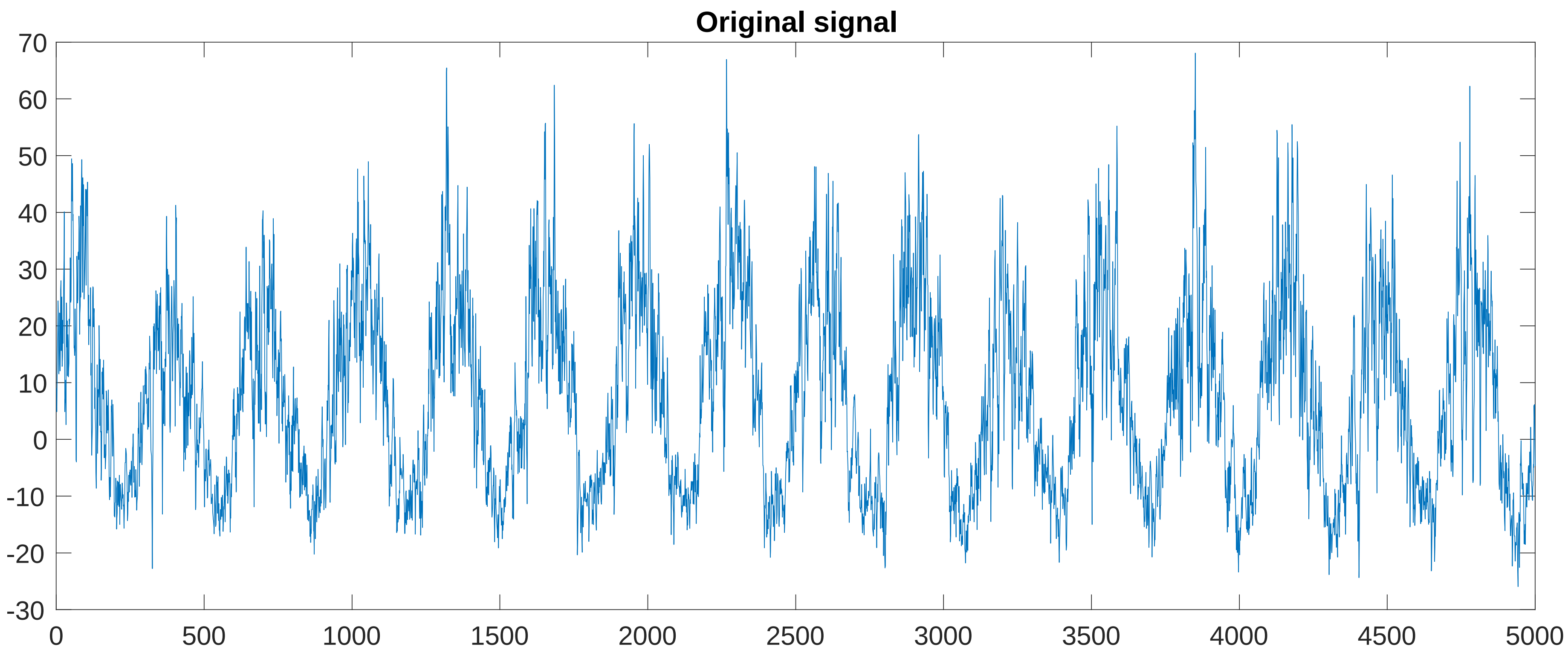
Water Resources Engineering and Management

Exercises Lecture 6: Time series analysis

24/03/2025

Exercise 1: time series diagnostic



Consider the non-stationary time series in the figure above.

a) Describe all features that may characterize the system underlying dynamics;

b) Illustrate the steps that should be done in order to obtain a stationary time series;

c) Explain and write how should one proceed in order to check if the stationary series of the residuals is pure noise or not

Exercise 1: time series diagnostic - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

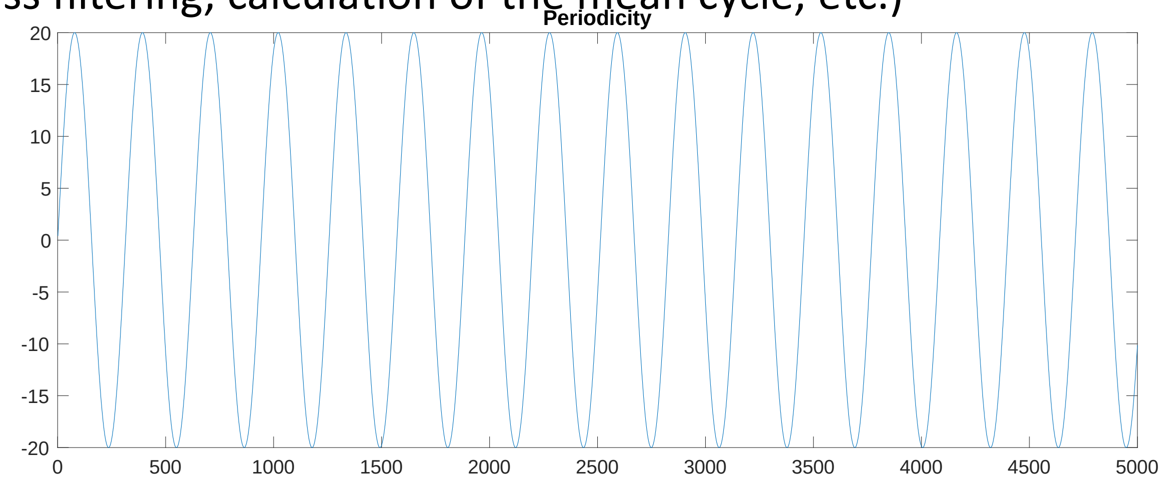
a) The following features seems to characterize the series:

- The time series clearly contains a periodic component which makes it non-stationary at the period time scale and possibly stationary at longer time scale;
- The mean of the time series defines the periodic component, however the std deviation seems also to have an underlying periodic component;
- Nothing can be said with respect to the autocorrelation of the detrended time series, which should be tested after a standardized time series is obtained. Nothing can be said whether the components are combined lineraly or nonlinearly. This might only emerge later if the result of partitioning is not satisfactory

b) In order to standardize the series one should proceed as follows:

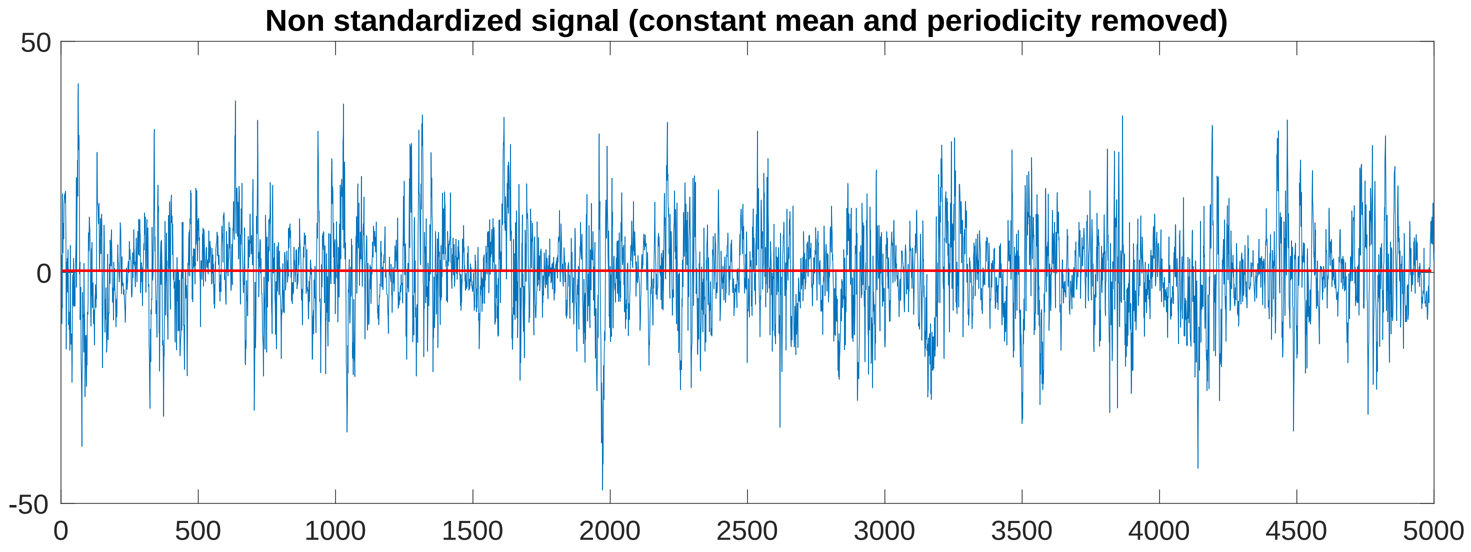
1. First step is to assess the global mean and the periodic trend. This can be done with different methods (e.g. moving average, Fourier analysis and then low-pass filtering, calculation of the mean cycle, etc.)

Supposing that the result is the periodic trend aside, then this can be removed from the series after removal of the constant average

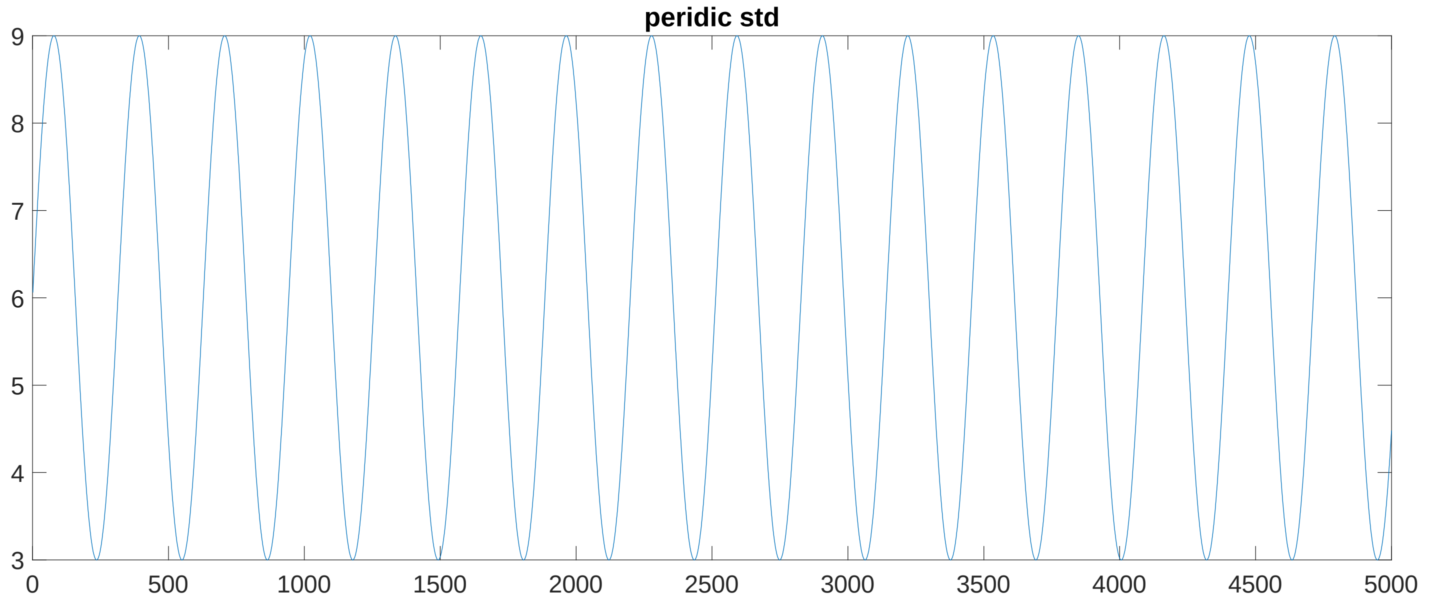


Exercise 1: time series diagnostic - SOLUTION

2. Remove the periodic trend, which shows that a residual periodic structure affects the signal, likely due to the std acting on the signal. This has to be assessed in order to be able to normalize the time series

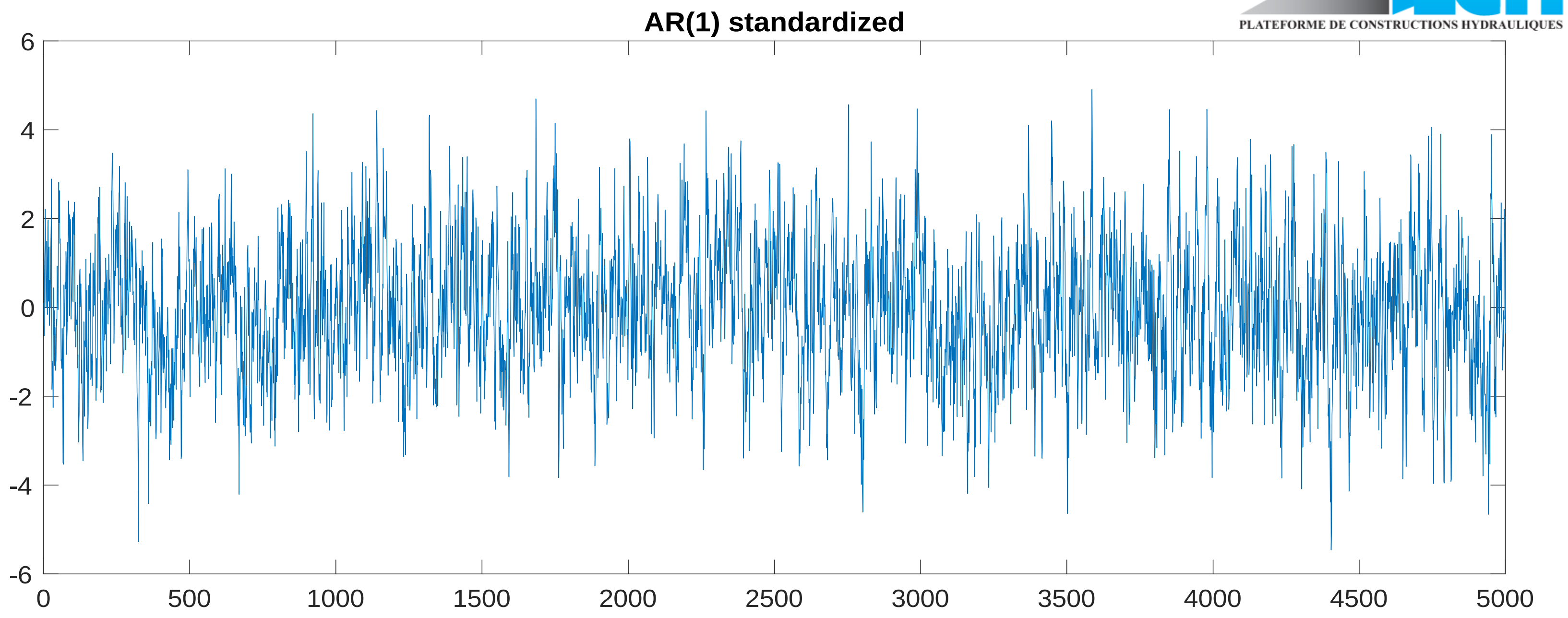


3. The standard deviation shows clearly periodic and with the same period of the trend (as suspected), probably due to intrinsic effects of the forcing (e.g., annual temperature on streamflows)

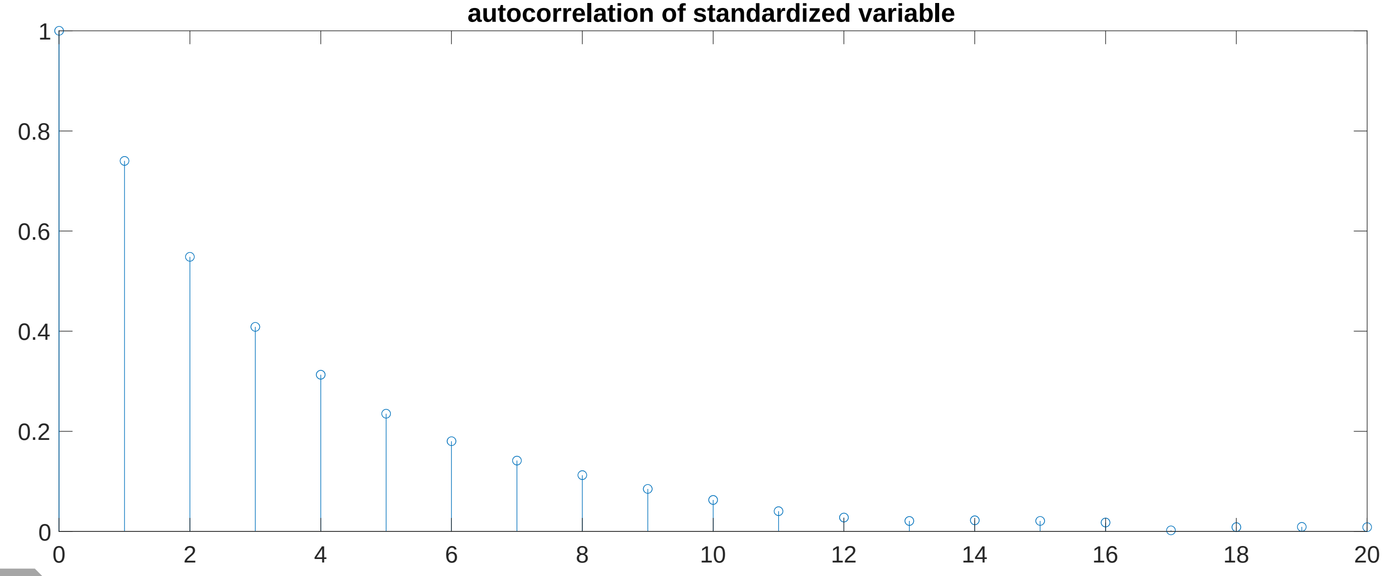


Exercise 1: time series diagnostic - SOLUTION

4. Standardize the series after removing the residual constant mean (already zero) and divide by the periodic std. The standardized series has now and std=1 and is stricktly stationary (none of the moments are time dependent). Still we do not know if the standardized time series is now pure noise or whether it still contains a serial (correlation) structure in it.

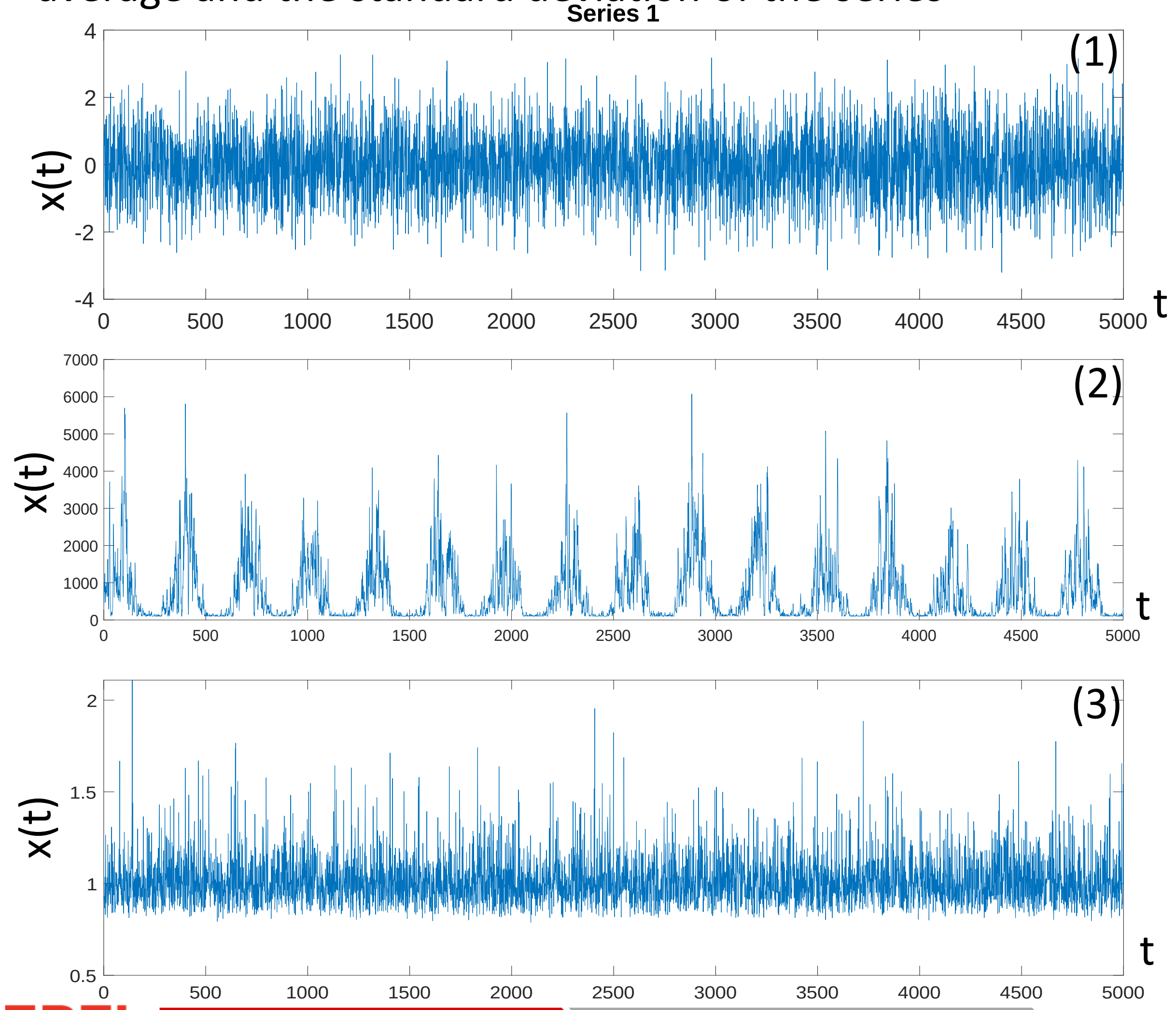


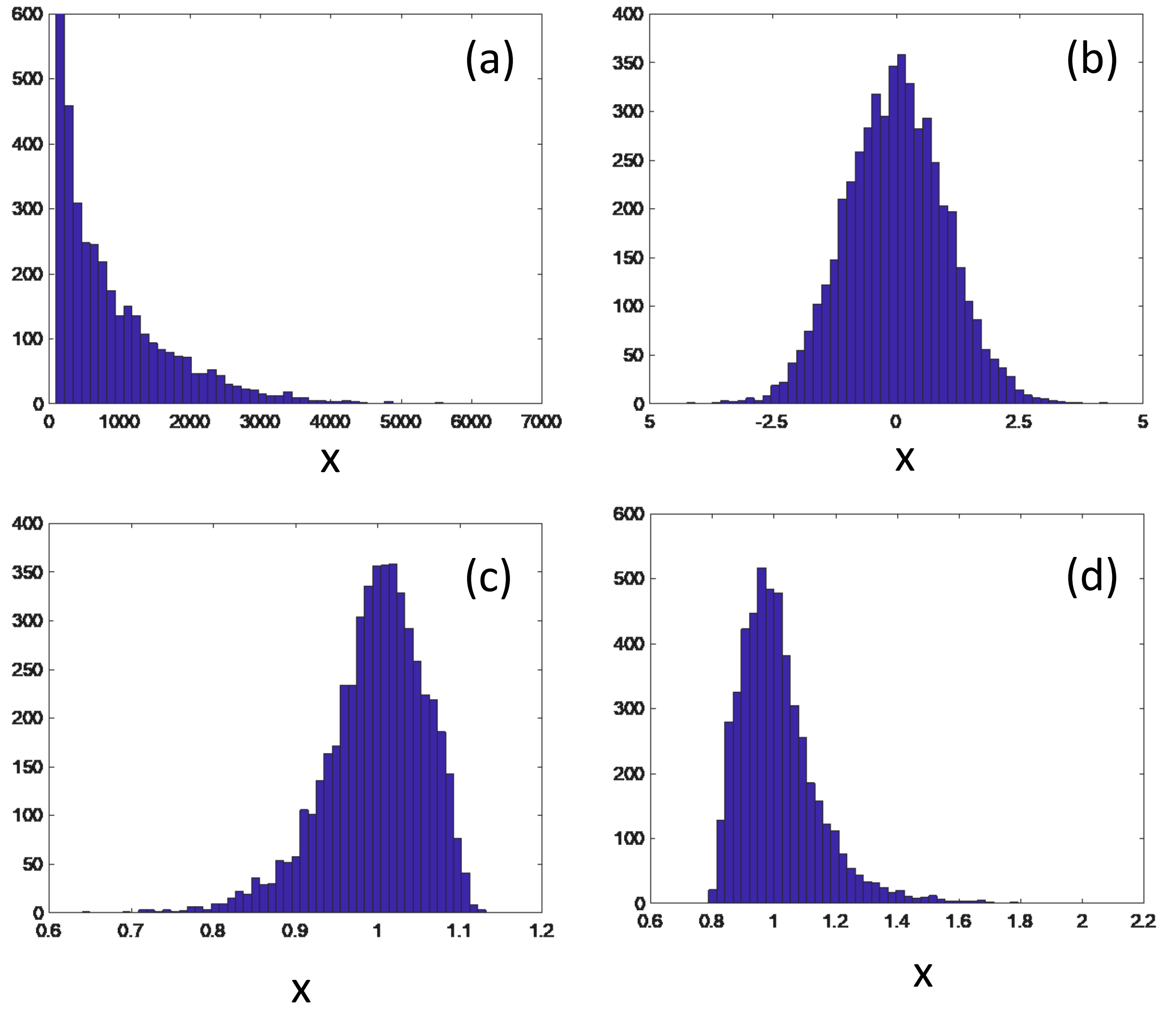
c) This can be done by computing the autocorrelation function, which decreases exponentially. This indicates a serial correlation structure and therefore the standardized time serie does not correspond to a white noise (Dirac-delta correlated). To further decompose it we need a stochastic model (see Lecture 7)



Exercise 2: time series distribution function

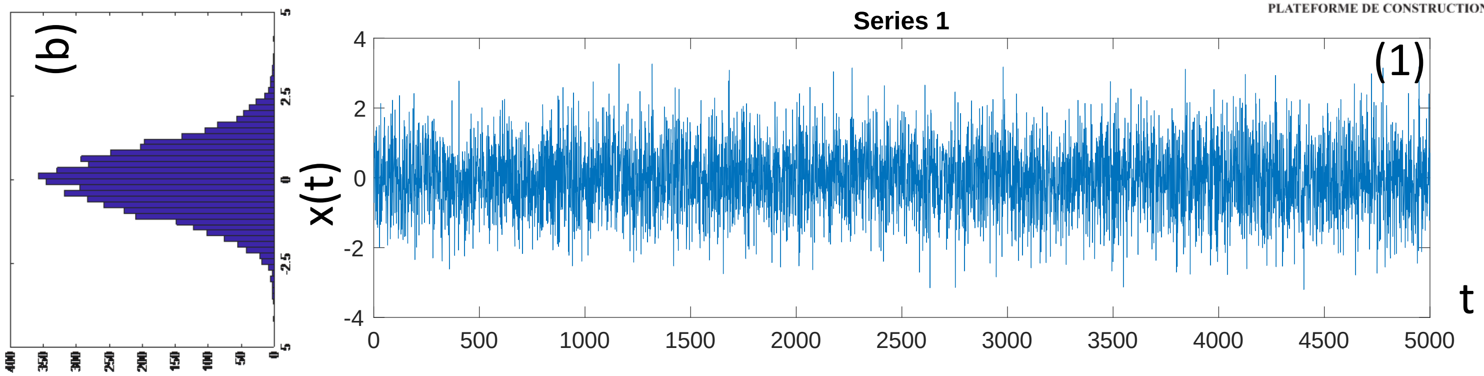
Consider the three time series (1-3) and the histograms (a-d). Recognize i) which time series corresponds to which histogram, and ii) assess whether the skewness coefficient is negative, null or positive; iii) Use the histograms to assess the average and the standard deviation of the series



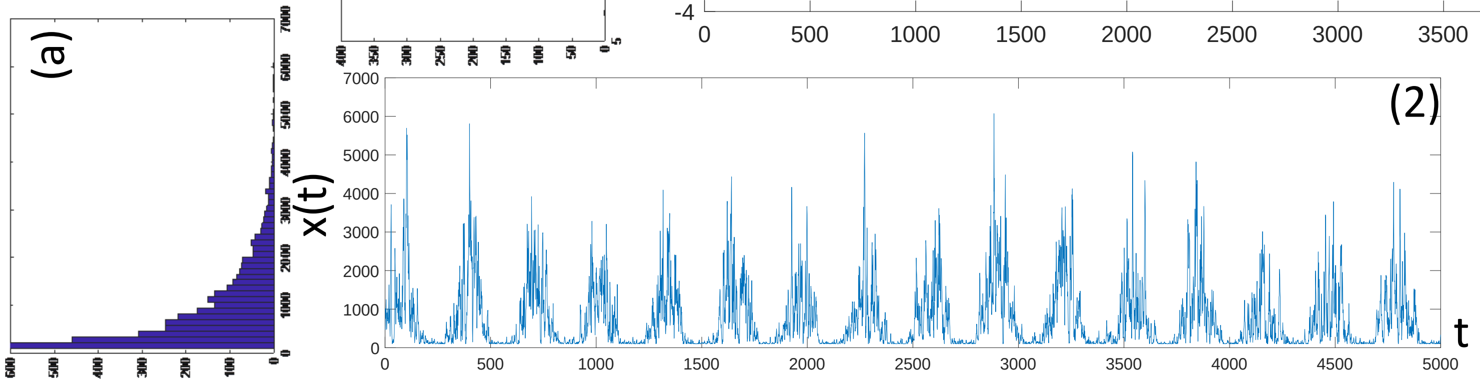


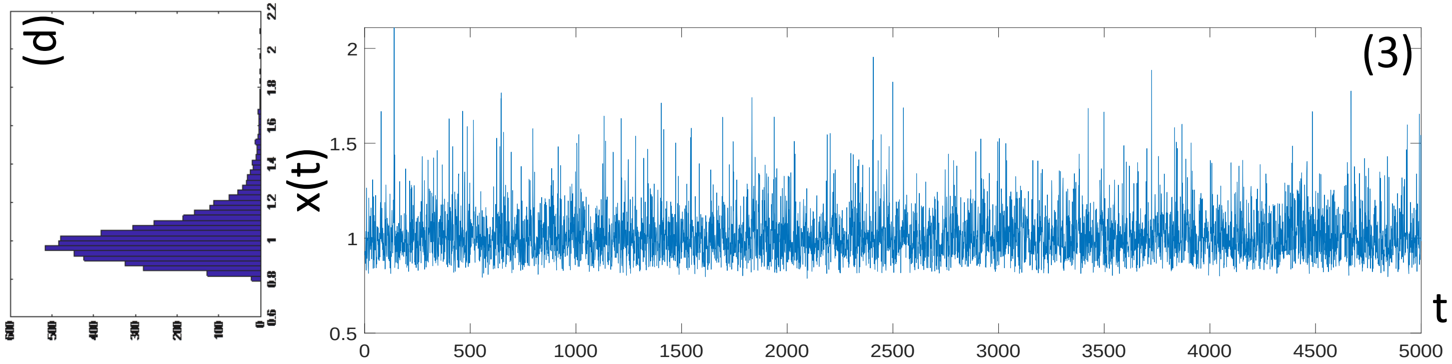
Exercise 2: time series distribution function – SOLUTION

This has skew coefficient zero (symmetric) and zero mean and std≅1



This has skew coefficient > zero (pos. asymmetric), mean ≅ 720 and std≅820





This has skew coefficient > zero (pos. asymmetric), mean ≅ 1 and std≅0.05

Exercise 3. time series analysis

Consider the given time series of daily streamflow Q(t). Use Matlab, Python or Excel or whatever software and try to smooth the series by using both the moving average and the exponential smoothing technique.

- Try different averaging windows and smoothing parameters (α) and draw your conclusions. What effects do you observe on the filtered series?
- How large should be the moving window to approximate the signal with a smoothed line that follows the lower signal trend (e.g., green line)?

Is this at all possible?