\ \mathrm{m}$$

- Compute the lake area when the level is equal to 10 meters

$$A = \frac{dV}{dy} = 2ay \rightarrow A(y = 10) = 20\ \mathrm{m}^2$$

Exercise 2: Water pumping

Water at 50°C is pumped from reservoir 1 to 2.
Reservoirs have constant water levels.
The pump head characteristic curve is
$h_p = 30 - 100\,Q^2$, where h_p [m] is the head and
Q [m^3/s] is the discharge.
The pump efficiency curve is $\eta = 0.75 - 20\,(Q - 0.35)^2$
where η [-] is the efficiency.
The pipeline has a total length L = 100 m, and a diameter D = 0.25 m. The pump is located halfway the pipeline.
The flow can be assumed to be fully turbulent with a friction factor λ = 0.025 [-].
Minor head loss at the entrance: ξ_{in} = 0.5.
Minor head loss at the exit: ξ_{out} = 1.



QUESTIONS

- Determine the discharge operated by the pump
- Draw the qualitative behavior of the energy and hydraulic lines
- Determine the daily energy demanded to drive the pump
- Compute the available Net Positive Suction Head (NPSH) at the inlet of the pump

Temperature (°C)	Density (kg/m^3)	Dynamic viscosity (mPa·s)	Heat of vaporization (MJ/kg)	Saturation vapor pressure (kPa)
0	999.8	1.781	2.499	0.611
5	1000.0	1.518	2.487	0.872
10	999.7	1.307	2.476	1.227
15	999.1	1.139	2.464	1.704
20	998.2	1.002	2.452	2.337
25	997.0	0.890	2.440	3.167
30	995.7	0.798	2.428	4.243
40	992.2	0.653	2.405	7.378
50	988.0	0.547	2.381	12.340
60	983.2	0.466	2.356	19.926
70	977.8	0.404	2.332	31.169
80	971.8	0.354	2.307	47.367
90	965.3	0.315	2.282	70.113
100	958.4	0.282	2.256	101.325

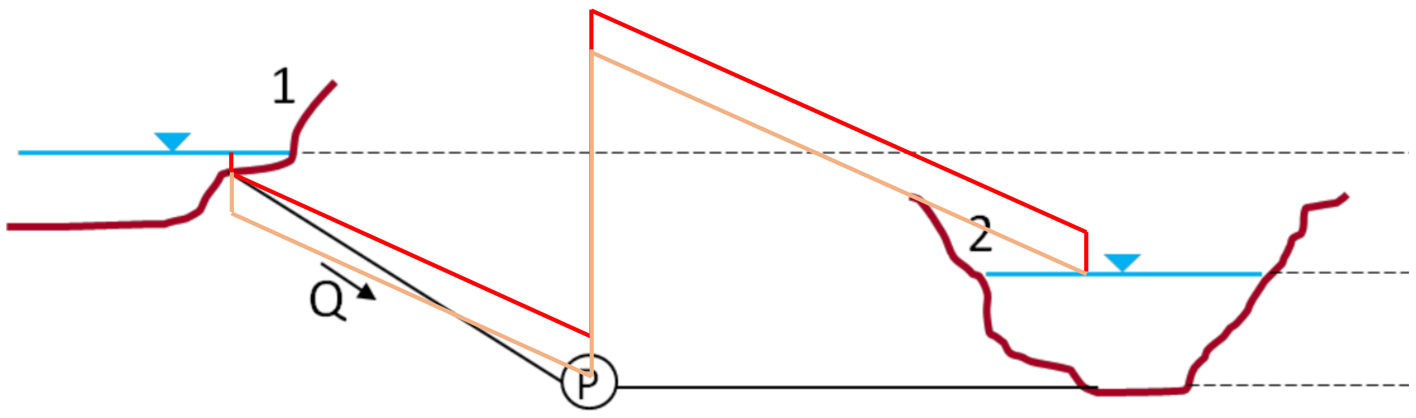
Exercise 2: Water pumping - SOLUTION

- Determine the discharge operated by the pump

$$z_1 - z_2 + h_p = \left(\xi_{\text{in}} + \xi_{\text{out}} + \frac{\lambda}{D}L\right)\frac{8\,Q^2}{g\pi^2 D^4} \quad \longrightarrow \quad 5 + 30 \; - 100\,Q^2 = 243.25\,Q^2$$

$$Q = \sqrt{\frac{35}{343.25}} = 0.319\,\frac{\text{m}^3}{\text{s}}; \qquad h_p = 19.81\ \text{m}; \qquad \eta = 0.731.$$

- Draw the qualitative behaviour of the energy line of the water in the pipe



Exercise 2: Water pumping - SOLUTION

- Determine the daily energy demanded to drive the pump

$$P = \frac{\rho\ g\ h_p\ Q}{\eta} = \frac{988\ \cdot 9.806\ \cdot 19.81\ \cdot 0.319}{0.731} = 83.8\ \text{KW}$$

$$E = P\ \cdot 24\ \text{h} = 2.01\ \text{MWh}$$

- Compute the available Net Positive Suction Head (NPSH) at the inlet of the pump

$$NPSH = \Delta z + \frac{p_{\text{atm}} - p_v}{\rho\ g} - \left(\xi_{\text{in}} + \frac{\lambda}{D}\frac{L}{2}\right)\frac{8\ Q^2}{g\pi^2 D^4}$$

$$NPSH = 10 + \frac{101325 - 12340}{988\ 9.806} - 11.91 = 7.32\ \text{m}$$

Exercise 3: Expected damage

The maximum annual discharge Q [m^3/s] in a floodplain can be expressed as a function of its probability of exceedance $P_e(Q)$ as $Q = 50 - 200 \ln[P_e(Q)]$.

$P_e(Q)$ [-] is the probability that in any year the maximum discharge is larger than Q.

The stage y [m] in the floodplain is related to the discharge Q by the following stage-discharge relationship $y = 0.5\ Q^{0.5}$.

A stage y in the floodplain produces a damage D [MCHF] that can be estimated as $D = 0.002\ y^2$.

To protect the area from floods, the construction of levees is evaluated. The design height of the levees is $y = 10$ m.

Assume that the damage for all stages $y<10$ m is null.

As a first approximation, secondary effects of the levees construction (i.e. the modification of the discharge-frequency and the stage-discharge relationships) can be neglected.

QUESTIONS

- Compute the expected annual damage at the current state
- Compute the expected annual damage after the construction of the levees

Exercise 3: Expected damage - SOLUTION

- Compute the expected annual damage at the current state

$$D = 0.025 - 0.1\ln(P_e)$$

$$\rightarrow E[D] = \int_0^1 D(P_e)dP_e = 0.025 - 0.1\int_0^1 \ln(P_e)dP_e$$

$$= 0.025 - 0.1\Big|P_e\ln(P_e) - P_e\Big|_0^1 = 0.125 \text{ MCHF}.$$

- Compute the expected annual damage after the construction of the levees

$$\bar{y} = 10 \text{ m} \qquad \bar{Q} = \left(\frac{\bar{y}}{0.5}\right)^{\frac{1}{0.5}}$$

$$\overline{P_e} = \exp\left(-\frac{\overline{Q} - 50}{200}\right) = 0.174$$

$$E[D] = \int_0^{\overline{P_e}} D(P_e)dP_e = \Big|0.025P_e - 0.1P_e\ln(P_e) + 0.1P_e\Big|_0^{\overline{P_e}} = 0.052 \text{ MCHF}.$$